
Contents

Preface	xi
---------	----

The Mathematical Core

Chapter 1	Fourier’s representation for functions on $\mathbb{R}$, $\mathbb{T}_p$, $\mathbb{Z}$, and $\mathbb{P}_N$	1
1.1	Synthesis and analysis equations	1
1.2	Examples of Fourier’s representation	12
1.3	The Parseval identities and related results	23
1.4	The Fourier–Poisson cube	31
1.5	The validity of Fourier’s representation	37
	Further reading	59
	Exercises	61
Chapter 2	Convolution of functions on $\mathbb{R}$, $\mathbb{T}_p$, $\mathbb{Z}$, and $\mathbb{P}_N$	89
2.1	Formal definitions of $f * g$, $f \star g$	89
2.2	Computation of $f * g$	91
2.3	Mathematical properties of the convolution product	102
2.4	Examples of convolution and correlation	107
	Further reading	115
	Exercises	116
Chapter 3	The calculus for finding Fourier transforms of functions on $\mathbb{R}$	129
3.1	Using the definition to find Fourier transforms	129
3.2	Rules for finding Fourier transforms	134
3.3	Selected applications of the Fourier transform calculus	147
	Further reading	155
	Exercises	156

viii *Contents*

Chapter 4	The calculus for finding Fourier transforms of functions on $\mathbb{T}_p$, $\mathbb{Z}$, and $\mathbb{P}_N$	173
4.1	Fourier series	173
4.2	Selected applications of Fourier series	190
4.3	Discrete Fourier transforms	196
4.4	Selected applications of the DFT calculus	212
	Further reading	216
	Exercises	217
Chapter 5	Operator identities associated with Fourier analysis	239
5.1	The concept of an operator identity	239
5.2	Operators generated by powers of $\mathcal{F}$	243
5.3	Operators related to complex conjugation	251
5.4	Fourier transforms of operators	255
5.5	Rules for Hartley transforms	263
5.6	Hilbert transforms	266
	Further reading	271
	Exercises	272
Chapter 6	The fast Fourier transform	291
6.1	Pre-FFT computation of the DFT	291
6.2	Derivation of the FFT via DFT rules	296
6.3	The bit reversal permutation	303
6.4	Sparse matrix factorization of $\mathcal{F}$ when $N = 2^m$	310
6.5	Sparse matrix factorization of H when $N = 2^m$	323
6.6	Sparse matrix factorization of $\mathcal{F}$ when $N = P_1 P_2 \cdots P_m$	327
6.7	Kronecker product factorization of $\mathcal{F}$	338
	Further reading	345
	Exercises	345
Chapter 7	Generalized functions on $\mathbb{R}$	367
7.1	The concept of a generalized function	367
7.2	Common generalized functions	379
7.3	Manipulation of generalized functions	389
7.4	Derivatives and simple differential equations	405
7.5	The Fourier transform calculus for generalized functions	413
7.6	Limits of generalized functions	427
7.7	Periodic generalized functions	440
7.8	Alternative definitions for generalized functions	450
	Further reading	452
	Exercises	453

Selected Applications

Chapter 8	Sampling	483
8.1	Sampling and interpolation	483
8.2	Reconstruction of f from its samples	487
8.3	Reconstruction of f from samples of $a_1 * f, a_2 * f, \dots$	497
8.4	Approximation of almost bandlimited functions	505
	Further reading	508
	Exercises	509
Chapter 9	Partial differential equations	523
9.1	Introduction	523
9.2	The wave equation	526
9.3	The diffusion equation	540
9.4	The diffraction equation	553
9.5	Fast computation of frames for movies	571
	Further reading	573
	Exercises	574
Chapter 10	Wavelets	593
10.1	The Haar wavelets	593
10.2	Support-limited wavelets	609
10.3	Analysis and synthesis with Daubechies wavelets	640
10.4	Filter banks	655
	Further reading	673
	Exercises	674
Chapter 11	Musical tones	693
11.1	Basic concepts	693
11.2	Spectrograms	702
11.3	Additive synthesis of tones	707
11.4	FM synthesis of tones	711
11.5	Synthesis of tones from noise	718
11.6	Music with mathematical structure	723
	Further reading	727
	Exercises	728

x	<i>Contents</i>	
Chapter 12	Probability	737
12.1	Probability density functions on $\mathbb{R}$	737
12.2	Some mathematical tools	741
12.3	The characteristic function	746
12.4	Random variables	753
12.5	The central limit theorem	764
	Further reading	780
	Exercises	780
Appendices		A-1
Appendix 1	The impact of Fourier analysis	A-1
Appendix 2	Functions and their Fourier transforms	A-4
Appendix 3	The Fourier transform calculus	A-14
Appendix 4	Operators and their Fourier transforms	A-19
Appendix 5	The Whittaker–Robinson flow chart for harmonic analysis	A-23
Appendix 6	FORTTRAN code for a radix 2 FFT	A-27
Appendix 7	The standard normal probability distribution	A-33
Appendix 8	Frequencies of the piano keyboard	A-37
Index		I-1