

LONDON MATHEMATICAL SOCIETY LECTURE NOTE SERIES

Managing Editor: Professor M. Reid, Mathematics Institute,
 University of Warwick, Coventry CV4 7AL, United Kingdom

The titles below are available from booksellers, or from Cambridge University Press at
www.cambridge.org/mathematics

- 280 Characters and automorphism groups of compact Riemann surfaces, T. BREUER
- 281 Explicit birational geometry of 3-folds, A. CORTI & M. REID (eds)
- 282 Auslander–Buchweitz approximations of equivariant modules, M. HASHIMOTO
- 283 Nonlinear elasticity, Y.B. FU & R.W. OGDEN (eds)
- 284 Foundations of computational mathematics, R. DEVORE, A. ISERLES & E. SÜLI (eds)
- 285 Rational points on curves over finite fields, H. NIEDERREITER & C. XING
- 286 Clifford algebras and spinors (2nd Edition), P. LOUNESTO
- 287 Topics on Riemann surfaces and Fuchsian groups, E. BUJALANCE, A.F. COSTA & E. MARTÍNEZ (eds)
- 288 Surveys in combinatorics, 2001, J.W.P. HIRSCHFELD (ed)
- 289 Aspects of Sobolev-type inequalities, L. SALOFF-COSTE
- 290 Quantum groups and Lie theory, A. PRESSLEY (ed)
- 291 Tits buildings and the model theory of groups, K. TENT (ed)
- 292 A quantum groups primer, S. MAJID
- 293 Second order partial differential equations in Hilbert spaces, G. DA PRATO & J. ZABCZYK
- 294 Introduction to operator space theory, G. PISIER
- 295 Geometry and integrability, L. MASON & Y. NUTKU (eds)
- 296 Lectures on invariant theory, I. DOLGACHEV
- 297 The homotopy category of simply connected 4-manifolds, H.-J. BAUES
- 298 Higher operads, higher categories, T. LEINSTER (ed)
- 299 Kleinian groups and hyperbolic 3-manifolds, Y. KOMORI, V. MARKOVIC & C. SERIES (eds)
- 300 Introduction to Möbius differential geometry, U. HERTRICH-JEROMIN
- 301 Stable modules and the D(2)-problem, F.E.A. JOHNSON
- 302 Discrete and continuous nonlinear Schrödinger systems, M.J. ABLOWITZ, B. PRINARI & A.D. TRUBATCH
- 303 Number theory and algebraic geometry, M. REID & A. SKOROBOGATOV (eds)
- 304 Groups St Andrews 2001 in Oxford I, C.M. CAMPBELL, E.F. ROBERTSON & G.C. SMITH (eds)
- 305 Groups St Andrews 2001 in Oxford II, C.M. CAMPBELL, E.F. ROBERTSON & G.C. SMITH (eds)
- 306 Geometric mechanics and symmetry, J. MONTALDI & T. RATIU (eds)
- 307 Surveys in combinatorics 2003, C.D. WENSLEY (ed.)
- 308 Topology, geometry and quantum field theory, U.L. TILLMANN (ed)
- 309 Corings and comodules, T. BRZEZINSKI & R. WISBAUER
- 310 Topics in dynamics and ergodic theory, S. BEZUGLYI & S. KOLYADA (eds)
- 311 Groups: topological, combinatorial and arithmetic aspects, T.W. MÜLLER (ed)
- 312 Foundations of computational mathematics, Minneapolis 2002, F. CUCKER *et al* (eds)
- 313 Transcendental aspects of algebraic cycles, S. MÜLLER-STACH & C. PETERS (eds)
- 314 Spectral generalizations of line graphs, D. CVETKOVIĆ, P. ROWLINSON & S. SIMIĆ
- 315 Structured ring spectra, A. BAKER & B. RICHTER (eds)
- 316 Linear logic in computer science, T. EHRHARD, P. RUET, J.-Y. GIRARD & P. SCOTT (eds)
- 317 Advances in elliptic curve cryptography, I.F. BLAKE, G. SEROUSSI & N.P. SMART (eds)
- 318 Perturbation of the boundary in boundary-value problems of partial differential equations, D. HENRY
- 319 Double affine Hecke algebras, I. CHEREDNIK
- 320 L-functions and Galois representations, D. BURNS, K. BUZZARD & J. NEKOVAŘ (eds)
- 321 Surveys in modern mathematics, V. PRASOLOV & Y. ILYASHENKO (eds)
- 322 Recent perspectives in random matrix theory and number theory, F. MEZZADRI & N.C. SNAITH (eds)
- 323 Poisson geometry, deformation quantisation and group representations, S. GUTT *et al* (eds)
- 324 Singularities and computer algebra, C. LOSSEN & G. PFISTER (eds)
- 325 Lectures on the Ricci flow, P. TOPPING
- 326 Modular representations of finite groups of Lie type, J.E. HUMPHREYS
- 327 Surveys in combinatorics 2005, B.S. WEBB (ed)
- 328 Fundamentals of hyperbolic manifolds, R. CANARY, D. EPSTEIN & A. MARDEN (eds)
- 329 Spaces of Kleinian groups, Y. MINSKY, M. SAKUMA & C. SERIES (eds)
- 330 Noncommutative localization in algebra and topology, A. RANICKI (ed)
- 331 Foundations of computational mathematics, Santander 2005, L.M. PARDO, A. PINKUS, E. SÜLI & M.J. TODD (eds)
- 332 Handbook of tilting theory, L. ANGELERI HÜGEL, D. HAPPEL & H. KRAUSE (eds)

