

Index

Abel–Dirichlet argument, 43
 absolutely integrable, 48–50, 57, 81–82,
 90, 122, 126–127, 164, 169–171,
 179
 absolutely summable, 38, 40–41, 127,
 210
 additive synthesis, *ix*
 of solution to PDE, 15, 524
 of tone, 707–711, 718–723, 730–732
 alias, 7, 486, 495, 506, 659, 717, 730
Almagast, 12, 699
 almost bandlimited, 505
 almost periodic, 88
 analysis equation
 for almost periodic f , 88
 for Dirac δ , 429
 of Fourier, 3, 6–7, 10
 for periodic generalized f , 446
 using sin, cos, 12–13, 66–68, 247
 for wavelets, 600, 640
 antiderivative
 of generalized function, 390
 of probability density, 379
 antiderivative rule, 164
 arginine spectrum, 22, 165
 arrow notation for δ , 381
 audio signal, *xiii*, 483–484, 509, 593,
 602, 672, 693–694
 autocorrelation, 109, 162, 238
 automobile suspension, 418
 averaging operator, 72

B-spline, 117, 164, 171, 193, 462, 794
 Bach, J.S., 725, 735
 backward difference, 230 (*see also*
 forward difference)
 bandlimited function, 426
 almost, 505
 bound for derivatives, 517
 convolution of, 511
 discrete, 512, 515
 for ear, 485, 728
 for eye, 510
 limit of sequence, 511
 Maclaurin series for, 486, 517
 multiplication by, 477, 487
 recovery from samples, 489, 491, 497,
 511
 recovery from filtered samples, 499,
 503
 series for derivative, 515
 structure of, 427, 485
 bar operator, 251
 tag, 252
 Beethoven sonata, 693
 bell curve, 771, 794
 bell tone, 709, 716, 732
 Benford's density for first digits, 798
 Bernoulli discretized string, 580

Bernoulli functions, 41–42, 177, 182,
 552
 for creating comb, 432
 discrete, 230
 and Eagle's method, 184, 479
 for Euler–Maclaurin formula, 212,
 225
 for evaluating sums, 221
 properties, 43–44, 182–183, 212, 225
 for synthesis of Fourier series, 184
 Bernstein's bound, 516
 Bessel functions, 224, 712–714, 732–734,
 760
 Bessel's inequality, 26, 77
 biorthogonality constraint, 666, 690
 bit reversal permutation, 303, 354
 algorithms, 305–307, 310
 via even-odd sorts, 303, 314–315, 351
 via Kronecker products, 342, 365
 operation count, 354
 via perfect shuffles, 314–315, 342
 Bluestein's chirp FFT, 358
 Bochner, S., 59, A1
 brass tone, 710
 analysis of Gibbs phenomenon, 80
 characterization of densities, 749
 Bohr, H., 88
 Born and Wolf, 553, 573, 587
 boundary conditions for PDE, 535, 545,
 550–552, 569, 585
 box function, 130
 Bracewell, R., 115
 bit reversal, 307–310
 FHT, 323–326, A1
 brass tone, 710
 bunched samples, 519
 Buneman's tricks, 308, 319–320

C for complex numbers, 41, 165
 cardinal series, 491, 513
 basis functions, 141, 160, 493,
 514–515
 convergence, 492–493
 generalization, 497, 515
 Carlson's theorem, 57
 carrier frequency, 711
 cascade algorithm, 682
 Cauchy–Schwartz inequality, 491, 507,
 521, 745, 756, 762, 792
 ceiling function $\lceil \cdot \rceil$, 229, 683
 central limit theorem, 771–775
 applications 775–779, 793
 for probability densities on \mathbb{P}_N , 796
 for probability densities on \mathbb{T}_p , 797
 centroid, 74, 756
 characteristic functions, 741
 Bochner's characterization, 749
 boundedness, 748
 for computing moments, 757
 continuity, 748

