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978-0-521-70084-9 - The Mathematics of OZ: Mental Gymnastics from Beyond the Edge

Clifford A. Pickover

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## THE MATHEMATICS OF OZ

### *Mental Gymnastics from Beyond the Edge*

Grab a pencil. Relax. Then take off on a mind-boggling journey to the ultimate frontier of math, mind, and meaning as acclaimed author Dr. Clifford Pickover, Dorothy, and Dr. Oz explore some of the oddest and quirkiest highways and byways of the numerically obsessed. Prepare yourself for a shattering odyssey as *The Mathematics of Oz* unlocks the doors of your imagination. The thought-provoking mysteries, puzzles, and problems range from zebra numbers and circular primes to Legion's number – a number so big that it makes a trillion pale in comparison. The strange mazes, bizarre consequences, and dizzying arrays of logic problems will entertain people at all levels of mathematical sophistication.

The tests devised by enigmatic Dr. Oz to assess human intelligence will tease the brain of even the most avid puzzle fan. Test your wits on a host of mathematical topics: geometry and mazes, sequences, series, sets, arrangements, probability and misdirection, number theory, arithmetic, and even several problems dealing with the physical world. With numerous illustrations, this is an original, fun-filled, and thoroughly unique introduction to numbers and their role in creativity, computers, games, practical research, and absurd adventures that teeter on the edge of logic and insanity. *The Mathematics of Oz* will have you squirming in frustration and begging for more.

Clifford A. Pickover received his Ph.D. from Yale University and is the author of over thirty highly-acclaimed books on topics such as computers and creativity, art, mathematics, black holes, human behavior and intelligence, time travel, alien life, and science fiction. Pickover is a prolific inventor with dozens of patents, is the associate editor for several journals, the author of colorful puzzle calendars, and puzzle contributor to magazines geared to children and adults. *The Los Angeles Times* wrote, "Pickover has published nearly a book a year in which he stretches the limits of computers, art, and thought." *Wired* magazine wrote, "Bucky Fuller thought big, Arthur C. Clarke thinks big, but Cliff Pickover outdoes them both." Pickover's computer graphics have been featured on the cover of many popular magazines and on TV shows. His web site, [www.pickover.com](http://www.pickover.com), has received over a million visits.

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*The Alien IQ Test*

*Black Holes: A Traveler's Guide*

*Chaos and Fractals*

*Chaos in Wonderland*

*Computers, Pattern, Chaos, and Beauty*

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# THE MATHEMATICS OF *OZ*

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*This book is dedicated to my uncle, Dr. Bruce Pickover,  
who stimulated my early interest in mathematical and other puzzles.*

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“Can’t you give me brains?” asked the Scarecrow.

“You don’t need them. You are learning something every day. A baby has brains, but it doesn’t know much. Experience is the only thing that brings knowledge, and the longer you are on earth the more experience you are sure to get.”

“That may all be true,” said the Scarecrow, “but I shall be very unhappy unless you give me brains.”

– The Scarecrow conversing with Oz, in L. Frank Baum’s  
*The Wonderful Wizard of Oz*

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## Travel Guide

He calmly rode on, leaving it to his horse's discretion to go which way it pleased, firmly believing that in this consisted the very essence of adventures.




– Miguel de Cervantes, *Don Quixote*

The road through this book is chaotic and takes many turns in order to surprise and delight you. However, if you wish to take your hovercraft and jump between mathematical puzzles of similar kinds, the following guide should help.



   **You are here**

- **Geometry** (Chapters 1, 3, 8, 14, 16, 17, 18, 23, 47, 50, 54, 55, 58, 61, 65, 84, 88, 96, 103, 104, 106)
- **Mazelike Problems** (Chapters 6, 12, 13, 15, 22, 24, 36, 46, 49, 52, 60, 83, 87, 97, 98, 101, 102)
- **Sequences, Series, Sets, and Arrangements** (Chapters 2, 4, 5, 9, 11, 25, 26, 30, 34, 37, 38, 41, 48, 50, 56, 59, 63, 64, 66, 69, 71, 72, 73, 74, 79, 80, 82, 85, 86, 89, 92, 93, 107)
- **Physical World** (Chapters 1, 3, 7, 40, 44, 45, 78, 102)
- **Probability and Misdirection** (Chapters 8, 14, 17, 19, 27, 31, 32, 51, 70, 81, 90)
- **Number Theory and Arithmetic** (Chapters 9, 10, 20, 21, 27, 33, 35, 37, 39, 42, 43, 45, 47, 53, 57, 62, 67, 68, 76, 77, 82, 91, 99, 100, 105, 108)

**End here**   

(Freedom for Dorothy and Ultimate Reader Enlightenment)

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B.C. MANSFIELD

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## Preface

"If you only had brains in your head you would be as good a man as any of them, and a better man than some of them. Brains are the only things worth having in this world, no matter whether one is a crow or a man."

– Crows talking to the Scarecrow in *The Wonderful Wizard of Oz*

Oz is a metaphor for mystery. Oz is a state of mind. Oz is a parallel universe that may exist side by side with our own in some ghostly fashion.

Published in 1900 by L. Frank Baum, *The Wonderful Wizard of Oz* starred young Dorothy of Kansas along with a magical array of characters ranging from a Scarecrow and a Tin Woodman to a phony Wizard who used magic to help Dorothy come to realize that there is no place like home.

