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Non-equilibrium Statistical Mechanics and Turbulence

JOHN CARDY, GREGORY FALKOVICH AND KRZYSZTOF GAWEDZKI

Edited by

SERGEY NAZARENKO AND OLEG V. ZABORONSKI

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Preface

Understanding of turbulence is one of the most challenging problems of modern mathematical and theoretical physics. Turbulence can be described as a chaotic, highly non-equilibrium state of a non-linear physical system. Defined this way, turbulence embraces a broader class of examples than the chaotic Navier-Stokes flow, - a system for which the concept of turbulence was originally introduced and developed. In particular, turbulence appears as far-from-equilibrium states in plasmas, solids, Bose-Einstein condensates and even in nonlinear optics. The characteristic feature of turbulence is the presence of significant energy exchange between many degrees of freedom, which renders most attempts of perturbative treatment of the problem useless. Still mathematicians, physicists and engineers invest a large effort in understanding of turbulence due to the unprecedented importance of turbulence both for theoretical and applied science.

Thanks to fundamental works of Richardson, Taylor, Kolmogorov and Obukhov, we have a phenomenology of turbulent cascades leading to the famous Kolmogorov spectrum. This theory has been successfully applied to a wide range of turbulent systems. What is still lacking, however, is a fundamental theory of turbulence, which would allow both finding a rigorous mathematical foundation for the Kolmogorov theory and understanding of its limitations. The stepping stones on the way to the construction of such a fundamental theory are the solved models of turbulent phenomena. The list of such models which have been understood theoretically, has been growing steadily over the past 60 years. The examples include the theory of wave turbulence, passive scalar advection, the theory of kinematic magnetic dynamo, Burgers turbulence and constant flux states in cluster-cluster aggregation. The success in solving all these problems came from the intensive use of machinery of non-equilibrium statistical physics, such as kinetic

theory, quantum field theory-inspired instanton analysis of rare fluctuations and the theory of non-equilibrium critical phenomena.

There is no doubt that non-equilibrium statistical mechanics will play an increasingly important role in further progress of theoretical turbulence. Unfortunately, the range of tools and methods of non-equilibrium statistical mechanics used in modern turbulence research is so wide and they are developing so fast that there is not a single text book which could introduce a graduate student to this area of research. The goal of the present volume of LMS Lecture Notes is to start filling this gap. The book contains three sets of lecture notes written by world-class experts in the non-equilibrium statistical mechanics, which introduce the reader to ideas, methods and applications of non-equilibrium statistical mechanics to turbulence.

The course by professor Gregory Falkovich (Weizmann Institute of Science) gives an introduction to turbulence from the perspective of a statistical physics.

The course by professor Krzysztof Gawedzki (ENS Lyon) gives a thorough introduction into the problem of passive advection using rigorous methods of statistical physics.

The course by professor John Cardy (Oxford University) introduces the reader to field theoretical methods for non-equilibrium critical phenomena.

All courses are equipped with worked-out exercises illustrating the subtle points raised in the lectures and deepen the reader's understanding of the presented theoretical material.

These lectures have been given at the LMS 2006 Summer School at Warwick as part of Warwick Turbulence Symposium. The lectures have been accompanied by example classes led by course assistants - Alexander Fouxon, Adam Gamsa and Peter Horvai. We are indebted to these gentlemen for their help in preparing the lecture notes, designing and typing the class problems and their solutions.

Sergey Nazarenko and Oleg Zaboronski, September 2007.