characteristic functions (*cont.*)
 convention for 2π , 741
 from expectation integrals, 759
 periodic, 752
 for probability densities on \mathbb{P}_N , 788
 for probability densities on \mathbb{T}_p , 797
 products of, 750
 smoothness, 786
 for sums of random variables, 765,
 767
 via weak limit, 748
 chirp signal, 109, 487, 706, 797
 for Bluestein's FFT, 358
 Chowning, J., *xiii*, 711, 716
 Chowning's rule, 713–717, 732
 circulant matrix, 122
 closure
 of \mathbb{G} , 390, 396, 459
 of \mathbb{S} , 375–376, 454
 of probability densities, 787
 of symmetries, 277
 coherent light, 560, 561
 comb function on \mathbb{P}_N , 197, 796
 comb function III on \mathbb{R} , 383, 393, 432,
 437, 448
 commensurate, 88
 commuting diagram, 37, 205, 256
 commuting operators, 122
 complex exponentials
 on \mathbb{R} , \mathbb{T}_p , \mathbb{Z} , \mathbb{P}_N , 2, 4, 6, 9
 as eigenfunctions of LTI systems, 18
 compression of signal
 via filter bank, 668–670
 via interpolation, 71, 234
 concatenation of vectors, 351, 352,
 607–608, 671, 675
Concerning Harmonics, 699
 conjugation, 62
 operator, 251
 tag, 292
 conjugation rule, 136, 177, 199, 413
 conservation of energy, 24, 531, 544,
 565
 convention for 2π , 63, 741
 convolution, 89–90
 algebraic properties, 103, 120, 122,
 126–127, 233
 of bandlimited functions, 511
 and correlation, 91
 differentiation, 106, 121, 126, 401
 via FFT, 113, 357
 via FHT, 265
 as filtering, 497
 of generalized functions, 398, 400,
 461
 via generating function, 101
 identity, 104, 118–120, 168, 196, 400,
 408 (*see also* Dirac delta)
 indirect, 103, 149, 158
 and LTI systems, 105, 282, 470, 481

I-2 Index

convolution (*cont.*)
 of ordinary functions, 89–90
 of periodic generalized functions, 447
 of probability densities, 110, 123, 150,
 751
 via sliding strip, 94
 as smearing, 107
 and smoothness, 107, 123, 126, 128,
 401
 for solving ODEs, 145, 470, 481
 for solving PDEs, 528, 534, 542, 548,
 559, 567
 square root, 105, 232
 support, 121
 translation, 120, 162
 convolution rule, 103, 143, 170, 177,
 199, 257, 286, 413
 for Hartley transforms, 265, 286
 for Hilbert transforms, 268, 289
 Cooley-Tukey, 295, 362, A1
 correlation, 90, 109, 162
 cos operator, 246, 287
 cos transform, 63, 66, 247, 287
 cosine signal, 511
 CSG function, 376

dagger operator, 251
 tag, 252
 d'Alembert, 523
 d'Alembert formula, 528, 574, 575
 Daubechies, I., *xiii*, 610, 614, 631
 Daubechies wavelets, 631, 639
 decay of Fourier transform, 153, 170,
 193, 376, 786
 decibel, 695
 decimation (downsampling) rule, 188,
 204, 258
 deconvolution, 108, 679
 definition chasing, 391, 471
 delta function on \mathbb{P}_N , 196
 density (*see* probability density)
 derivative operator, 121, 251, 285
 tag, 251
 derivative rule
 for convolution, 106, 401
 for Fourier transform, 141, 169, 413
 for generalized functions, 377, 405,
 413
 for Hartley transform, 265, 285
 for Hilbert transform, 269
 of Leibnitz, 107
 detail for wavelet approximation, 600,
 640
 computation of, 606, 608, 649, 652
 DFT, 11, 291
 eigenvalues, 279
 fast algorithms (*see* FFT)
 via geometric progression, 198, 227
 via Horner's algorithm, 293
 via paper strips, 294, 349
 via Poisson's relations, 209
 for problems of harmonic analysis,
 348
 of real vector, 276, 352
 via rules, 199–208
 via summation, 196
 table, A12
 DHT, 249
 fast algorithms (*see* FHT)

dice, 110, 123, 793
 loaded, 796
 tetrahedral, 780
 differential equation, 163, 281, 408
 via convolution, 470
 for finding Fourier transform, 132,
 164, 420–425
 homogeneous, 408, 416, 460
 inhomogeneous, 146, 367, 408,
 410–412, 417, 419, 461, 470,
 477
 differentiation of
 generalized function, 377, 405
 piecewise smooth function, 406–408,
 525
 diffraction, 553
 from edge, 588
 of gaussian beam, *ii*, 561–564
 from periodic source, 568–571
 from slits, 560–561, 588
 diffraction equation, 524
 with boundary conditions, 569
 conservation of energy, 565
 derivation, 553–558, 587
 Fourier synthesis, 524, 567, 571–572
 Fraunhofer approximation, 565, 569
 initial condition, 557, 559, 566, 587
 kernel, 466, 559, 566, 586
 no extreme value principle, 590
 symmetry, 590
 diffusion equation, 15, 524
 with boundary conditions, 545, 550,
 551, 552, 585
 conservation of energy, 544
 derivation, 540
 extreme value principle, 545
 forced, 546, 585–586
 Fourier synthesis, 524, 549, 571–572
 initial condition, 541, 548, 587
 kernel, 542, 548, 586
 polynomial solutions, 544, 584
 smoothness of solutions, 542, 584
 symmetry, 584
 digitized sound file, *xiii*, 483–484,
 508–509, 593, 602, 672, 693–694
 dilate of wavelet, 594
 dilation equation, 163, 281, 598, 609
 and multiresolution analysis, 599,
 645–646, 650
 solution via infinite product, 615,
 677
 symmetry, 678
 uniqueness of solution, 616, 677
 dilation operator, 258, 283, 656, 659
 dilation rule, 138, 140, 187, 219, 205–
 207, 236, 257, 413
 dimensional analysis, 69
 dipoles, 429
 Dirac delta, 371, 380, 392–393
 approximations, 28, 77–78, 168, 474
 dilation property, 392
 discrete, 196
 and Eagle's method, 421–425, 479
 as identity for convolution product,
 400
 not an ordinary function, 168, 453
 sifting property, 392
 as solution of dilation equation, 610
 Dirichlet conditions, 57, 77
 Dirichlet kernel, 66, 187, 219, 474