- 333 Synthetic differential geometry (2nd Edition), A. KOCK
- 334 The Navier–Stokes equations, N. RILEY & P. DRAZIN
- 335 Lectures on the combinatorics of free probability, A. NICA & R. SPEICHER
- 336 Integral closure of ideals, rings, and modules, I. SWANSON & C. HUNEKE
- 337 Methods in Banach space theory, J.M.F. CASTILLO & W.B. JOHNSON (eds)
- 338 Surveys in geometry and number theory, N. YOUNG (ed)
- 339 Groups St Andrews 2005 I, C.M. CAMPBELL, M.R. QUICK, E.F. ROBERTSON & G.C. SMITH (eds)
- 340 Groups St Andrews 2005 II, C.M. CAMPBELL, M.R. QUICK, E.F. ROBERTSON & G.C. SMITH (eds)
- 341 Ranks of elliptic curves and random matrix theory, J.B. CONREY, D.W. FARMER, F. MEZZADRI & N.C. SNAITH (eds)
- 342 Elliptic cohomology, H.R. MILLER & D.C. RAVENEL (eds)
- 343 Algebraic cycles and motives I, J. NAGEL & C. PETERS (eds)
- 344 Algebraic cycles and motives II, J. NAGEL & C. PETERS (eds)
- 345 Algebraic and analytic geometry, A. NEEMAN
- 346 Surveys in combinatorics 2007, A. HILTON & J. TALBOT (eds)
- 347 Surveys in contemporary mathematics, N. YOUNG & Y. CHOI (eds)
- 348 Transcendental dynamics and complex analysis, P.J. RIPPON & G.M. STALLARD (eds)
- 349 Model theory with applications to algebra and analysis I, Z. CHATZIDAKIS, D. MACPHERSON, A. PILLAY & A. WILKIE (eds)
- 350 Model theory with applications to algebra and analysis II, Z. CHATZIDAKIS, D. MACPHERSON, A. PILLAY & A. WILKIE (eds)
- 351 Finite von Neumann algebras and masas, A.M. SINCLAIR & R.R. SMITH
- 352 Number theory and polynomials, J. MCKEE & C. SMYTH (eds)
- 353 Trends in stochastic analysis, J. BLATH, P. MÖRTERS & M. SCHEUTZOW (eds)
- 354 Groups and analysis, K. TENT (ed)
- 355 Non-equilibrium statistical mechanics and turbulence, J. CARDY, G. FALKOVICH & K. GAWEDZKI
- 356 Elliptic curves and big Galois representations, D. DELBOURGO
- 357 Algebraic theory of differential equations, M.A.H. MACCALLUM & A.V. MIKHAILOV (eds)
- 358 Geometric and cohomological methods in group theory, M.R. BRIDSON, P.H. KROPHOLLER & I.J. LEARY (eds)
- 359 Moduli spaces and vector bundles, L. BRAMBILA-PAZ, S.B. BRADLOW, O. GARCÍA-PRADA & S. RAMANAN (eds)
- 360 Zariski geometries, B. ZILBER