In *The Mathematics of Oz* Dorothy is certainly far from home. Abducted by mathematically obsessed aliens, Dorothy tries to solve a baffling array of brainteasers that often center around numbers and mathematics. The aliens' obsession with mathematics probably sounds silly to many of you, but numerical challenges are a great way to transcend space and time. Mathematics is a universal language, and numbers might be our first means of communication with intelligent alien races.

Dorothy, Dr. Oz (her abductor), and Mr. Plex (Oz's assistant) have a limited attention span, and they don't want you readers to wade through pages of background before getting to the essential ingredients. To avoid this problem, each chapter in this book is just a few pages in length. One advantage of this format is that you can jump right in to experiment and have fun. The book is not intended for mathematicians looking for formal mathematical explanations; however, additional material can be found in the "Further Exploring" and "Further Reading" sections.

Prepare yourself for a strange journey as *The Mathematics of Oz* unlocks the doors of your imagination. The mysteries, puzzles, and prob-

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lems range from building a yellow-brick road that crosses America, to zebra numbers and circular primes, to Legion's number – a number so big that it makes a trillion pale in comparison – to “The Problem of the Bones,” a fiendishly difficult mathematical problem involving probability and shattering of leg bones.

Grab a pencil. Do not fear. Some of the topics in the book may appear to be curiosities, with little practical application or purpose. However, I have found these experiments to be useful and educational – as have the many students, educators, and scientists who have written to me. Throughout history, experiments, ideas and conclusions originating in the play of the mind have found striking and unexpected practical applications. Or as mathematician Gottfried Wilhelm Leibniz once said, “Les hommes ne sont jamais plus ingénieux que dans l'invention des jeux.” (“Men are never more ingenious than when they are inventing games.”)

\* \* \*

This book is for anyone who wants to enter new mental worlds. If you are a teacher, you may want to use the mathematical brain teasers to stimulate students. Have them design their own puzzles similar to the ones in this book. Computer programmers may want to create or solve similar puzzles using a computer, although a computer is definitely not necessary to attack and solve the problems in this book.

To help you assess your level of performance during your journey through this book, I have assigned difficulty ratings to the various puzzles:

- \* Challenging
- \*\* Very challenging
- \*\*\* Extremely difficult
- \*\*\*\* Outrageously difficult: probably impossible for Dorothy and other *Homo sapiens* to solve.

To retain the playful spirit of the book, and its sense of crazy adventure, puzzles with different difficulty levels are scattered randomly through the book – as if the puzzles had been tossed about by a tornado. Browse from the mathematical smorgasbord and feed your mind.

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## Acknowledgments

Every now and again one comes across an astounding result that closely relates two foreign objects which seem to have nothing in common. Who would suspect, for example, that on the average, the number of ways of expressing a positive integer  $n$  as a sum of two integral squares,  $x^2 + y^2 = n$ , is  $\pi$ .

– Ross Honsberger, *Mathematical Gems III*

I thank Brian C. Mansfield for his wonderful cartoon diagrams, used throughout the book. Over the years, Brian has been helpful beyond compare.

Numerous people have provided very useful feedback and information for the solutions to my puzzles: Dennis Gordon, Robert Stong, David T. Blackston, Dennis Yelle, Balakumar Jothimohan Balasubramaniam, Ilan Mayer, Ed Murphy, Jim Gillogly, Dan Tilque, Bill Ryan, James Van Buskirk, “R.E.S.,” Dennis Gordon, Dharmashankar Subramanian, Richard Heathfield, Al Zimmerman, Risto Lankinen, Seth Breidbart, Darrell Plank, David A. Karr, Jason Earls, Ken Inoue, and others.

I thank Samuel Marcius for the symbol ☹, which represents Mr. Plex, and for other alienlike symbols used in this book. The animal font is a freeware font by Alan Carr. Ann Stretton designed the font that contains symbols such as 🐾. Symbols like 🐾 are part of freeware from Omega Font Labs. Michael Lee and Josh Dixon designed the font that looks like ☺☻☹!!!.

All Oz quotations come from L. Frank Baum’s classic novels, *The Wizard of Oz* (originally published in 1900 as *The Wonderful Wizard of Oz*), *The Land of Oz* (originally published in 1904 as *The Marvelous Land of Oz*), *Ozma of Oz* (1907), *Dorothy and the Wizard in Oz* (1908), *The Road to Oz* (1909), *The Emerald City of Oz* (1910), *The Patchwork Girl of Oz* (1913), *Tik-Tok of Oz* (1914), *The Scarecrow of Oz* (1915), *Rinkitink in Oz* (1916), *The Lost Princess of Oz* (1917), *The Tin Woodman of Oz* (1918), *The*

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*Magic of Oz* (1919), and *Glinda of Oz* (1920). For more information on Oz, see Eric P. Gjovaag's "Oz" Web site: <http://www.eskimo.com/~tiktok/>.

*Note:* As many readers are aware, Internet Web sites come and go. Sometimes they change addresses or completely disappear. The Web site addresses (URLs) listed in this book provided valuable background information when this book was written. You can, of course, find numerous other sites relating to many of the mathematical puzzles and topics in this book by using search engines such as [www.google.com](http://www.google.com).