discrete function (*see also* function
 on \mathbb{Z})
 Fourier transform (*see* DFT)
 wavelet transform, 594
 discretization (sampling), 32, 483–484
 dispersion
 of heat, 543
 of light waves, 566
 of water waves, 591
 distribution (tempered), *x*, 368, 451
 distribution function, 740
 via limits, 784
 for max, min of random variables,
 790
 for standard normal density, 755,
 A33
 dual rule, 260, 280
 duality for Fourier analysis and
 synthesis, 8, 66
 DuBois-Reymond, 57
 DWT, 607, 675
 via herringbone, 608, 646
 operation count, 608, 692
 using operators, 646
 in place, 676
 dyadic dilate, 594

Eagle's method, 184, 422, 479
 eavesdrop, 512, 520
 echo location, 109
 eigenfunctions of operators, 18, 282
 of \mathcal{F}_N , 279
 eigenvalues, 279–280, 282
 of \mathcal{F} , 151, 167, 212
 of Kronecker product, 363
 of LTI system, 18, 282
 end padding operator, 234, 284, 688
 equidistribution of arithmetic sequence,
 194
 error function, 582, A36
 errors
 for computation of ω^k , 357
 for computing sum with round off,
 793
 for fast arithmetic, 124
 for FFT, 301
 for frame, detail coefficients, 649–655,
 685–686
 for least squares, 75–76, 84
 for sampling theorems, 492, 497, 499,
 507
 Euclidean algorithm for gcd, 227
 even
 function, 62, 64, 247
 generalized function, 397, 455
 projection, 245
 expectation integral, 720, 738, 755–761,
 764
 for independent random variables,
 764
 for spectral density, 720
 Euler
 gamma function, 164
 identity for sin, cos, 1, 67
 Maclaurin sum formula, 213
 exponent notation, 313
 via Kronecker products, 339
 exponential operators, 245
 extreme value principle, 545

factorial powers, 440

factorization

- of convolution operator, 257
- of DFT matrix, 314, 329, 338
- of DHT matrix, 324, 359
- of $\mathbf{L}_\pm, \mathbf{H}_\pm$ from filter bank, 656
- of Q for filter bank, 666

fast arithmetic, 113–114, 124

fast convolution, 113, 357

FBI filters, 672, 691

Fejér kernel, 78, 474

Fejér example of divergent Fourier series, 57, 85

Fermat theorem, 236

FFT

- Bluestein's scheme, 358
- Cooley–Tukey, 295, 349
- decimation in frequency, 299, 318, 338
- decimation in time, 296, 316, 322, 331
- via DFT rules, 296, 299, 353
- FORTRAN code, A27
- for frames of movies, 572
- Gauss discovery, 70, 295, 360
- impact, 294–295, A1
- via flow chart, 348, A23
- via Kronecker product, 344, 365–366
- via Mason flow diagram, 355
- via matrix factorization, 310, 329, 356
- operations, 295, 323, 332, 353, 358
- in place, 311
- with precomputed sines, 318, 322
- via recursive algorithm, 301, 350, 351
- via segments, 352
- for spectral factorization, 637
- for spectrogram, 705
- Stockham's autosort, 344, 366
- three loop algorithm, 313, 318, 322
- two loop algorithm, 365
- via zipper identity, 312, 314, 328

FHT

- advantages, 325, 327
- patent, 325, A1
- three loop algorithm, 326
- via zipper identity, 324, 356, 359
- filter
 - via convolution, 497
 - FBI, 672, 691
 - for filter bank, 659
 - high-, low-pass, 281, 511, 659
 - for sampling, 497, 519
 - for shaping noise, 721
 - translation, 663
- filter bank, 655
 - Fourier analysis, 658
 - perfect reconstruction, 661, 689
 - using up, down sampling, 656
- Fletcher–Munson contours, 695–696
- floor function $\lfloor \cdot \rfloor$, 124, 200, 229, 383, 467, 683, 703, 717, 731
- FM synthesis of tone, 711–717
 - parallel and cascade, 734
- forward difference, 118, 440, 480
- Fourier
 - analysis and synthesis, 3–11, 15, 73, 86