- 361 Words: Notes on verbal width in groups, D. SEGAL
- 362 Differential tensor algebras and their module categories, R. BAUTISTA, L. SALMERÓN & R. ZUAZUA
- 363 Foundations of computational mathematics, Hong Kong 2008, F. CUCKER, A. PINKUS & M.J. TODD (eds)
- 364 Partial differential equations and fluid mechanics, J.C. ROBINSON & J.L. RODRIGO (eds)
- 365 Surveys in combinatorics 2009, S. HUCZYNSKA, J.D. MITCHELL & C.M. RONEY-DOUGAL (eds)
- 366 Highly oscillatory problems, B. ENGQUIST, A. FOKAS, E. HAIRER & A. ISERLES (eds)
- 367 Random matrices: High dimensional phenomena, G. BLOWER
- 368 Geometry of Riemann surfaces, F.P. GARDINER, G. GONZÁLEZ-DIEZ & C. KOUROUNOTIS (eds)
- 369 Epidemics and rumours in complex networks, M. DRAIEF & L. MASSOULIÉ
- 370 Theory of p -adic distributions, S. ALBEVERIO, A.YU. KHRENNIKOV & V.M. SHELKOVICH
- 371 Conformal fractals, F. PRZYTYCKI & M. URBAŃSKI
- 372 Moonshine: The first quarter century and beyond, J. LEPOWSKY, J. MCKAY & M.P. TUIITE (eds)
- 373 Smoothness, regularity and complete intersection, J. MAJADAS & A. G. RODICIO
- 374 Geometric analysis of hyperbolic differential equations: An introduction, S. ALINHAC
- 375 Triangulated categories, T. HOLM, P. JØRGENSEN & R. ROUQUIER (eds)
- 376 Permutation patterns, S. LINTON, N. RUŠKUC & V. VATTER (eds)
- 377 An introduction to Galois cohomology and its applications, G. BERHUY
- 378 Probability and mathematical genetics, N.H. BINGHAM & C.M. GOLDIE (eds)
- 379 Finite and algorithmic model theory, J. ESPARZA, C. MICHAUX & C. STEINHORN (eds)
- 380 Real and complex singularities, M. MANOEL, M.C. ROMERO FUSTER & C.T.C. WALL (eds)
- 381 Symmetries and integrability of difference equations, D. LEVI, P. OLVER, Z. THOMOVA & P. WINTERNITZ (eds)
- 382 Forcing with random variables and proof complexity, J. KRAJÍČEK
- 383 Motivic integration and its interactions with model theory and non-Archimedean geometry I, R. CLUCKERS, J. NICAISE & J. SEBAG (eds)
- 384 Motivic integration and its interactions with model theory and non-Archimedean geometry II, R. CLUCKERS, J. NICAISE & J. SEBAG (eds)
- 385 Entropy of hidden Markov processes and connections to dynamical systems, B. MARCUS, K. PETERSEN & T. WEISSMAN (eds)

