

Position, displacement, distance

Vectors and scalars

Scalars are physical quantities with magnitude only, e.g. distance, speed, time, energy, mass, volume, work, power, electric charge.

Vectors are physical quantities with magnitude and direction, e.g. displacement, velocity, acceleration, force, weight, momentum, electric field strength.

Position

Position is the place occupied by an object. It is defined relative to a reference point. When an object changes its position, motion occurs. The positive direction for motion must always be indicated.

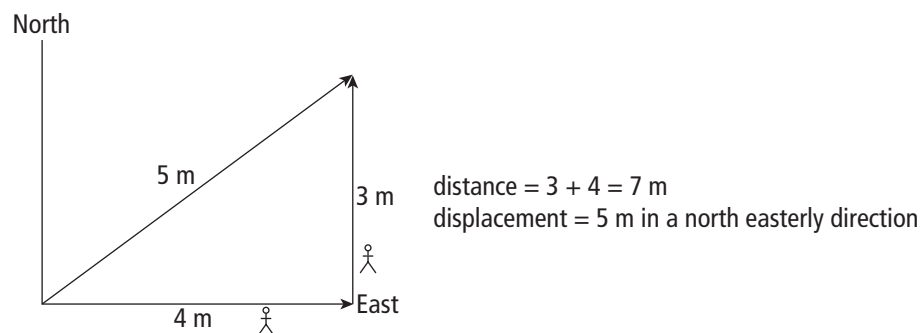
Distance and displacement

Distance and displacement is a change in position.

Distance is the actual path length that an object moves away from its original position. Distance is a scalar. We use the symbol d for distance.

Displacement is the straight-line path between the starting point and the endpoint of a journey – i.e. the distance moved in a particular direction. Displacement is a vector. Displacement can be positive or negative, depending on which direction was taken as positive. Negative displacement is displacement in the direction opposite to the positive direction. We use the symbol Δx to indicate displacement. (Δ is used for ‘change in’.)

Example



Speed, average velocity, instantaneous velocity

Speed

Speed is a measurement of the distance covered during a specific time period.

$$\text{average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

Speed is a scalar.

Instantaneous speed is how fast an object is moving at a particular moment.

Unit 1 • Motion in one dimension

3

Velocity

Velocity is a measurement of the rate of change in displacement – the speed in a particular direction. We use the symbol v to indicate velocity and t to indicate time.

$$\text{velocity} = \frac{\text{displacement}}{\text{time taken}} \qquad v = \frac{\Delta x}{\Delta t}$$

Velocity is a vector and has the unit $\text{m}\cdot\text{s}^{-1}$.

Instantaneous velocity is the velocity at a particular moment. To calculate instantaneous velocity you need to know the change in position, Δx , over a very short time interval, t .

Average velocity is the displacement for the whole motion divided by the time taken for the whole motion.

Uniform or constant velocity is the velocity of an object covering equal distances in equal time intervals, with the magnitude and/or direction not changing.

Positive velocity is velocity in the positive direction.

Negative velocity is velocity in the direction opposite to the positive direction.

Example

The journey in the example on the previous page takes 2 s. Calculate the average speed and average velocity.

$$\begin{aligned} \text{average speed} &= \frac{\text{distance travelled}}{\text{time taken}} \\ &= \frac{7 \text{ m}}{2 \text{ s}} \\ &= 3,5 \text{ m}\cdot\text{s}^{-1} \end{aligned}$$

$$\begin{aligned} \text{average velocity} &= \frac{\text{total displacement}}{\text{time taken}} \quad v = \frac{\Delta x}{\Delta t} \\ &= \frac{5 \text{ m}}{2 \text{ s}} \\ &= 2,5 \text{ m}\cdot\text{s}^{-1} \text{ in a north easterly direction} \end{aligned}$$

Conversion between units

Distance and displacement are measured in metres (m). To convert from other units to metres, first decide if the unit is greater or smaller than metres. If the unit is greater, multiply to get metres, e.g. $5 \text{ km} = 5 \times 1\,000 = 5\,000 \text{ m}$. If the unit is smaller than metres, divide, e.g. $1\,000 \text{ cm} \div 100 = 10 \text{ m}$

Refer to page xi of the Introduction to see how to convert in the decimal system.