Fourier (*cont.*)

- validity, 37–58
- big pixel image, 510
- and dimensional analysis, 69
- and heat conduction, 15, 72, 541
- impact of work, A1–A3
- quote, *iii*
- sketch, *xv*, 134
- spoken word, 483, 509

FOURIER, *ii*, *xiv*, *xv* (check author's web site for additional details)

Fourier coefficient, 5, 10, 441, 446

rate of decay, 193, 441

Fourier–Poisson cube, 31, 36–37, 205

Fourier series, 5, 173

- via Bernoulli functions, 184, 218, 437

convergence, 39–48, 75, 77

via differentiation, 422, 479

for generalized functions, 440–441

via integration, 174

via Laurent series, 185

via Poisson's formula, 179, 478

via Riemann sum, 79

via rules, 176–179, 191–192

to solve PDEs, 532, 535, 538, 549, 567

uniqueness, 30, 78

weak convergence, 58, 433, 441

Fourier transform ($\mathbb{R}, \mathbb{T}_p, \mathbb{Z}, \mathbb{P}_N$), 3, 6, 7, 11

rules, A14–A18

tables, A1–A13

Fourier transform (\mathbb{R})

calculus, 129, 114, 146

decay at infinity, 153, 170

via differentiation, 132, 421–425

of generalized function, 413

via integration, 129–131

of periodic functions, 444

of probability density, 746–752

via rules, 134–147, 413

smoothness, 48, 81, 153, 169, 786

table, A1–A7

Fourier transform operator, 240, 243

tag, 251

Fourier transform of operator, 255, 259

fractional derivatives, 154

fragmentation of Π , 495, 518

frame for wavelet approximation, 598, 640

computation, 606, 608, 649, 652

illustrations, 601, 603, 650, 669

Fraunhofer approximation, 565, 569, 571, 588–589

frequency

carrier-modulation, 711

of concert A, 700

function for tune, 723

local, 706, 730

for piano keyboard, A37

and pitch, 694

via spectrogram, 705

units for, 73

of vibrating string, 536

via wavelet coefficients, 596

Fresnel

approximation, 555

convolution equation, 558

function, 165, 420, 560, 588

Fresnel (*cont.*)

discrete, 214

integrals, 165, 215, 420, 466

FT-NMR spectrum, 22, 165, A2

function

on $\mathbb{R}, \mathbb{T}_p, \mathbb{Z}, \mathbb{P}_N$, 3–5, 8

bandlimited, 426

CSG, 376

entire, 517

frequency, 724

generalized, 58, 368, 378–379, 524

locally integrable, 389, 467

of operator, 244, 272, 277–278, 281, 283

probability density, 739

of random variable, 758–760, 789

Schwartz, 372–374

slowly growing, 376

support-limited, 426

functional, 369

continuous, 451

fundamental, 370, 377–378

integral notation, 372, 378

linear, 390, 451

FWT

via coefficients, 606

via matrix factorization, 675

via operators, 645–646

in place, 676

G for generalized functions, 390

Gauss

asteroid orbit, 14, 70

discovery of FFT, 71, 295, 360

interpolation, 8, 14, 360

law of errors, 771, 794

Poisson sum formula, 179

signature, 669–670

sums, 215

gaussian function, 132, 741

for mollification and tapering, 743

gaussian laser beam, 561

interfering, 564

pointing, 562

spreading, 562

gcd, 206, 227

generalized function, 367–372, 378

bandlimited, 426

closure, 390, 459

“continuity”, 431

convolution, 398–399

via CSG functions, 378

division, 402–405, 459

Fourier transform rules, 413

as functional, 369–372, 376, 382, 384, 388, 451

integral notation, 371, 378, 390–391

via limit of Schwartz functions, 450, 482

limits, 427–439

via locally integrable functions, 389, 467

multiplication, 398–399, 402

as ordinary function, 58, 376, 379

partial derivatives, 438–439, 525

periodic, 440–448

as probability densities, 739

as scaling function, 616, 623

I-4 Index

generalized function (*cont.*)
 as solution to PDE, 524–525, 528,
 541, 559, 587
 special structure, 408–410, 427, 441,
 459–464
 table of Fourier transforms, A4–A6
 transformations, 389–405, 458
 “values”, 378, 382
 generating function for
 Bernoulli polynomials, 225
 Bessel functions, 224
 function on \mathbb{Z} , 101
 Hermite polynomials, 166
 Hermite function, 167
 geometric progression, 25, 65–66, 228
 GFT, 450
 Gibbs phenomenon, 44, 47–48, 80, 84
 for wavelets, 624, 687
 glissando of Risset, 725
 Goertzel algorithm, 347
 grouping operator, 659
 grouping rule, 187

