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978-0-521-72236-0 - The Higher Arithmetic: An Introduction to the Theory of Numbers, Eighth Edition

H. Davenport

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