Acceleration

Acceleration is the rate of change in velocity. An object accelerates when the speed and/or the direction of the object changes, e.g. an object that is moving in a circle at constant speed is accelerating, because its direction is changing constantly. We use the symbol a to indicate acceleration.

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{time taken}} \qquad a = \frac{\Delta v}{\Delta t}$$

Acceleration is a vector and its unit is metres per second squared ($\text{m}\cdot\text{s}^{-2}$).

Acceleration can be positive or negative:

Positive acceleration

- * an object's velocity is increasing in the positive direction – it is going forwards faster
- * an object's velocity is decreasing in a negative direction – it is going backwards more slowly

Negative acceleration

- * an object's velocity is decreasing in the positive direction – it is going forwards more slowly
- * an object's velocity is increasing in the negative direction – it is going backwards faster

Uniform or constant acceleration is the increase of velocity of an object by the same amount in each time interval. We will discuss only motion at uniform acceleration.

Description of motion

Describing motion in words

The motion of an object can be described in words.

- * Always describe the type of motion or shape of a graph in terms of velocity and acceleration.
- * Distinguish between motion at constant (uniform) velocity and motion at constant (uniform) acceleration.
- * State if displacement, velocity and acceleration are positive or negative.
- * Use positive and negative signs to show the direction of displacement, velocity and acceleration.
- * Give values where possible.

Describing motion in diagrams

Diagrams can be used to clarify and enhance the explanation of motion in words.

Describing motion in graphs

There are three types of graph: position–time graphs, velocity–time graphs and acceleration–time graphs.

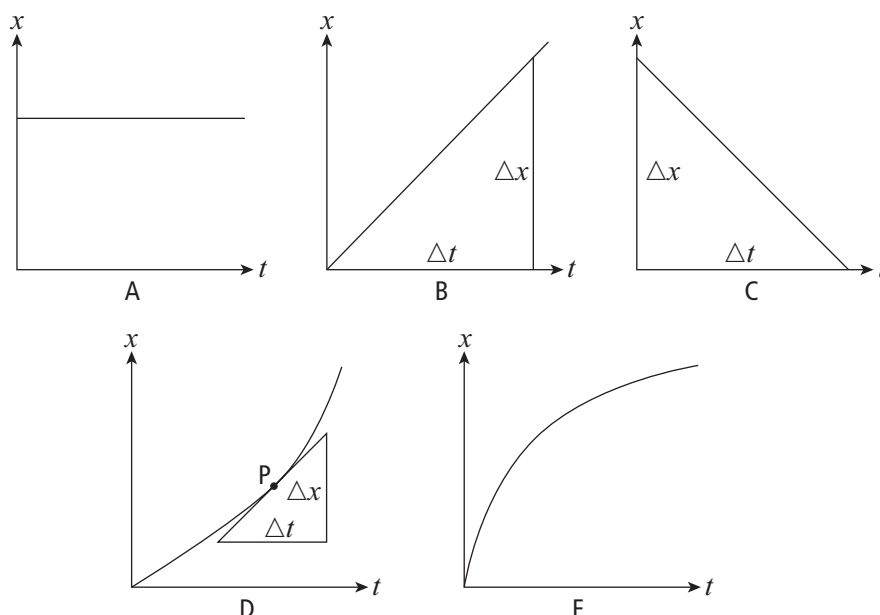
Unit 1 • Motion in one dimension

5

Position–time graphs

The y -axis depicts the change in position (distance or displacement) of an object and the x -axis shows the time taken.