Haar wavelet, 594
 analysis, 600, 606
 Fourier transform, 596
 scaling function, 597
 synthesis, 594, 606
 hanning window, 229
 Hartley transform, 248, 249
 advantages, 248, 255, 263, 279
 via Fourier transform, 249, 251
 via rules, 263, 265–266, 285
 tag, 251
 heat flow, *xiii*, 15, 72, 540–553,
 582–587
 Heaviside function, 116, 131, 380, 424
 Helmholtz, H., 707
 Hermite functions, 151, 166
 discrete, 211
 Hermite polynomials, 151, 160
 hermitian conjugation, 62
 operator, 251
 tag, 252
 hermitian conjugation rule, 136
 Hilbert transform, 266, 471
 via analytic function, 270
 via Kramers-Kronig relations, 269
 via rules, 267, 288
 for sampling theorem, 519
 tag, 267
 Hipparchus-Ptolemy model, 12, 70
 Horner’s algorithm
 for DFT, 293
 for other tasks, 345–347
 Huygens synthesis of waves, 555

i for $\sqrt{-1}$, 2
 impulse response, 282, 481
 automobile suspension, 418
 mass on spring, 367–368, 408, 417,
 453
 for ODE, 419, 470, 471
 for PDE (*see kernel*)
 independent random variables, 764
 infinite product, 615
 infinite series, 1, 5
 of bandlimited functions, 489, 491,
 497, 503, 505
 of generalized functions, 431, 435

infinite series (*cont.*)
 of sinusoids, 5, 440
 of solutions for PDE, 16, 532, 549,
 567
 weak convergence of, 431, 440
 initial “conditions” for PDEs, 587
 integral notation, 371, 378
 interpolation
 of bandlimited function, 485, 489,
 491, 497, 503, 505
 using FFT, 234, 284
 of Pallas orbit, 14, 70, 360
 by piecewise linear function, 145, 234,
 485
 by trigonometric polynomials, 70, 284,
 360, 514
 inverse power function, 387, 395
 inversion rule, 141, 174, 199, 240, 413
 involution, 251, 279
 isoperimetric inequality, 226, 235
jumps in f, f', \dots , 46, 55, 123, 184
 and Eagle’s method, 184, 479
 and generalized derivatives, 406
 and Gibbs phenomenon, 47, 84, 624
 removing, 46, 55

Kasner’s problem, 125
 kernel
 de la Vallée-Poussin, 27
 diffraction, 466, 559, 566, 586
 diffusion, 542, 548, 586
 Dirichlet, 66, 187, 474
 Fejér, 78, 474
 Poisson, 65
 wave, 528, 532, 575, 576
 Kramers-Kronig relations, 269, 289–290
 Kronecker product, 338
 algebraic properties, 339, 363
 eigenvalues, 363
 rearrangement, 341
 Kronecker rule, 174

Laplace’s equation, 525, 591
 Laplace function, 135
 laser beam, 553
 Laurent series
 for ellipse, 222
 for Fourier series, 185, 187, 218
 law of large numbers, 778
 least squares approximation
 and Fourier synthesis, 75–76, 84
 and sampling theory, 492, 497, 522
 Legendre function, 237
 Leibnitz notation, 372
 Leibnitz rule for differentiation, 107,
 375, 398, 455, 457, 459
 Liberal arts, 698
 Lighthill, M.J., 451–452
 likelihood function, 794
 lolipop plots, 6
 Lorenzian, 22
 LTI system, 16, 72, 282, 470, 522, 738
Maclaurin series, 1, 522, 738
 for bandlimited function, 486, 517
 and weak convergence, 438
 Mallat’s herringbone algorithm, 606–607,
 645–646 (*see also FWT*)

Mars orbit, 13, 70
 Mason flow diagram for FFT, 355
 max bound
 for generalized function, 784
 for probability density, 745, 783
 max flat trigonometric polynomial, 633
 Maxwell density, 737, 791
 mean μ , 756, 759
 for sum of random variables,
 766–767
 Mersenne’s formula for frequency, 536
 mesa function, 373, 454, 754 (*see also*
 tapered box)
 Michelson and Stratton harmonic
 analyzer, 87
 midpoint regularization, 45
 mirror
 for boundary condition, 569
 for Fourier Transform rules, 134
 for reverse carry algorithm, 305
 mnemonic, 137, 177, 200
 modulation
 frequency, 711
 index, 711, 715
 modulation rule, 137, 177, 199, 260,
 413
 modulus of continuity, 598, 643
 moments
 for probability density and
 smoothness, 757, 785
 for sum of random variables, 766
 for wavelet, 618, 621, 680–681
 monochord, 698, 729
 monotonicity relation, 744
 Monticello, 107
 mother wavelet, 594
 movies
 computing frames with FFT, 571
 for diffracting laser beam, 572, 590
 for heat flow, 572
 for vibrating string, 572, 575, 577,
 579–581
 for water waves, 591
 multiplication using FFT, 113
 multiplication of generalized functions,
 398
 multiplication rule, 144, 170, 177, 199,
 413
 multiresolution analysis, 597, 642,
 684
 music
 as mathematics, 698
 beat, 700
 interval, 694, 697–699
 loudness, 696, 701
 pictogram, 700
 pitch, 694, 700, 702, A37
 samples, 693, 707–708, 715, 732
 scales, 697–700
 score, 693, 701–703
 for wavelets, 597, 600, 674
 spectrogram, 703, 724
 timbre, 707
 transformation, 725, 735
 musical tone, 694
 via additive synthesis, 73, 707–711
 for bell, 709, 716
 for brass, 710
 via computer, 694, 700, 715, 718
 via FM synthesis, 711–717, 734

