

# Index

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- Upper semicontinuous, 557
  
- $V$ -norm, 392, 395
- Vague topology, 560
  
- Weak convergence, 558
- Weak Feller property, 125
- Weak topology, 290, 558
- Weakly, 140

## Symbols

- $A \rightsquigarrow B$ , Uniformly accessibility, 86  
 $A_+(x)$ , States reachable from  $x$  by  $\text{CM}(F)$ , 148  
 $A_+^k(x)$ , States reachable from  $x$  at time  $k$  by  $\text{CM}(F)$ , 147  
 $C_{x_0}^k$ , Generalized controllability matrix, 153  
 $*$ , Convolution operator, 68  
 $1 \otimes \pi$ , Outer-product kernel, 392  
 $\rightsquigarrow^a B$ ,  $B$  is uniformly accessible using  $a$  from  $A$ , 116  
 $A \otimes B$ , 415  
 $A^0$ , Points from which  $A$  is inaccessible, 86  
 $C^\infty$ , Functions whose derivatives of arbitrary order exist, 26  
 $C_V(r) := \{x : V(x) \leq r\}$ , Sublevel set of  $V$ , 188  
 $C_n$ , Controllability matrix, 90  
 $Co$ , Complex plane, 90  
 $F_k$ , Output maps for the linear control model, 27  
 $I_B$ , Indicator kernel of  $B$ , 66  
 $I_g$ , Multiplication kernel using  $g$ , 126  
 $K_{a_\varepsilon}$ , Resolvent kernel, 62  
 $L(x, A) := \mathbb{P}_x(\tau_A < \infty)$ , 64  
 $L(x, h) = U_h(x, h)$ , 296  
 $M_n(g)$ , Martingale derived from  $g$ , 446  
 $N(t)$ , Number of customers in queue at time  $t$ , 40  
 $N(t)$ , Number of customers in queue immediately before  $n$ th arrival, 41  
 $N_n^*$ , Number of customers in queue immediately after  $n$ th service time is completed, 43  
 $O_w$ , Control set, 149, 153  
 $O_w$ , Supports input in  $\text{CM}(F)$  and  $\text{NSS}(F)$  models, 29  
 $P(x, A)$ ,  $n$ -step transition probability, 61  
 $P(x, A)$ , One-step transition probability, 52, 61  
 $P^n(x, A)$ ,  $n$ -step transition probability, 53  
 $P_h(x, A)$ , Kernel for “process on  $h$ ”, 295  
 $Q(x, A)$ , Probability that chain enters  $A$  i.o. from  $x$ , 199  
 $Q(x, h) = \mathbb{P}_x\{\Psi_k \in A_h \text{ i.o.}\}$ , 296  
 $R(x)$ , Conditional emptying time for dam model, 45  
 $R_n$ , Residual service time immediately after customer arrival, 71  
 $S_n(g)$ , Partial sum of  $g(\Phi_k)$ , 421  
 $T(x, A)$ , Continuous component, 124  
 $T_{ab} = \min\{j : Z_a(j) = Z_b(j) = 1\}$ , 321  
 $U_h$ , Resolvent kernel, 293  
 $V^+(n)$ , Forward recurrence time chain, 39  
 $V^+(t) := \inf(Z_n - t : Z_n > t, n \geq 1)$  Forward recurrence time process, 69  
 $V_\delta^+(n) = V^+(n\delta)$ , Forward recurrence time  $\delta$ -skeleton, 69  
 $V^-(t) := \inf(t - Z_n : Z_n \leq t, n \geq 1)$ , Backward recurrence time process, 69  
 $V_\delta^-(n) = V^-(n\delta)$ , Backward recurrence time  $\delta$ -skeleton, 69  
 $V_C$ , Minimal solution to (V2), 267  
 $\Delta = P - I$ , Drift operator, 172  
 $\Gamma$ , Distribution of a disturbance variable, 22  
 $\Lambda_i^*(x, A) = P(i, x; 0, A)$ , Ladder chain transition probability, 70  
 $\Lambda_{i-j+1}(x, A) = P(i, x; j, A)$ , Ladder chain transition probability, 70  
 $\Omega = X^\infty$ , Sequence space, 49  
 $\Omega_+(C)$ , Omega limit set for  $\text{NSS}(F)$ , 154  
 $\Phi_n$ , Markov chain value at time  $n$ , 3  
 $\Sigma_\mu$ ,  $\sigma$ -field of  $\mathbb{P}_\mu$ -invariant events, 423  
 $\|\mu\| := \sup_{A \in \mathcal{B}(X)} \mu(A) - \inf_{A \in \mathcal{B}(X)} \mu(A)$ , Total variation norm, 315  
 $\delta_x(A) = P^0(x, A)$ , Dirac measure, 61  
 $\gamma_g^2$ , Asymptotic variance in the CLT, 422  
 $\lambda^*$ , Split measure on  $bcx$ , 99  
 $\leftrightarrow$ , Communicates with, 78  
 $\bar{p}(M)$ , Upper tail of renewal sequence, 465  
 $\mathbb{I}_B$ , Indicator function of  $B$ , 62  
 $\mathbb{R}$ , Real line, 553  
 $\mathbb{R}^n$ ,  $n$ -dimensional Euclidean space, 6  
 $\mathbb{Z}_+$ , non-negative integers, 3  
 $\mu^{\text{Leb}}$ , Lebesgue measure, 88  
 $\mu^{\text{Leb}}$ , Lebesgue measure on  $\mathbb{R}$ , 553  
 $\rightarrow^w \mu_\infty$ , Weak convergence of  $\mu_k$  to  $\mu_\infty$ , 139  
 $\nabla^\Phi$ , Sensitivity process, 163  
 $\bar{A}$ , Points from which  $A$  is accessible, 86  
 $\bar{A}(m)$ , 86  
 $\pi$ , Invariant measure, 229  
 $\psi$ , Maximal irreducibility measure, 83