London Mathematical Society Lecture Notes series: 379

Finite and Algorithmic Model Theory

Edited by

JAVIER ESPARZA

Technische Universität München, Germany

CHRISTIAN MICHAUX

Université de Mons, Belgium

CHARLES STEINHORN

Vassar College, New York, USA



CAMBRIDGE
UNIVERSITY PRESS

Cambridge University Press & Assessment
 978-0-521-71820-2 — Finite and Algorithmic Model Theory
 Edited by Javier Esparza, Christian Michaux, Charles Steinhorn
 Frontmatter
[More Information](#)



CAMBRIDGE
UNIVERSITY PRESS

Shaftesbury Road, Cambridge CB2 8EA, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of Cambridge University Press & Assessment,
 a department of the University of Cambridge.

We share the University's mission to contribute to society through the pursuit of
 education, learning and research at the highest international levels of excellence.

www.cambridge.org

Information on this title: www.cambridge.org/9780521718202

© Cambridge University Press & Assessment 2011

This publication is in copyright. Subject to statutory exception and to the provisions
 of relevant collective licensing agreements, no reproduction of any part may take
 place without the written permission of Cambridge University Press & Assessment.

First published 2011

A catalogue record for this publication is available from the British Library

Library of Congress Cataloging-in-Publication data

Finite and algorithmic model theory / edited by Javier Esparza, Christian Michaux,
 Charles Steinhorn.

p. cm. – (London Mathematical Society lecture note series ; 379)
 Includes index.

ISBN 978-0-521-71820-2 (pbk.)

1. Finite model theory. 2. Computer science – Mathematics. I. Esparza, Javier.
 II. Michaux, Christian. III. Steinhorn, Charles. IV. Title. V. Series.

QA9.7.F565 2011

511.3'4 – dc22 2010048062

ISBN 978-0-521-71820-2 Paperback

Cambridge University Press & Assessment has no responsibility for the persistence
 or accuracy of URLs for external or third-party internet websites referred to in this
 publication and does not guarantee that any content on such websites is, or will
 remain, accurate or appropriate.

Contents

	<i>Preface</i>	<i>page vii</i>
	<i>Javier Esparza, Christian Michaux and Charles Steinhorn</i>	
1	Automata-based presentations of infinite structures	1
	<i>Vince Bárány, Erich Grädel and Sasha Rubin</i>	
2	Logical aspects of spatial databases	77
	<i>Bart Kuijpers and Jan Van den Bussche</i>	
3	Some connections between finite and infinite model theory	109
	<i>Vera Koponen</i>	
4	Definability in classes of finite structures	140
	<i>Dugald Macpherson and Charles Steinhorn</i>	
5	Algorithmic meta-theorems	177
	<i>Stephan Kreutzer</i>	
6	Model theoretic methods for fragments of FO and special classes of (finite) structures	271
	<i>Martin Otto</i>	

Preface

This volume is based on the satellite workshop on *Finite and Algorithmic Model Theory* that took place at the University of Durham, January 9–13, 2006, to inaugurate the scientific program *Logic and Algorithms* held at the Isaac Newton Institute for Mathematical Sciences during the first six months of 2006. The goal of the workshop was to explore the emerging and potential connections between finite and infinite model theory, and their applications to theoretical computer science. The primarily tutorial format introduced researchers and graduate students to a number of fundamental topics. The excellent quality of the tutorials suggested to the program organizers, Anuj Dawar and Moshe Vardi, that a volume based on the workshop presentations could serve as a valuable and lasting reference. They proposed this to the workshop scientific committee; this volume is the outcome.

The *Logic and Algorithms* program focused on the connection between two chief concerns of theoretical computer science: (i) how to ensure and verify the correctness of computing systems; and (ii) how to measure the resources required for computations and ensure their efficiency. The two areas historically have interacted little with each other, partly because of the divergent mathematical techniques they have employed. More recently, areas of research in which model-theoretic methods play a central role have reached across both sides of this divide. Results and techniques that have been developed have found applications to fields such as database theory, complexity theory, and verification.

