A Student's Guide to Maxwell's Equations

Maxwell's Equations are four of the most influential equations in science: Gauss's law for electric fields, Gauss's law for magnetic fields, Faraday's law, and the Ampere–Maxwell law. In this guide for students, each equation is the subject of an entire chapter, with detailed, plain-language explanations of the physical meaning of each symbol in the equation, for both the integral and differential forms. The final chapter shows how Maxwell's Equations may be combined to produce the wave equation, the basis for the electromagnetic theory of light.

This book is a wonderful resource for undergraduate and graduate courses in electromagnetism and electromagnetics. A website hosted by the author, and available through www.cambridge.org/9780521877619, contains interactive solutions to every problem in the text. Entire solutions can be viewed immediately, or a series of hints can be given to guide the student to the final answer. The website also contains audio podcasts which walk students through each chapter, pointing out important details and explaining key concepts.

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Preface

This book has one purpose: to help you understand four of the most influential equations in all of science. If you need a testament to the power of Maxwell's Equations, look around you – radio, television, radar, wireless Internet access, and Bluetooth technology are a few examples of contemporary technology rooted in electromagnetic field theory. Little wonder that the readers of *Physics World* selected Maxwell's Equations as "the most important equations of all time."

How is this book different from the dozens of other texts on electricity and magnetism? Most importantly, the focus is exclusively on Maxwell's Equations, which means you won't have to wade through hundreds of pages of related topics to get to the essential concepts. This leaves room for in-depth explanations of the most relevant features, such as the difference between charge-based and induced electric fields, the physical meaning of divergence and curl, and the usefulness of both the integral and differential forms of each equation.

You'll also find the presentation to be very different from that of other books. Each chapter begins with an "expanded view" of one of Maxwell's Equations, in which the meaning of each term is clearly called out. If you've already studied Maxwell's Equations and you're just looking for a quick review, these expanded views may be all you need. But if you're a bit unclear on any aspect of Maxwell's Equations, you'll find a detailed explanation of every symbol (including the mathematical operators) in the sections following each expanded view. So if you're not sure of the meaning of $\vec{E} \circ \hat{n}$ in Gauss's Law or why it is only the enclosed currents that contribute to the circulation of the magnetic field, you'll want to read those sections.

As a student's guide, this book comes with two additional resources designed to help you understand and apply Maxwell's Equations: an interactive website and a series of audio podcasts. On the website, you'll find the complete solution to every problem presented in the text in interactive format – which means that you'll be able to view the entire solution at once, or ask for a series of helpful hints that will guide you to the final answer. And if you're the kind of learner who benefits from hearing spoken words rather than just reading text, the audio podcasts are for you. These MP3 files walk you through each chapter of the book, pointing out important details and providing further explanations of key concepts.

Is this book right for you? It is if you're a science or engineering student who has encountered Maxwell's Equations in one of your textbooks, but you're unsure of exactly what they mean or how to use them. In that case, you should read the book, listen to the accompanying podcasts, and work through the examples and problems before taking a standardized test such as the Graduate Record Exam. Alternatively, if you're a graduate student reviewing for your comprehensive exams, this book and the supplemental materials will help you prepare.

And if you're neither an undergraduate nor a graduate science student, but a curious young person or a lifelong learner who wants to know more about electric and magnetic fields, this book will introduce you to the four equations that are the basis for much of the technology you use every day.

The explanations in this book are written in an informal style in which mathematical rigor is maintained only insofar as it doesn't get in the way of understanding the physics behind Maxwell's Equations. You'll find plenty of physical analogies – for example, comparison of the flux of electric and magnetic fields to the flow of a physical fluid. James Clerk Maxwell was especially keen on this way of thinking, and he was careful to point out that analogies are useful not because the *quantities* are alike but because of the corresponding *relationships between quantities*. So although nothing is actually flowing in a static electric field, you're likely to find the analogy between a faucet (as a source of fluid flow) and positive electric charge (as the source of electric field lines) very helpful in understanding the nature of the electrostatic field.

One final note about the four Maxwell's Equations presented in this book: it may surprise you to learn that when Maxwell worked out his theory of electromagnetism, he ended up with not four but *twenty* equations that describe the behavior of electric and magnetic fields. It was Oliver Heaviside in Great Britain and Heinrich Hertz in Germany who combined and simplified Maxwell's Equations into four equations in the two decades after Maxwell's death. Today we call these four equations Gauss's law for electric fields, Gauss's law for magnetic fields, Faraday's law, and the Ampere–Maxwell law. Since these four laws are now widely defined as Maxwell's Equations, they are the ones you'll find explained in the book.

Acknowledgments

This book is the result of a conversation with the great Ohio State radio astronomer John Kraus, who taught me the value of plain explanations. Professor Bill Dollhopf of Wittenberg University provided helpful suggestions on the Ampere–Maxwell law, and postdoc Casey Miller of the University of Texas did the same for Gauss's law. The entire manuscript was reviewed by UC Berkeley graduate student Julia Kregenow and Wittenberg undergraduate Carissa Reynolds, both of whom made significant contributions to the content as well as the style of this work. Daniel Gianola of Johns Hopkins University and Wittenberg graduate Melanie Runkel helped with the artwork. The Maxwell Foundation of Edinburgh gave me a place to work in the early stages of this project, and Cambridge University made available their extensive collection of James Clerk Maxwell's papers. Throughout the development process, Dr. John Fowler of Cambridge University Press has provided deft guidance and patient support.