- * Position–time graphs for constant motion ($v = \text{constant}$; $a = 0$) are straight lines as shown in diagrams B and C.
- * Position–time graphs for accelerating objects (v increasing or decreasing; $a = \text{constant positive or negative}$) form curves as shown in diagrams D and E.



- A: object stationary
 B: object increasing its displacement at a constant rate
 C: object decreasing its displacement at a constant rate
 D: object increasing its displacement at an accelerated rate
 E: object decreasing its displacement at a decelerated rate

We calculate the slope of the line to determine the **constant velocity** of the object in diagrams B or C.

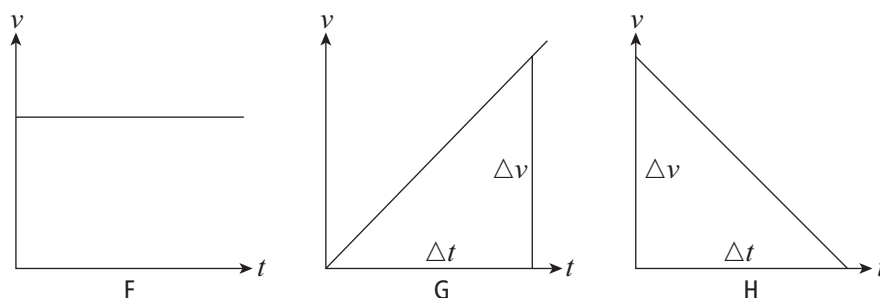
$$\text{slope (gradient)} = \frac{\text{displacement}}{\text{time taken}} \quad v = \frac{\Delta x}{\Delta t}$$

B has a positive slope and positive velocity; C has a negative slope and negative velocity.

To determine the **instantaneous velocity** at point P during the motion, we draw a tangent to the graph at point P and find its gradient.

Velocity–time graphs

The y -axis depicts the change in velocity of an object and the x -axis shows the time taken. There are three possibilities:



F: velocity is constant, $a = 0$

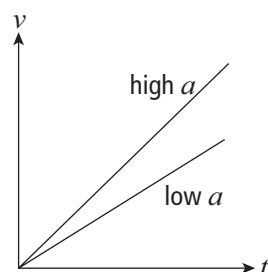
G: velocity is increasing at a constant rate

H: velocity is decreasing at a constant rate

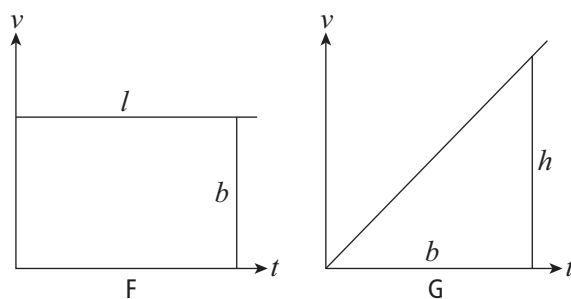
We calculate the slope of the line to determine the **acceleration** from the velocity–time graph.

$$\text{slope (gradient)} = \frac{\text{change in velocity}}{\text{time taken}} \quad a = \frac{\Delta v}{\Delta t}$$

The slope of the graph shows how fast the velocity is changing.



We calculate the area under the graph to determine **displacement** from a velocity–time graph.



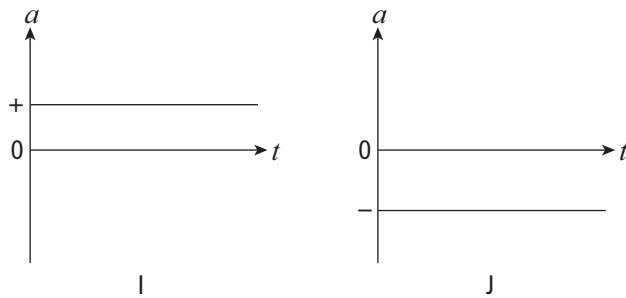
F: displacement = area under graph = $l \times b$

