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978-0-521-70043-6 - Maths Trails: Excel at Problem Solving
Graeme Brown and Jennifer Piggott
Excerpt
More information

Make a copy, make a chain

Ready-made spreadsheets to explore mathematical ideas

Prerequisite knowledge

- Multiplication facts

Why do this unit?

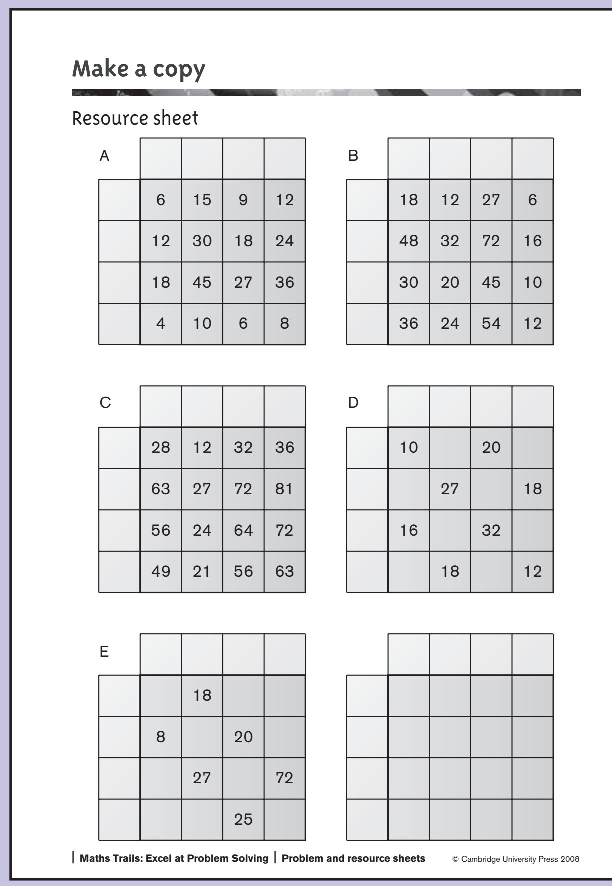
The 'Make a copy' activity (resource sheet) is a good way to maintain and extend pupil familiarity with tables. 'Make a chain' (problem sheet) offers an opportunity for pupils to pose their own challenges.

Time

One lesson

Resources

CD-ROM: spreadsheet, problem sheet, resource sheet
NRICH website (optional): www.nrich.maths.org, November 2007, 'Make a copy'; March 2007, 'Factor-multiple chains'


Make a copy

Resource sheet

A

	6	15	9	12
	12	30	18	24
	18	45	27	36
	4	10	6	8

B

	18	12	27	6
	48	32	72	16
	30	20	45	10
	36	24	54	12

C

	28	12	32	36
	63	27	72	81
	56	24	64	72
	49	21	56	63

D

	10		20	
		27		18
	16		32	
		18		12

E

		18		
	8		20	
		27		72
			25	

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Introducing the unit

Show the class the sheet 'Grid A' and ask:

- How are the numbers in the grid produced by the blue numbers at the edge? [by multiplication]
- How do the spinner buttons change the blue values? [increase or decrease]
- Can you choose particular blue numbers so that the grid on the left is the same as the grid on the right?

Allow plenty of time for pupils to share their reasoning.

Grids B to E are similar puzzles. Later puzzles need solving using less given information.

The resource sheet 'Make a copy' contains a version of the same puzzles and one blank grid for pupils to make a challenge of their own.

Main part of the unit

Display the sheet 'Factor multiple chains'.

A complete line like $3 \leftrightarrow 6 \leftrightarrow 30 \leftrightarrow 90$ is referred to as a 'chain' because adjacent numbers are related factor to multiple. The values in each blue box can range from 2 to 100. Check that pupils understand how the sheet works by creating other chains.

Encourage use of the terms 'factor' and 'multiple' by making statements like '30 is a multiple of 6' and inviting pupils to offer the other. [6 is a factor of 30]

Ask pupils to work in pairs on the challenges posed on the problem sheet 'Make a chain', pausing for discussion at intervals.

Invite pupils to pose and pursue similar challenges of their own.

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Plenary

Discuss the reasoning used for the challenges on the problem sheet and for pupils' own challenges.