- musical tone (*cont.*)
 information content, 728
 local frequency, 705–706
 loudness, 695, 728
 from monochord, 729
 from noise, 718–723
 for string, 710, 731
- N₂ molecule**, 738
 Newton's "method", 1
 Newton's iteration, 114, 636
 Nobel prize, 22, A2
 noise
 filtered, 721
 sound of, 720, 723
 white, 718
 normal density, 139, 150, 739, 772, 775, 794, 795
 distribution function, 740, A33
 normal vibration modes, 536, 702
 Nyquist condition, 486, 705, 717
- odd**
 function, 62, 64, 247
 generalized function, 397, 455
 projection, 245
 Ohm's law of acoustics, 696, 729
 orbit
 for cardioid, rose, . . . , 225
 for Mars, 13, 70
 for Pallas, 14, 70
 symmetry of, 225
 for vibrating string, 579
 order of approximation, 620, 642–644
 operation, 292
 operation count
 for additive, FM synthesis of tone, 732
 for bit reversal permutation, 354
 for FFT, 299, 323, 327, 358
 for FHT, 325
 for FWT, 608, 646, 692
 for naive DFT, 292, 294
 operators, 16, 239, A19–A22
 blanket hypotheses, 241–242
 from complex conjugation, 251–253
 factorization of, 257
 for filter bank, 656–660
 Fourier transform of, 255–256, A19–A22
 Hartley transform of, 263
 LTI, 16, 72, 282, 470
 for Mallat's herringbone, 645
 from powers of \mathcal{F} , 243
 for pre-, post-processing, 652
 projection, 245, 253–254, 277
 symmetry preserving, 254, 282
 orthogonal projection, 642
 orthogonality relations
 via centroid, 74
 for complex exponentials, 24–25
 for Hermite functions, 166
 for sin, cos, 75
 for sinc functions, 160
 for wavelets, 602, 625, 642
- P_N for polygon**, 8
 Paley–Wiener theorem, 517
 Pallas, 14, 70, 361
- Papoulis sampling theorem, 503–504, 519
 parallel operation for FFT, 343
 Parseval identities, 23–24, 73–74
 for evaluating integrals, 83, 149
 for evaluating sums, 190, 221, 226
 for generalized functions, 391, 466
 link to convolution, 170
 validity, 24, 82 (*see also* Plancherel)
 partial derivative of generalized
 function, 438
 partial fractions, 143, 416
 partition of unity, 171, 446, 622
 PDE, *xiii*, 523, 587
 periodic function, 4, 8, 33, 440
 for ear, 697
 periodization, 32, 535, 550, 569, 671
 phase deaf, 697, 729
 pi, computation of, 113–114
 piano
 equitempered scale, 699
 for harmonic synthesis, 73
 keyboard frequencies, A37
 piecewise
 constant, 81
 continuous, 26, 76–77, 81, 121–123, 126–128
 polynomial, 117, 145, 170, 380, 382–383, 463, 623
 smooth, 39, 42, 45, 55, 57, 83–85, 123, 406, 491, 505, 623
 pitch perception, 694
 Plancherel identities, 24
 via autocorrelation, 162
 for evaluating integrals, 148
 for evaluating sums, 190, 221, 224, 226, 491
 validity, 30, 76, 82–83
 Poisson probability density, 781–782
 Poisson process, 782
 Poisson relations, 33–36
 for evaluating sums, 149
 for finding Fourier series, 179, 262, 478, 488
 for unifying Fourier analysis, 36–37
 Poisson sum formula, 39, 50, 393, 488
 polarization identity, 24, 74
 polygon function, 191
 power functions, 329, 395, 401, 456
 truncated, 382
 power scaling rule, 142, 413
 primitive root, 238
 probability density function, 122, 154, 739 (*see also* random variable)
 Benford, 798
 Bernoulli, 767, 769, 776, 779
 binomial, 239
 bivariate, 764
 Cauchy, 758, 760, 775
 from characteristic function, 741
 chi squared, 791
 closure, 787
 coin flip, 740, 742, 777
 convolution of, 765
 die-toss, 739, 742, 754, 756, 773
 Dirac, 757, 775
 from distribution function, 740
 gamma, 781
 Laplace, 739, 781
 Maxwell, 737, 791
- probability density function (*cont.*)
 Poisson, 739, 781
 standard normal, 139, 150, 739–740, 754, 758, 768
 for sum of random numbers, 765–766, 768
 truncated exponential, 742, 757, 771, 777
 uniform, 739, 742
 products
 of generalized functions, 398
 of probability functions, 750
 projection operators, 245, 253–254, 277–278, 281, 642
 Ptolemy, C., 12, 699
 pulse amplitude, 511
 Pythagoras and music, 698, 729–730
- quadratic residue**, 237
 quantization of samples, 484
 quantum mechanics
 Schrödinger equation, 558
 uncertainty relation, 762
 wave packet for free particle, 563
- R for real numbers**, 3
 ramp function, 227, 381
 random number generator, 194, 718
 random variables, 753
 characteristic function for, 759, 767
 generation of, 194, 718, 795
 independent, 764
 joint density, 764
 max, min of, 790
 via probability density, 253
 sum of independent, 764
 random walk, 777
 rational function, 143, 159, 416–419, 465
 real world sampling theorem, 507
 reciprocity relations, 69
 recursion (*see* recursion)
 recursive algorithm for FFT, 350–351
 reduced wave function, 556
 reflection of light at mirror, 569
 reflection operator, 240, 243
 tag, 251
 reflection rule, 135, 177, 199, 413
 regular tails, 48, 53, 55, 82, 85
 relatively prime, 206, 236
 repeat rule, 202, 232, 257, 261
 response of LTI system, 282, 419, 470, 471
 Riemann sum, 39, 44
 for Fourier coefficient, 79
 for Fourier transform, 79
 and Gibbs phenomenon, 44
 Riemann–Lebesgue lemma, 81, 458
 Risset's glissando, 725
 Rodrigues formula, 151
 rules
 for derivatives of generalized
 functions, 405
 for Fourier transforms of functions
 on \mathbb{P}_N , 199–212, A17–A18
 on \mathbb{R} , 132–147, A14
 on \mathbb{T}_p , \mathbb{Z} , 176–182, 187–190, A15–A16