Some brief historical remarks help situate the context for this volume. The study of the model-theoretic properties of finite structures emerged initially as a branch of classical model theory, with its focus primarily on first-order logic. Beginning in the late 1980s, however, research concerning logics on finite structures diverged sharply from work in classical model theory. Classical model theory, with its emphasis on infinite structures, had made dramatic advances

both theoretically and in applications to other areas of mathematics. Work on finite structures focused on connections with discrete complexity theory and verification. Indeed, the connections between finite model theory, descriptive complexity theory, parameterized complexity, and state machine verification are now so strong that boundaries between them are hard to distinguish.

The methods employed in these two facets of model theory also grew apart during this period. Probabilistic techniques and machine simulations have played a prominent role in the study of finite structures, and stand in contrast to the geometric, algebraic, and analytic methods that pervade classical (infinite) model theory. Although both classical and finite model theory deal with restricted classes of structures, the conditions by which such classes are delimited also have been quite different. Finite model theory and verification typically concentrate on classes linked to particular computing formalisms, or to which decomposition methods from finite graph theory can be applied. In contrast, infinitary model theory usually places restrictions on combinatorial or geometric properties of the definable sets of a structure.

Yet, there are recent indications of a re-convergence of classical model theory and logical aspects of computer science. This has resulted both from the interest of computer scientists in new computing and specification models that make use of infinitary structures, and from the development of powerful model-theoretic techniques that provide insight into finite structures. If there is an overarching theme, it is how various “tameness” hypotheses used to delimit classes of structures *and* logics have deeply impacted the study of those aspects of theoretical computer science in which model-theory naturally comes into play. The chapters that comprise this volume survey many of the common themes that have emerged and gained attention, and point to the significant potential for wider interaction.

The chapter of Bárány, Grädel, and Rubin, *Automata-based presentations of infinite structures* develops what the authors call *algorithmic model theory*. The authors direct their attention to the “tame” class of *automatic structures*, that is structures that have a presentation in a precise sense by automata operating on finite or infinite words or trees. The goal of this work, to extend algorithmic and logical methods from finite structures to finitely presented infinite structures, has been a focal point for research in computer science, combinatorics, and mathematical logic. This point of view allows structures to be viewed alternately from both a finite and infinite model theoretic perspective. The theory that has emerged makes use of techniques both from classical model theory and theoretical computer science, and has found appealing applications to several areas, including database theory, complexity theory and verification.

Classical model theory by and large concentrates on the analysis of the first-order definable sets over a structure, that is, those sets of n -tuples of the universe whose definition is given by a first-order formula. This analysis has predominantly taken two forms. The first is based on the “structural complexity” of the formula, e.g., the number of alternations of blocks of existential and universal quantifiers appearing in its prenex normal form. This theme is best illustrated by *quantifier elimination*, in which definable sets over a structure are shown to have quantifier-free definitions. The second involves assigning a dimension (with a corresponding notion of independence) to the definable sets that is combinatorially, algebraically, or geometrically motivated. Stability theory, with its combinatorial/algebraic account of dimension and independence, is perhaps the most widely known and longest-studied exemplar, its development traceable to Morley’s seminal work in the 1960’s and to Shelah’s deep and extensive work in the 1970’s. More recently, o-minimality, and in particular its focus on o-minimal expansions of the ordered field of real numbers, provides another important class of examples. The imposition of “tameness” assumptions in classical model theory such as stability and o-minimality – often verified in examples by quantifier elimination – make the analysis of the structures satisfying these hypotheses not only tractable but also amenable to applications in mathematics outside of logic.

Tarski’s quantifier-elimination for real-closed fields which thereby (effectively) equates the first-order definable sets over the field of real numbers with the semialgebraic sets, has long proved a fertile ground for framing and addressing computational issues. Kuijpers and Van den Bussche, in their chapter, *Logical aspects of spatial databases*, model spatial data via semialgebraic subsets of n -dimensional Euclidean space, and investigate the expressive power of several logic-based languages to query these databases. They first characterize the topological properties of planar spatial databases that are first-order expressible over the usual language for the ordered field of real numbers – of interest from the point of view of geographical information systems, for example – in terms of the query language “cone logic”. The second half of their chapter deals with query languages that extend first-order logic over the real field by some form of recursion, including spatial Datalog, and first-order logic extended with a while loop or with a transitive closure operator.

