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Elements of the Representation Theory of Associative Algebras

Volume 3 Representation-Infinite Tilted Algebras

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To our Wives

Sabina and Mirosława

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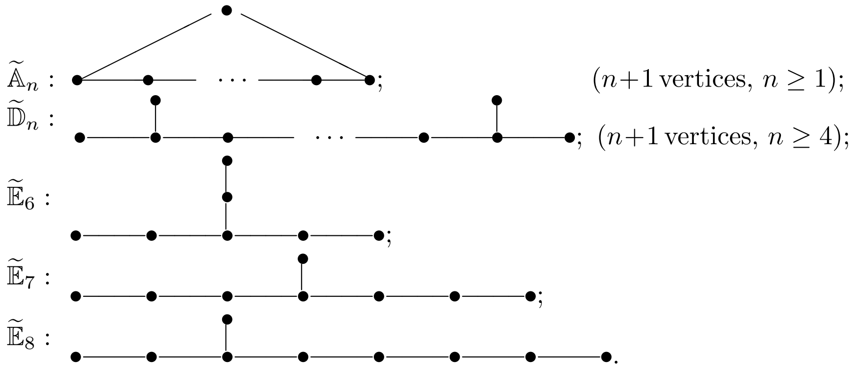
Introduction

The first volume serves as a general introduction to some of the techniques most commonly used in representation theory. The quiver technique, the Auslander–Reiten theory and the tilting theory were presented with some application to finite dimensional algebras over a fixed algebraically closed field. In particular, a complete classification of those hereditary algebras that are representation-finite (that is, admit only finitely many isomorphism classes of indecomposable modules) is given. The result, known as Gabriel's theorem, asserts that a basic connected hereditary algebra A is representation-finite if and only if the quiver Q_A of A is a Dynkin quiver.

In Volume 2 we study in detail the indecomposable modules and the shape of the Auslander–Reiten quiver $\Gamma(\operatorname{mod} A)$ of the class of hereditary algebras A that are representation-infinite and minimal with respect to this property. They are just the hereditary algebras of Euclidean type, that is, the path algebras KQ, where Q is a connected acyclic quiver whose underlying non-oriented graph $\overline{Q}$ is one of the following Euclidean diagrams



In Volume 2, we also study in detail the indecomposable modules and the shape of the Auslander–Reiten quiver $\Gamma(\operatorname{mod} B)$ of concealed algebras of Euclidean type, that is, the tilted algebras B of the form

$$B = \operatorname{End} T_{KQ},$$

where KQ is a hereditary algebra of Euclidean type and T_{KQ} is a postprojective tilting KQ-module.

The main aim of the first part of Volume 3 is to study arbitrary representation-infinite tilted algebras $B = \operatorname{End} T_{KQ}$ of a Euclidean type Q, where T_{KQ} is a tilting T_{KQ}-module, and to give a fairly complete description of their indecomposable modules, their module categories $\operatorname{mod} B$, and the Auslander–Reiten quivers $\Gamma(\operatorname{mod} B)$.

For this purpose, we introduce in Chapters XV-XVII some concepts and tools that allow us to give in Chapter XVII a complete description of arbitrary representation-infinite tilted algebras B of Euclidean type and

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their module categories mod B, due to Ringel [525]. In particular, we show that:

- the Auslander–Reiten quiver $\Gamma(\mathrm{mod} B)$ of any such an algebra B has a disjoint union decomposition

$$\Gamma(\mathrm{mod} B) = \mathcal{P}(B) \cup \boldsymbol{\mathcal{T}}^B \cup \mathcal{Q}(B),$$

where $\mathcal{P}(B)$ is a unique postprojective component, $\mathcal{Q}(B)$ is a unique preinjective component, and

$$\boldsymbol{\mathcal{T}}^B = \{\mathcal{T}_\lambda^B\}_{\lambda \in \mathbb{P}_1(K)}$$

is a $\mathbb{P}_1(K)$-family of pairwise orthogonal standard ray or coray tubes $\mathcal{T}_\lambda^B$ separating $\mathcal{P}(B)$ from $\mathcal{Q}(B)$;

- the module category mod B of a tilted algebra B of Euclidean type is controlled by the Euler quadratic form $q_B : K_0(B) \longrightarrow \mathbb{Z}$ of B, and

- the number of the isomorphism classes of tilted algebras of Euclidean type of any fixed dimension is finite.