G: displacement = $\frac{1}{2}bh$

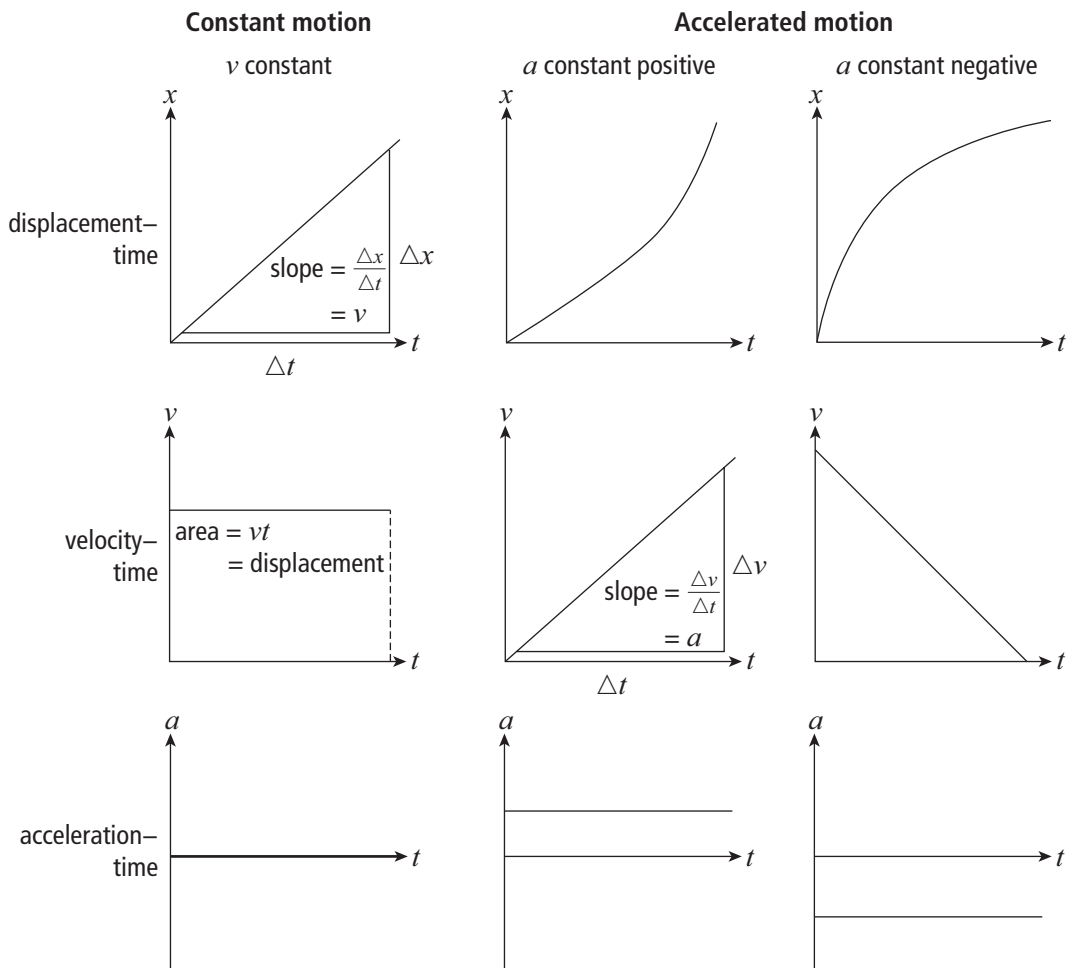
Acceleration–time graphs

Motion with constant acceleration gives a horizontal line with a positive value. A horizontal line with a negative value indicates deceleration.