Solution notes

Make a copy

A

	2	5	3	4
3	6	15	9	12
6	12	30	18	24
9	18	45	27	36
2	4	10	6	8

B

	6	4	9	2
3	18	12	27	6
8	48	32	72	16
5	30	20	45	10
6	36	24	54	12

C

	7	3	8	9
4	28	12	32	36
9	63	27	72	81
8	56	24	64	72
7	49	21	56	63

D

	2	3	4	2
5	10		20	
9		27		18
8	16		32	
6		18		12

E

	2	3	5	8
6		18		
4	8		20	
9		27		72
5			25	

Make a chain

$2 \leftrightarrow 4 \leftrightarrow 8 \leftrightarrow 16$ is the smallest complete chain.

The chain $5 \leftrightarrow 25 \leftrightarrow 50 \leftrightarrow 100$ produces the largest possible number in the last three positions but $12 \leftrightarrow 24 \leftrightarrow 48 \leftrightarrow 96$ contains the largest number possible in the first position.

26 cannot be in a chain. If it were possible, the latest it could appear would be position two with either 2 or 13 in position one but position four is limited to a number up to 100 and so cannot offer a value to make a chain. Also prime numbers must occupy position one in any chain in which they appear.

88 is the maximum difference between adjacent numbers in a chain. [$2 \leftrightarrow 4 \leftrightarrow 8 \leftrightarrow 96$]

For the greatest possible range $2 \leftrightarrow 4 \leftrightarrow 8 \leftrightarrow 96$ looks promising [94] but $5 \leftrightarrow 25 \leftrightarrow 50 \leftrightarrow 100$ is greater still [95].

The minimum range is 14 produced by $2 \leftrightarrow 4 \leftrightarrow 8 \leftrightarrow 16$.

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Multiples grid

Ready-made spreadsheets to explore mathematical ideas

Prerequisite knowledge

- Multiplication facts

Why do this unit?

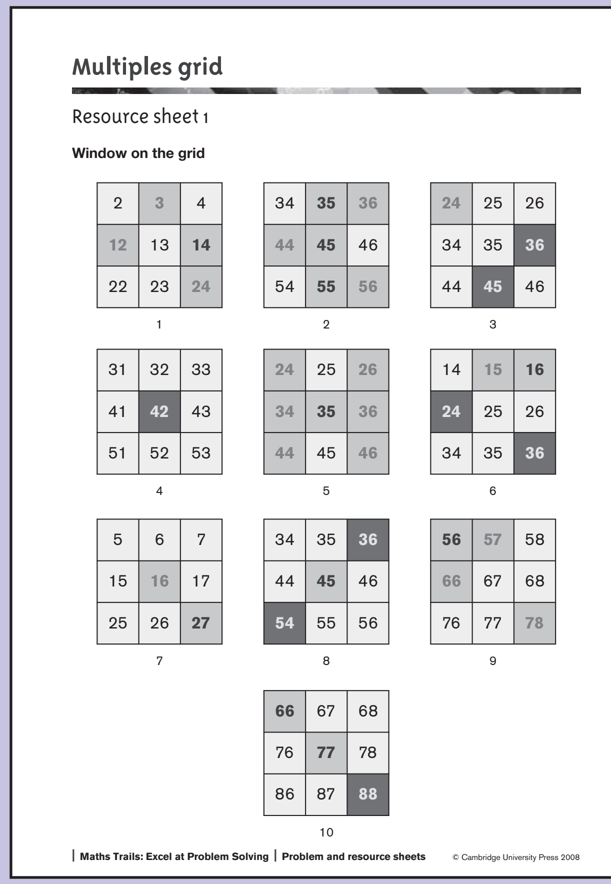
This unit reinforces the concepts of multiples and common multiples and leads to an exploration of the underpinning structure.

Time

One lesson

Resources

Squared paper
CD-ROM: spreadsheet, resource sheets 1–6
NRICH website (optional):
www.nrich.maths.org, March 2007, 'Multiples grid'



Multiples grid

Resource sheet 1

Window on the grid

2	3	4
12	13	14
22	23	24

1

34	35	36
44	45	46
54	55	56

2

24	25	26
34	35	36
44	45	46

3

31	32	33
41	42	43
51	52	53

4

24	25	26
34	35	36
44	45	46

5

14	15	16
24	25	26
34	35	36

6

5	6	7
15	16	17
25	26	27

7

34	35	36
44	45	46
54	55	56

8

56	57	58
66	67	68
76	77	78

9

66	67	68
76	77	78
86	87	88

10

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Introducing the unit

Ask pupils to work in pairs at a computer for five minutes on the sheet 'Number grid multiples'. At the end of that time ask pupils to offer their observations to the whole group. Pupils may observe that multiples of the chosen numbers are coloured pink or blue, and common multiples are coloured maroon.