I-6 Index

- rules (*cont.*)
 for Fourier transforms of generalized functions, 413
 for manipulation of generalized functions, 389
- S** for Schwartz functions, 372
 same sign shift, 137 (*see also* mnemonic)
 sample-sum rule, 210, 212, 284
 samples for Daubechies wavelet, 652, 687, 689
 sampling, 32, 483
 rate for audio, 484
 for wavelet analysis, 649, 668, 685
 sampling function (*see* comb function)
 sampling rule, 210, 214, 262
 sampling theorem
 for almost bandlimited functions, 505
 when F is piecewise smooth, 491, 497
 using filters, 498, 501, 503
 using fragments of Π , 495
 for generalized functions, 487
 for real-world signals, 507
 scale for music, 697–700, 729
 scale for wavelet approximation, 595–596
 scaling function for wavelets, 597, 609
 Schoenberg, I., 19, 72, 125
 Schrödinger's equation, *xiii*, 558
 Schwartz, L., *xii*, 368, 451
 Schwartz functions, 372–374, 482
 closure of, 375
 semitone, 699
 Shah function (*see* comb function)
 Shannon, C., *xiii*, 484, 491
 Shannon's sampling theorem, 491
 shift rule (*see* translation rule)
 shuffle permutation, 312, 332
 action, 315, 332, 341–342
 operator identities, 314, 333, 342, 364
 products for FFT, 314, 365
 signum function, 228, 380, 424
 for Hilbert transform, 267
 sin operator, 63, 67, 246–247
 sin transform, 63, 67, 247, 287
 sinc function, 130
 properties, 141, 160, 493, 514–515
 singularity function
 on \mathbb{R} , 51–52, 55, 82–83
 on \mathbb{T}_p , 41–42, 46
 slowly growing function, 376
 slowly growing sequence, 444
 smoothness of
 B-spline, 117, 193
 convolution product, 107, 123, 126, 128, 401
 Fourier transform, 48, 81, 153, 169, 454
 solution of diffusion equation, 542
 solution of dilation equation, 616–619
 soil temperature, 547, 586
 sparse matrix factorization, 311, 675
 spectral
 density, 719–720
 enrichment, 715, 731
 factorization, 634, 682
- spectrogram
 for bell tone, 616, 709
 generation of, 702–704
 for Risset's glissando, 726
 for shaped noise, 721, 723
 for Twinkle, Twinkle, 703
- spectroscopy, 21
- spectrum of arginine, 22
- standard deviation σ , 756
- standard normal probability distribution, A33
- step response, 470, 471
- Stockham's autosort FFT, 344, 366
- Strang, G., *xii*, A1
- structure of this book, *xi*
- suitably regular, 3
- sum of independent random variables, 765
 and central limit theorem, 771–779
 mean, variance for, 767
 probability density for, 766
- summation rule, 204, 209, 258, 262
- support-limited function, 426
- support-limited wavelets, 609
- synthesis
 using bandlimited functions, 75–76, 84, 489, 491, 497, 499, 503
 using cas, 249
 using complex exponentials, 3, 5, 7, 10, 430, 440
 using sin, cos, *ix*, 67, 247
 using solutions of PDE, 15, 524, 582
 using wavelets, 594, 597, 684
- symmetry
 for boundary conditions, 535, 545, 550–552, 569, 585
 for deriving the Maxwell density, 791
 via Fourier transforms, 62, 64, 66–67, 477
 via operators, 239, 254–256, 277
 for solutions of PDEs, 575, 584, 590
- T_p** for circle, 4
- tag notation, 251–252, 267, 286, 607
- tapered box, 445–447, 488
- Tartaglia formula for roots of cubic, 19, 72
- Taylor's formula, 122, 624, 685, 786
- temperature, 15–16, 540–553