Unit 1 • Motion in one dimension



I: uniform positive acceleration
 J: uniform negative acceleration

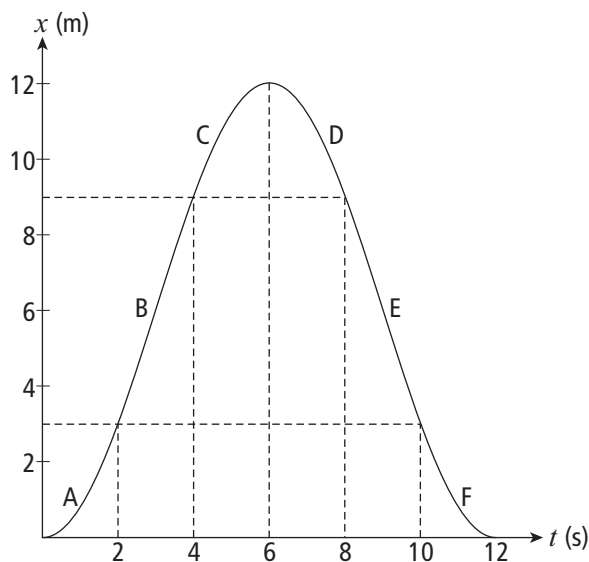


Summary of graphs

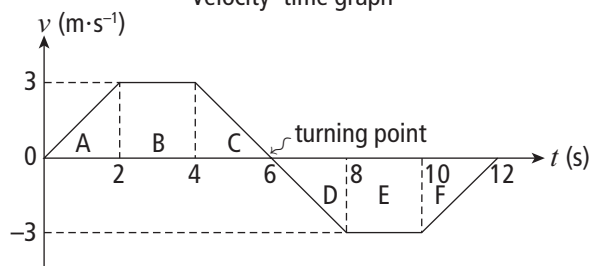
Example

Sarah mounts her bicycle to ride to school. She pulls away from home and accelerates down the road when she remembers that she forgot to pack her science homework. She slows down, turns around and rides back home. Her journey can be represented graphically.

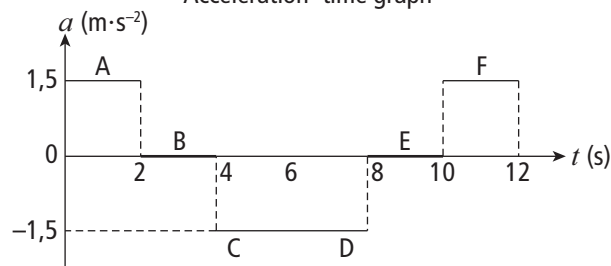
Position–time graph



Velocity–time graph



Acceleration–time graph



From the velocity–time graph we can calculate Sarah's acceleration:
 acceleration = gradient of the line

For section A:

$$\begin{aligned}
 a &= \frac{\Delta v}{\Delta t} \\
 &= \frac{3 \text{ m}\cdot\text{s}^{-1}}{2 \text{ s}} \\
 &= 1,5 \text{ m}\cdot\text{s}^{-2}
 \end{aligned}$$

Unit 1 • Motion in one dimension

9

We can also calculate the distance travelled and her displacement:

distance = total area under the graph

$$\begin{aligned}
 &= \text{area A} + \text{area B} + \text{area C} + \text{area D} + \text{area E} + \text{area F} \\
 &= \frac{1}{2}(3 \text{ m}\cdot\text{s}^{-1})(2 \text{ s}) + (3 \text{ m}\cdot\text{s}^{-1})(2 \text{ s}) + \frac{1}{2}(3 \text{ m}\cdot\text{s}^{-1})(2 \text{ s}) + \frac{1}{2}(3 \text{ m}\cdot\text{s}^{-1})(2 \text{ s}) \\
 &\quad + (3 \text{ m}\cdot\text{s}^{-1})(2 \text{ s}) + \frac{1}{2}(3 \text{ m}\cdot\text{s}^{-1})(2 \text{ s}) \\
 &= 24 \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 \text{displacement} &= \text{dist. in a positive direction} - \text{dist. in a negative direction} \\
 &= 12 \text{ m} - 12 \text{ m} \\
 &= 0
 \end{aligned}$$

Describing motion in equations

The equations of motion are a set of equations that allow us to calculate the quantities involved when an object is moving with a constant acceleration.