- $\rho(F)$ , Maximum eigenvalue modulus, 137  
 $\rightarrow$ , Leads to, 78  
 $\sigma_A := \min\{n \geq 0 : \Phi_n \in A\}$ , 64  
 $\succ$ , Absolute continuity, 75  
 $\tau_A := \min\{n \geq 1 : \Phi_n \in A\}$ , 64  
 $\tau_A$ , First entry time to  $A$ , 14  
 $\tau_A(1) := \tau_A$ , 64  
 $\tau_A(k) := \min\{n > \tau_A(k-1) : \Phi_n \in A\}$ , 64  
 $\theta^k, k^{th}$  order shift operator, 63  
 $\varphi$ , Irreducibility measure, 81  
 $h_Y$ , Almost everywhere invariant function, 423  
 $m_n(t)$ , Interpolation of  $M_n(g)$ , 447  
 $q_j$ , Prob. of  $j$  arrivals in one service in M/G/1 queue, 58  
 $s_j(f)$ , Sum of  $f(\Phi_i)$  between visits to atom, 428  
 $s_n(t)$ , Interpolation of  $S_n(\bar{g})$ , 447  
 $\mathcal{B}(\mathbb{R})$ , Borel  $\sigma$ -field, 553  
 $\mathcal{B}^+(X)$ , Sets with  $\psi(A) > 0$ , 84  
 $\mathcal{F}_\zeta^\Phi := \{A \in \mathcal{F} : \{\zeta = n\} \cap A \in \mathcal{F}_n^\Phi, n \in \mathbb{Z}_+\}$ , 66  
 $\mathcal{F}_n^\Phi := \sigma(\Phi_0, \dots, \Phi_n)$ , 63  
 $\mathcal{G}^+(\gamma)$ , Distributions with transform convergent in  $[0, \gamma]$ , 399  
 $\mathcal{M}$ , Borel probability measures, 17  
 $\hat{P}(x_i, A)$  Split transition function, 99  
 $\hat{g}$ , Solution to Poisson's equation, 443  
 $\bar{\pi}\{A\}$ , 425  
 $\bar{P}_k(x, \cdot)$ , Cesaro average of  $P^k$ , 288  
 $\Phi$ , Markov chain, 3, 61  
 $\Phi^m$ ,  $m$ -skeleton chain, 62  
 $\Phi_a$ , Chain with transition function  $K_a$ , 115  
 $\Phi_a$ , Sampled chain, 116  
 $\text{vec}(B)$ , 415  
 $\mathbb{P}_x$ , Prob. conditional on  $\Phi_0 = x$ , 14  
 $X$ , State space, 50  
 $\mathcal{B}(X)$ , Borel  $\sigma$ -field on  $X$ , 49  
 $\mathcal{C}(X)$ , Bounded continuous functions, 557  
 $\mathcal{C}(X)$ , Continuous bounded functions on  $X$ , 125  
 $\mathcal{C}_0(X)$ , Continuous functions vanishing at  $\infty$ , 560  
 $\mathcal{C}_c(X)$ , Continuous functions on  $X$ , compact support, 140  
 ${}_A P^n(x, B) := \mathbb{P}_x(\Phi_n \in B, \tau_A \geq n)$ , 67  
 $\text{Var}(u)$ , Variation in a renewal sequence, 330  
 $\tilde{\Phi}$ , Split chain, 99