Koponen, in her chapter, *Some connections between finite and infinite model theory*, discusses how stability theoretic considerations, as well as other properties and techniques from classical model theory such as smooth approximation, can be imported successfully into the study of finite structures by restricting to bounded variable logic, that is, first-order logic under the restriction that there is

a fixed value k such that only formulas in which no more than k variables occur. In particular, Koponen investigates when a theory in bounded variable logic with an infinite model has arbitrarily large finite models and isolates conditions for effectively determining least upper bounds for the size of the smallest such finite model.

The chapter of Macpherson and Steinhorn, *Definability in classes of finite structures*, contains two distinct threads that draw their motivation from classical model theory. The first, inspired by the model theory of finite and pseudofinite fields, concerns asymptotic classes of finite structures. These are non-elementary classes of finite structures whose first-order definable sets asymptotically satisfy cardinality constraints that permit the assignment of a dimension and measure, and have an intimate connection in classical model theory to so-called simple theories. The second theme concerns so-called *robust classes* of finite structures, whose origin lies in attempting to “finitize” classical model-theoretic tameness conditions, such as o-minimality, that are provably excluded in asymptotic classes. Robust classes consist of directed systems of finite structures in which the truth value of a formula requires “looking ahead” into a larger structure in the system.

For the model theory of finite structures that has been developed with great success within theoretical computer science, “tameness” assumptions do not apply only to isolate classes of structures that are well-behaved with respect to a preferred logic, such as first-order logic. Research has prospered by striking a balance between appropriate logics or fragments thereof and classes of finite structures: that is, tame logics matched with tame classes. This theme appears already in Koponen’s chapter, with its emphasis on bounded variable logic combined with classical tameness assumptions, and strongly emerges in the chapters of Otto and Kreutzer. As these chapters furthermore show, this point of view can furnish significant computational insights.

Kreutzer’s chapter, *Algorithmic meta-theorems*, discusses how constraining both classes of (finite) structures and logics yields a wealth of algorithmic results. An algorithmic meta-theorem has the form that every computational problem that can be expressed in some logic can be solved efficiently on every class of structures that satisfy certain constraints. This is usually accomplished by showing that the model-checking problem for formulas in some logic – typically first-order or monadic second-order – is what is called *fixed-parameter tractable* for a class of structures, typically based on graphs with well-behaved tree decompositions. This point of view goes back to well-known work of Courcelle and his collaborators.

Otto takes as the focus of his chapter the application of game-oriented methods and explicit model constructions in the analysis of fragments of first-order

logic restricted to well-behaved (non-elementary) classes of structures, particularly finite structures. Whereas the model-theoretic compactness theorem plays an essential role in the classical setting, paradigmatically in proving *expressive completeness* results such as the Łos-Tarski theorem characterizing those formulas preserved under extensions as the existential formulas, its failure for restricted classes of structures, e.g., classes of finite structures, motivates the introduction of the methods and techniques that Otto places at the center of the chapter. The chapter also surveys how by restricting to classes of finite structures defined by tree-width and locality considerations, expressive completeness results that fail for the class of finite structures can be regained.

The workshop organizer was Professor Iain Stewart (Durham). The members of the Scientific Committee for the workshop included : Michael Benedikt (Oxford), Javier Esparza (Munich), Bradd Hart (McMaster), Christian Michaux (Mons-Hainaut), Charles Steinhorn (Vassar), and Katrin Tent (Münster). Financial support from the Newton Institute and EPSRC is gratefully acknowledged. We also wish to express our appreciation to the staff at Cambridge University Press, in particular Clare Dennison, our maths/computer science editor, and Sabine Koch, our production editor, for their remarkable thoughtfulness, patience, and efficiency throughout the process of bringing this volume into print.

Javier Esparza
Christian Michaux
Charles Steinhorn