- Can any selected number in the grid be made pink, blue, maroon or uncoloured? [Yes, except for 1, which is always uncoloured.]

Main part of the unit

Window on the grid

Work together on the first two grids from 'Window on the grid' on resource sheet 1.

- What settings could result in these two grids? [Pink is 3, blue is 7 or 14; pink is 4, blue is 5.]
- Can you justify that? Check the solution using the sheet. [Notice how sometimes there is more than one right answer.]

It is important to allow plenty of time for discussion, drawing attention to efficient methods of covering all cases. For example, if the 16 on the grid is blue then the blue setting must be 2, 4, 8 or 16 (factors of 16).

When the group is confident and ready to move on, pupils can try the remaining 'windows' on the resource sheet and perhaps generate examples of their own with which to challenge each other.

Printing wallpaper

Use the image from 'Printing wallpaper, Pattern 1' on resource sheet 2. Discuss the structure of the pattern. 'Pattern 1 without numbers' (resource sheet 3) may help pupils identify patterns more easily.

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- What do you see? [For example, part of the pattern goes blue–pink–blue vertically, or the maroon steps down evenly, or …]
- What could the setting numbers have been? [pink 6, blue 4]

This pattern with coloured blocks can be made using a basic unit as a stamp, and stamping repeats of that unit, side by side, until the whole grid space is covered.

- What would the stamp unit look like? [any rectangle of width two and height six]

Discuss the structure of 'Pattern 2' (resource sheets 4 and 5) similarly. [The settings are pink 2, blue 5, with a single whole row as the repeating unit.]

Invite pupils, working in pairs, to find the basic stamp unit for other settings and explain any relationships they notice between the basic stamp and the setting numbers used.

Ask each pair of pupils to choose one grid pattern to print, superimposing their stamp unit and adding their conjectures and reasoning. Use these printouts to make a group display.

Plenary

Invite pupils to look at the patterns on display and to talk about any common features they have noticed.

Solution notes

Window on the grid

The table below shows the pink and blue settings which produce each window on resource sheet 1.

Window	Pink	Blue
1	3	7 or 14
2	4	5
3	3	9
4	6 7, 14, 21 or 42	7, 14, 21 or 42 6
5	2	7
6	3	4
7	4, 8 or 16	9 or 27
8	6	3 or 9
9	3	8
10	8	11

Printing wallpaper

If one of the setting choices is 2, for all choices of the other setting (n, $n<10$), the stamps are all 2 wide and n deep giving an area of $2n$ square units with the exception of 5×5. The rule breaks down here because multiples of 5 are vertical and therefore do not cut across the vertical lines of the multiples of 2.

By considering the slope of the diagonals of multiples of other n it is also possible to justify the 'heights' of the stamps.

For $3\times n$ $(n<7)$ and $4\times n$ $(n<7)$, similar justifications for the arrangement of stamps can be made.

If the two numbers have common factors this affects the ability to make a rectangular stamp. So 3×9 and 4×8 work.

It is possible to look at other grid widths to examine what stamps are made.

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Interactive number patterns I

Ready-made spreadsheets to explore mathematical ideas

Prerequisite knowledge

- Use of symbols to represent unknown or varying values
- Use of algebraic expressions to represent relationships

Why do this unit?

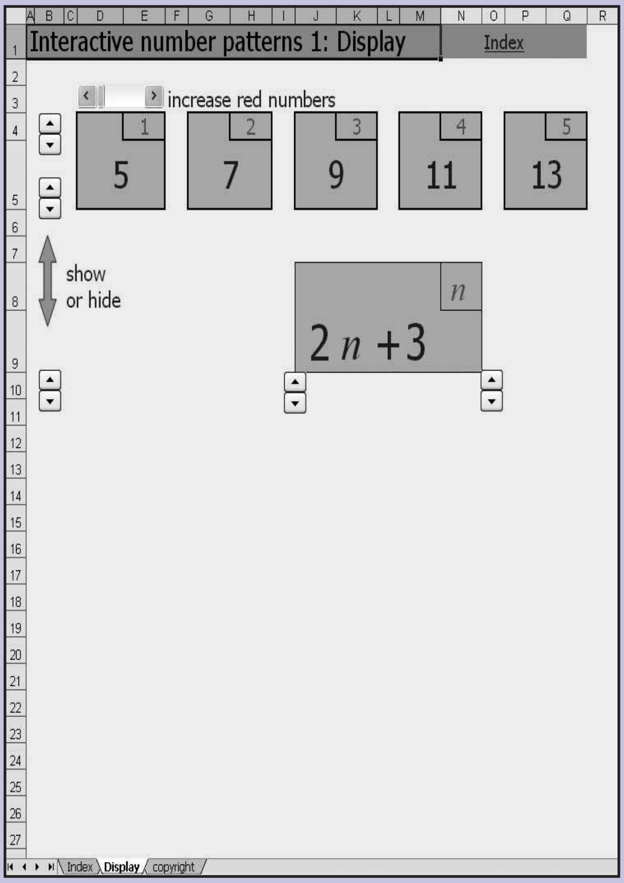
This interactive environment allows pupils to explore number sequences and their algebraic representation. 'Interactive number patterns 2' extends the work here to quadratics.