In Chapter XVIII, we turn our attention to the representation theory of wild hereditary algebras $A = KQ$, where Q is an acyclic quiver such that the underlying graph is neither a Dynkin nor a Euclidean diagram. The shape of the components of the regular part $\mathcal{R}(A)$ of $\Gamma(\mathrm{mod}\, A)$ is described and, for any such an algebra A, a wild behaviour of the category mod A is established. Moreover, an important theorem on homomorphisms between the regular modules over a wild hereditary algebra, due to Baer [35] and Kerner [343], is proved.

An essential rôle in the investigation is played by the notion of a perpendicular category associated to a partial tilting module, introduced by Geigle and Lenzing [247] and Schofield [559].

We also exhibit some classes of tilted algebras B of wild type and we discuss the structure of their module categories mod B. In particular, we prove a theorem of Ringel [526] on the existence of a regular tilting module over a hereditary algebra, and we present an efficient procedure of Baer [35], [36] allowing us to construct regular tilting modules over any wild hereditary algebra A with at least three pairwise non-isomorphic simple modules.

In Chapter XIX, we introduce the concepts of tame representation type and of wild representation type for algebras, and we discuss the tame and the wild nature of module categories mod B. We prove that the concealed algebras of Euclidean type are of tame representation type, and the concealed algebras of wild type are of wild representation type.

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In the final Chapter XX, we present (without proofs) selected results of the representation theory of finite dimensional algebras that are related to the material discussed in the previous chapters. This, together with a rather long list of complementary references, should provide the reader with the right tives for further study and interesting research directions.

Unfortunately, many important topics from the theory have been left out. Among the most notable omissions are covering techniques, the use of derived categories and partially ordered sets. Some other aspects of the theory presented here are discussed in the books [34], [53], [54], [242], [318], [276], [575], and especially [525].

We assume that the reader is familiar with Volumes 1 and 2, but otherwise the exposition is reasonably self-contained, making it suitable either for courses and seminars or for self-study. The text includes many illustrative examples and a large number of exercises at the end of each of the Chapters XV-XIX.

The book is addressed to graduate students, advanced undergraduates, and mathematicians and scientists working in representation theory, ring and module theory, commutative algebra, abelian group theory, and combinatorics. It should also, we hope, be of interest to mathematicians working in other fields.

Throughout this book we use freely the terminology and notation introduced in Volumes 1 and 2. We denote by K a fixed algebraically closed field. The symbols $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, and $\mathbb{C}$ mean the sets of natural numbers, integers, rational, real, and complex numbers. The cardinality of a set X is denoted by $|X|$. Given an algebra A, the A-module means a finite dimensional right A-module. We denote by $\operatorname{Mod} A$ the category of all right A-modules, by $\operatorname{mod} A$ the category of finite dimensional right A-modules, and by $\Gamma(\operatorname{mod} A)$ the Auslander–Reiten translation quiver of A. The ordinary quiver of an algebra A is denoted by Q_A. Given a matrix $C = [c_{ij}]$, we denote by C^t the transpose of C.

A finite quiver $Q = (Q_0, Q_1)$ is called a **Euclidean quiver** if the underlying graph $\overline{Q}$ of Q is any of the Euclidean diagrams $\widetilde{\mathbb{A}}_m$, with $m \geq 1$, $\widetilde{\mathbb{D}}_m$, with $m \geq 4$, $\widetilde{\mathbb{E}}_6$, $\widetilde{\mathbb{E}}_7$, and $\widetilde{\mathbb{E}}_8$. Analogously, Q is called a **Dynkin quiver** if the underlying graph $\overline{Q}$ of Q is any of the Dynkin diagrams $\mathbb{A}_m$, with $m \geq 1$, $\mathbb{D}_m$, with $m \geq 4$, $\mathbb{E}_6$, $\mathbb{E}_7$, and $\mathbb{E}_8$.

We take pleasure in thanking all our colleagues and students who helped us with their useful comments and suggestions. We wish particularly to express our appreciation to Ibrahim Assem, Sheila Brenner, Otto Kerner, and Kunio Yamagata for their helpful discussions and suggestions. Particular thanks are due to Dr. Jerzy Białkowski and Dr. Rafał Bocian for their help in preparing a print-ready copy of the manuscript.

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