- test functions, 451
- thick coin, 779
- timbre of tone, 707
- transformation of music theme, 724–725
- transformation shift rule (*see* modulation rule)
- translation invariant, 17, 282 (*see also* LTI)
- translation rule, 136, 140, 177, 199, 413
- trapazoid rule
 and Euler–Maclaurin formula, 213
 and Fourier coefficients, 234
- tree diagram for FFT, 298, 302
- triangle function, 144
- truncated exponential, 131, 143, 165, 470
- truncated power function, 382
- uncertainty relation**, 736, 761, 792
- unification of Fourier analysis
 via Fourier–Poisson cube, 36
 via generalized functions, 448–550
- unit gaussian function, 132
- units for s , x , k , n , 68
- universal constant β , 244
- upsampling (*see* zero packing)
- validity of Fourier's representation**
 for generalized functions, 413, 440–441, 450
 impact on mathematics, A1–A3
 for ordinary functions, 37–58, 77, 81–82, 85
- validity of wavelet representation, 600, 684
- variance σ^2 , 756, 759
 for sum of random variables, 766–767
- vector operation for FFT, 343
- velocity of traveling wave
 group, 589
 phase, 566, 589
 on string, 528
 on water, 591
- vibrating string, 523
 bowed, plucked, struck, 537, 582
 discretized, 580
 equation of motion, 526
 frequency, 536
 normal vibrational modes, 536
 overtones, 578
 send message, 526
 shake to rest, 577
 with stiffness, 581
 tone synthesis, 539, 710, 731
- Vietta's formula, 570
- water waves**, 554, 591
- wave equation, 523, 527
 with boundary conditions, 532, 535
 conservation of energy, 531
 derivation, 526
- Fourier synthesis, 524, 532
 initial conditions, 527, 531, 587
- kernel, 528, 532, 575, 576
- polynomial solutions, 574
- symmetry, 575
- traveling solution, 529–530, 534, 575–576
- velocity, 592
- wavelet, 594
 analysis equation, 600, 640
 coefficients, 609
 continuity, 683
 Daubechies, 610, 614
 via dilation equation, 609
 frame-detail, 598–604
 Haar's prototype, 594
 mother-father, 594, 597
 music score, 597, 600, 674
 samples, 687

-
- | | | |
|---|---|--|
| <p>wavelet (<i>cont.</i>)</p> <ul style="list-style-type: none"> scaling function, 597, 609 having smoothness, 616, 683 support-limited, 594, 616, 677 synthesis equation, 594, 684 vs wave, 593 <p>weak limit, 428</p> <ul style="list-style-type: none"> for central limit theorem, 772 for “continuity”, 431 for derivative, 430 for Fourier series, 440–441, 450 for initial conditions, 587 for partial derivative, 438 | <p>weak limit (<i>cont.</i>)</p> <ul style="list-style-type: none"> for sampling theorem, 489 for solving dilation equation, 614 for solving PDEs, 587 transformations of, 434 <p>Weierstrass theorem, 26, 29, 76, 78</p> <p>Weierstrass tone, 731</p> <p>Weyl's equidistribution theorem, 195</p> <p>Whittaker–Robinson flowchart for harmonic analysis, 349, A23</p> <p>Wiener's series for Fourier analysis, 167</p> | <p>window for DFT, 229, 705</p> <p>Wirtinger's inequality, 226, 231</p> <p>Young's double slit, 560, 588</p> <p>Z for integers, 5</p> <p>zero packing (upsampling), 187, 201, 232, 257, 261, 656</p> <p>Zipper identity, 314</p> <ul style="list-style-type: none"> for FFT, 312, 327 for FHT, 323, 359 as Kronecker product, 344 |
|---|---|--|