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Groups and Analysis

The legacy of Hermann Weyl

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Universität Bielefeld

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Contents

Preface

This volume grew out of the conference in honour of Hermann Weyl that took place in Bielefeld in September 2006.

Weyl was born in 1885 in Elmshorn, a small town near Hamburg. He studied mathematics in Göttingen and Munich, and obtained his doctorate in Göttingen under the supervision of Hilbert. After taking a teaching post for a few years, he left Göttingen for Zürich to accept a Chair of Mathematics at the ETH Zürich, where he was a colleague of Einstein just at the time when Einstein was working out the details of the theory of general relativity. Weyl left Zürich in 1930 to become Hilbert's successor at Göttingen, moving to the new Institute for Advanced Study in Princeton, New Jersey after the Nazis took power in 1933. He remained there until his retirement in 1951. Together with his wife, he spent the rest of his life in Princeton and Zürich, where he died in 1955.

The Collaborative Resarch Centre (SFB 701) *Spectral Structures and Topological Methods in Mathematics* has manifold connections with the areas of mathematics that were founded or influenced by Weyl's work. These areas include geometric foundations of manifolds and physics, topological groups, Lie groups and representation theory, harmonic analysis and analytic number theory as well as foundations of mathematics.

In 1913, Weyl published *Die Idee der Riemannschen Fläche* ('The Concept of a Riemann Surface'), giving a unified treatment of Riemann surfaces.

He described the development of relativity theory in his *Raum, Zeit, Materie* ('Space, Time, Matter') from 1918, which reached a fourth edition in 1922. In 1918, he introduced the concept of gauge and gave the first example of what is now known as a gauge theory.

From 1923 to 1938, Weyl developed the theory of compact groups in terms of matrix representations and proved a fundamental character formula for compact Lie groups. His book *Classical Groups* opened new directions in invariant theory. It covered symmetric groups, general

linear groups, orthogonal groups, and symplectic groups, and results on their invariants and representations.

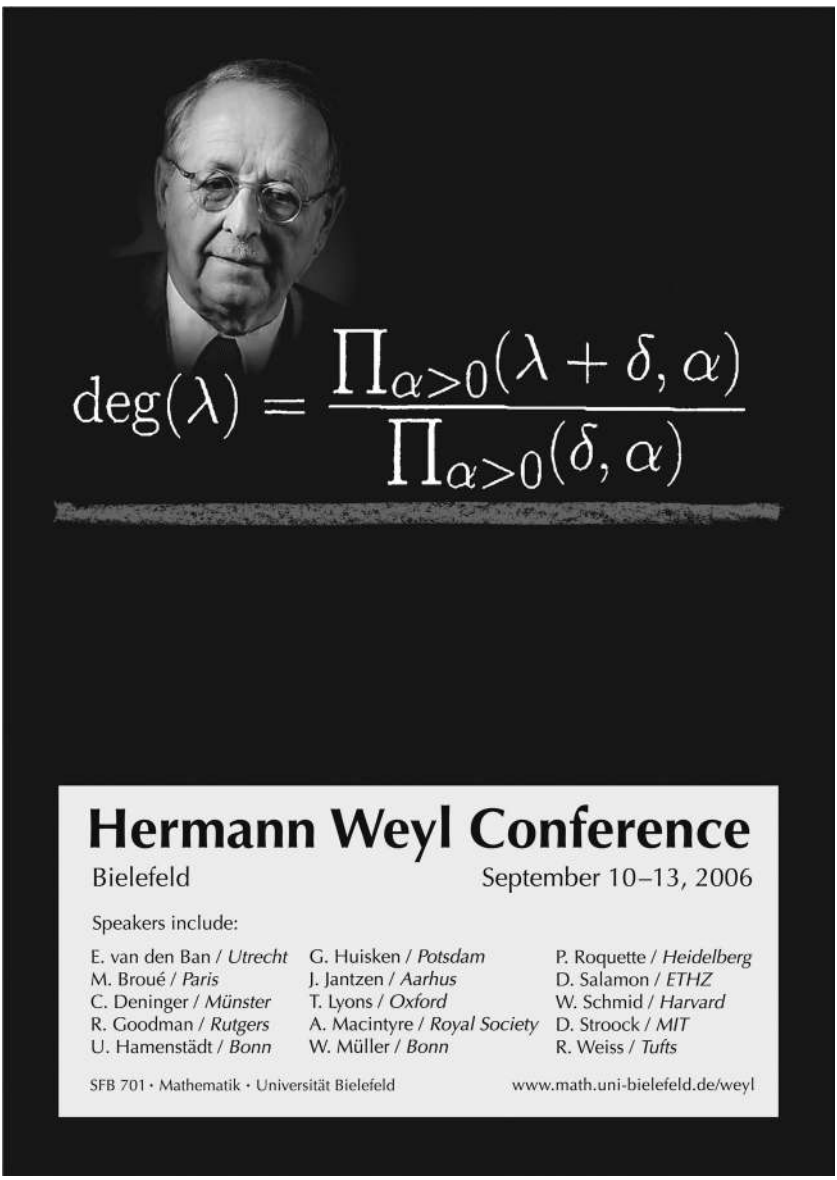
In *The Continuum*, Weyl developed the logic of classical analysis along the lines of Brouwer's intuitionism. However, he later decided that this radical constructivism puts too much of a restriction on his mathematics and reconciled himself with the more formalistic ideas of Hilbert.