The four equations of motion are:

$$\begin{aligned}
 v_f &= v_i + a\Delta t & \text{or} & & v &= u + a\Delta t \\
 v_f^2 &= v_i^2 + 2a\Delta x & \text{or} & & v^2 &= u^2 + 2a\Delta x \\
 \Delta x &= v_i \Delta t + \frac{1}{2}a\Delta t^2 & \text{or} & & \Delta x &= u\Delta t + \frac{1}{2}a\Delta t^2 \\
 \Delta x &= \left(\frac{v_i + v_f}{2}\right) \Delta t & \text{or} & & \Delta x &= \left(\frac{u + v}{2}\right) \Delta t
 \end{aligned}$$

v_i or u = initial velocity

v_f or v = final velocity

a = acceleration

Δx = displacement

Δt = time

These equations only apply:

- * for rectilinear motion (motion in a straight line)
- * to an object moving with a constant acceleration.

Use positive and negative signs to show direction of displacement, velocity and acceleration.

Note: When an object is travelling at constant velocity, its acceleration is zero. Then we use the equation $v = \frac{\Delta x}{\Delta t}$ to calculate constant velocity, displacement or time.

Follow this procedure when solving problems with equations of motion:

Step 1: Write down the quantities given and the quantity that must be calculated.

Step 2: Choose the equation that links these quantities. This will be the one where all the quantities are known, except for the unknown asked quantity that you need to find.

Step 3: Substitute the values into the equation. You can leave out the units when substituting into the equation.

Step 4: Calculate the unknown quantity. It is sometimes easier to substitute first and then to change the subject of the formula.

Example

A car is travelling at $17 \text{ m}\cdot\text{s}^{-1}$ when the driver notices that the traffic light has turned to orange. The driver takes $0,7 \text{ s}$ to react before he starts to brake. The car then decelerates at $9 \text{ m}\cdot\text{s}^{-2}$ to stop in time before the traffic light turns red.

- How fast is the car going in $\text{km}\cdot\text{h}^{-1}$ when the driver sees the traffic light?
- How far will the car travel before the driver starts to apply the brakes?
- What distance does the car travel while the driver is applying the brakes to stop?
- What is the total distance needed for the driver to stop the car from when he sees the traffic light until he comes to a halt?
- What is the total time needed to stop the car from when the driver sees the traffic light until he comes to a halt?

Solution:

a $17 \text{ m}\cdot\text{s}^{-1} \times 3,6 = 61,2 \text{ km}\cdot\text{h}^{-1}$

b $\Delta x = v\Delta t$
 $= (17 \text{ m}\cdot\text{s}^{-1})(0,7 \text{ s})$
 $= 11,9 \text{ m}$

c $v_f^2 = v_i^2 + 2a\Delta x$ $v_i = 17 \text{ m}\cdot\text{s}^{-1}$
 $0 = (17 \text{ m}\cdot\text{s}^{-1})^2 + 2(9 \text{ m}\cdot\text{s}^{-2})\Delta x$ $v_f = 0$
 $\therefore \Delta x = 16 \text{ m}$ $a = 9 \text{ m}\cdot\text{s}^{-2}$

d total $x = 11,9 \text{ m} + 16 \text{ m}$
 $= 27,9 \text{ m}$

e $v_f = v_i + a\Delta t$
 $0 = 17 \text{ m}\cdot\text{s}^{-1} + (9 \text{ m}\cdot\text{s}^{-2})\Delta t$
 $\therefore \Delta t = 1,9 \text{ s}$
 total $t = 0,7 \text{ s} + 1,9 \text{ s}$
 $= 2,6 \text{ s}$

Frames of reference

We need to choose an origin (starting point or zero point) and a set of directions before we can determine an object's displacement or velocity. Any measurement of displacement or velocity must be made with respect to a frame of reference. Use the sets of directions on the next page: