

Cambridge University Press

978-0-521-70138-9 - Applied Complex Variables for Scientists and Engineers, Second Edition

Yue Kuen Kwok

Table of Contents

[More information](#)

Contents

<i>Preface</i>	<i>page ix</i>
1 Complex Numbers	1
1.1 Complex numbers and their representations	1
1.2 Algebraic properties of complex numbers	4
1.2.1 De Moivre's theorem	7
1.3 Geometric properties of complex numbers	13
1.3.1 n th roots of unity	16
1.3.2 Symmetry with respect to a circle	17
1.4 Some topological definitions	23
1.5 Complex infinity and the Riemann sphere	29
1.5.1 The Riemann sphere and stereographic projection	30
1.6 Applications to electrical circuits	33
1.7 Problems	36
2 Analytic Functions	46
2.1 Functions of a complex variable	46
2.1.1 Velocity of fluid flow emanating from a source	48
2.1.2 Mapping properties of complex functions	50
2.1.3 Definitions of the exponential and trigonometric functions	53
2.2 Limit and continuity of complex functions	54
2.2.1 Limit of a complex function	54
2.2.2 Continuity of a complex function	58
2.3 Differentiation of complex functions	61
2.3.1 Complex velocity and acceleration	63
2.4 Cauchy–Riemann relations	64
2.4.1 Conjugate complex variables	69

2.5	Analyticity	70
2.6	Harmonic functions	74
2.6.1	Harmonic conjugate	75
2.6.2	Steady state temperature distribution	80
2.6.3	Poisson's equation	84
2.7	Problems	85
3	Exponential, Logarithmic and Trigonometric Functions	93
3.1	Exponential functions	93
3.1.1	Definition from the first principles	94
3.1.2	Mapping properties of the complex exponential function	97
3.2	Trigonometric and hyperbolic functions	97
3.2.1	Mapping properties of the complex sine function	102
3.3	Logarithmic functions	104
3.3.1	Heat source	106
3.3.2	Temperature distribution in the upper half-plane	108
3.4	Inverse trigonometric and hyperbolic functions	111
3.5	Generalized exponential, logarithmic, and power functions	115
3.6	Branch points, branch cuts and Riemann surfaces	118
3.6.1	Joukowski mapping	123
3.7	Problems	126
4	Complex Integration	133
4.1	Formulations of complex integration	133
4.1.1	Definite integral of a complex-valued function of a real variable	134
4.1.2	Complex integrals as line integrals	135
4.2	Cauchy integral theorem	142
4.3	Cauchy integral formula and its consequences	151
4.3.1	Derivatives of contour integrals	153
4.3.2	Morera's theorem	157
4.3.3	Consequences of the Cauchy integral formula	158
4.4	Potential functions of conservative fields	162
4.4.1	Velocity potential and stream function of fluid flows	162
4.4.2	Electrostatic fields	175
4.4.3	Gravitational fields	179
4.5	Problems	183

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Table of Contents

[More information](#)

	<i>Contents</i>	vii
5 Taylor and Laurent Series		194
5.1 Complex sequences and series		194
5.1.1 Convergence of complex sequences		194
5.1.2 Infinite series of complex numbers		196
5.1.3 Convergence tests of complex series		197
5.2 Sequences and series of complex functions		200
5.2.1 Convergence of series of complex functions		201
5.2.2 Power series		206
5.3 Taylor series		215
5.4 Laurent series		221
5.4.1 Potential flow past an obstacle		230
5.5 Analytic continuation		233
5.5.1 Reflection principle		236
5.6 Problems		238
6 Singularities and Calculus of Residues		248
6.1 Classification of singular points		248
6.2 Residues and the Residue Theorem		255
6.2.1 Computational formulas for evaluating residues		257
6.3 Evaluation of real integrals by residue calculus		268
6.3.1 Integrals of trigonometric functions over $[0, 2\pi]$		268
6.3.2 Integrals of rational functions		269
6.3.3 Integrals involving multi-valued functions		271
6.3.4 Miscellaneous types of integral		275
6.4 Fourier transforms		278
6.4.1 Fourier inversion formula		279
6.4.2 Evaluation of Fourier integrals		285
6.5 Cauchy principal value of an improper integral		288
6.6 Hydrodynamics in potential fluid flows		295
6.6.1 Blasius laws of hydrodynamic force and moment		295
6.6.2 Kutta–Joukowski’s lifting force theorem		299
6.7 Problems		300
7 Boundary Value Problems and Initial-Boundary Value Problems		311
7.1 Integral formulas of harmonic functions		312
7.1.1 Poisson integral formula		312
7.1.2 Schwarz integral formula		319
7.1.3 Neumann problems		324
7.2 The Laplace transform and its inversion		326
7.2.1 Bromwich integrals		330

7.3	Initial-boundary value problems	336
7.3.1	Heat conduction	337
7.3.2	Longitudinal oscillations of an elastic thin rod	341
7.4	Problems	346
8	Conformal Mappings and Applications	358
8.1	Conformal mappings	358
8.1.1	Invariance of the Laplace equation	364
8.1.2	Hodograph transformations	372
8.2	Bilinear transformations	375
8.2.1	Circle-preserving property	378
8.2.2	Symmetry-preserving property	381
8.2.3	Some special bilinear transformations	390
8.3	Schwarz–Christoffel transformations	399
8.4	Problems	409
<i>Answers to Problems</i>		419
<i>Index</i>		434