Weyl also showed how to use exponential sums in diophantine approximation, with his criterion for uniform distribution modulo one, which was a fundamental contribution to analytic number theory.

During the conference, his lasting influence on current mathematics became evident through a series of impressive talks often connecting theorems of Weyl with the most current results in dynamical systems, invariant theory, or partial differential equations. We are happy that so many speakers agreed to contribute to this volume.

The conference was funded by the Collaborative Research Center (SFB 701) 'Spectral structures and topological methods in mathematics'. We gratefully acknowledge support by the German Research Foundation (DFG). Thanks are also due to Philip Herrmann for editing this volume, and to Markus Rost and Ulf Rehmann.

Bielefeld, December 2007



List of speakers and talks

M. Broue (Paris)
Complex Reflection Groups as Weyl Groups

C. Deninger (Münster)
Determinants in von Neumann algebras and the entropy of noncommutativ group actions

R. Goodman (Rutgers)
Harmonic Analysis on Compact Symmetric Spaces - the Legacy of H.Weyl and E.Cartan

U. Hamenstädt (Bonn)
Mixing properties of the Teichmüller flow

G. Huisken (Potsdam)
The concept of Mass in General Relativity

J.C. Jantzen (Aarhus)
Character formulae from Weyl to the present

H. Kalf (München)
Weyl's work on singular Sturm-Liouville operators

A. Laptev (Stockholm)
Lieb–Thirring Inequalities. Recent results

T. Lyons (Oxford)
Inverting the signature of a path-extensions of a theorem of Chen

A. Macintyre (London)
Some model theory of Lie groups

W. Müller (Bonn)
Weyl's law and the theory of automorphic forms

N. Nadirashvili (Marseille)
Rearrangements and eigenvalues

M. Pflaum (Frankfurt)
From Weyl quantization and Weyl asymptotics to modern index theory.

P. Roquette (Heidelberg)
Hermann Weyl and Emmy Noether (Some observations from the correspondence Hasse–Noether and other documents)

M. Rost (Bielefeld)
On Galois cohomology, norm functions and cycles

D. Salamon (Zürich)
Pseudoholomorphic curves in symplectic topology

T.A. Springer (Utrecht)
A short history of the theory of Weyl groups

D.W. Stroock (MIT)
Weyl's Lemma, the original elliptic regularity result

E. van den Ban (Utrecht)
Weyl, eigenfunction expansions and analysis on non-compact symmetric spaces

R.M. Weiss (Tufts)
Affine Buildings