Time

One lesson

Resources

CD-ROM: spreadsheet
NRICH website (optional): www.nrich.maths.org, September 2007, 'Interactive number patterns 1'



Introducing the unit

Open the spreadsheet and familiarise the group with it. Check that pupils understand:

- the slider at the top changes the red numbers;
- the spinner buttons on the edge of the formula box change the numbers in the formula;
- the three spinner buttons on the left of the screen hide or show parts of the display.

The purpose of this part of the activity is to familiarise pupils with substituting values into given formulae and to predict the sequence of blue numbers.

Set the formula to $2n+3$ and set the red numbers to read 1 to 5.

- Why is the first blue number 5? [Substituting $n=1$ in the formula gives $2\times 1+3=5$.]
- Why are the other blue numbers 7, 9, 11 and 13? [similar substitutions of 2, 3, 4 and 5 into the formula]

Hide the blue numbers and invite pupils to suggest changes to the formula. Ask pupils to predict the hidden blue values and record their answers. Repeat this process with different formulae, keeping the red values 1 to 5. Have pupils propose formulae, predict the blue values and then check. Extend the activity by changing the red values using the slider but keep the calculations easily within the ability of the group so that their focus remains on the algebra not the calculation.

Continue this activity but hide the red values instead of the blue.

Pupils will be solving equations mentally, though they probably will not think of it like that, and now is a good opportunity to invite pupils to share their methods.

Make sure that these two processes of substituting and solving to find unknown or hidden values are well practised before moving to the main activity.

Main part of the unit

The purpose in this part of the activity is for pupils to arrive at a method for determining the formula which has been used to generate the terms within a displayed sequence. There are a number of ways that the spreadsheet can help. Here are two approaches.

Open investigation

Ask pupils to work in pairs to devise a method that can successfully predict the hidden formula. The method is then shared with the group and the presenters are asked to explain why their method works. If a justification is not possible for the presenters, other pupils may be asked if they can see and explain why that method works. Invite the group to explain how the offered methods compare. Ask pupils to say which method they like best and why.

Whole group

Set the red numbers to start at 1 and set the formula box to $2n$.

- Do you notice any pattern in the blue numbers? [They go up by 2.]

Discuss how the $2n$ makes this happen and then look at $3n$, $4n$ and $5n$ similarly. Some pupils might use phrases like '$2n$ is the algebra for the two times table'.

Return the formula box to $2n$, drawing attention to the blue values ascending by 2, and change the formula to $2n + 1$. Point out that the blue numbers still increase by 2 as before but with every value getting a 'bonus 1' from the $+1$ in the formula. Extend this to cover any constant between -10 and 10.

If time allows, pupils can work in pairs to test each other's understanding by taking turns to hide various elements of the spreadsheet for the partner to complete, before checking.

Plenary

Working with the whole class, ask pupils to 'look away' while one pupil changes the formula and then hides it so that the red and blue values are all that show.

Model to the group how you would determine the formula. For example: 'First I notice the step size – the increase between blue values – that tells me the value of the number of n in the formula. Next I think what it would be if it was that many n and nothing else (like a times table). I compare that with the actual blue number and that tells me how much bonus or loss every term is getting.'

Invite pupils to 'look away' with you each time. Offer your model initially but move towards pupils offering their own descriptions of the process.

Ensure that a good range of red values is used.

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Interactive number patterns 2

Ready-made spreadsheets to explore mathematical ideas

Prerequisite knowledge

- Substitution into formulae
- Linear equations
- Quadratic expressions

Why do this unit?

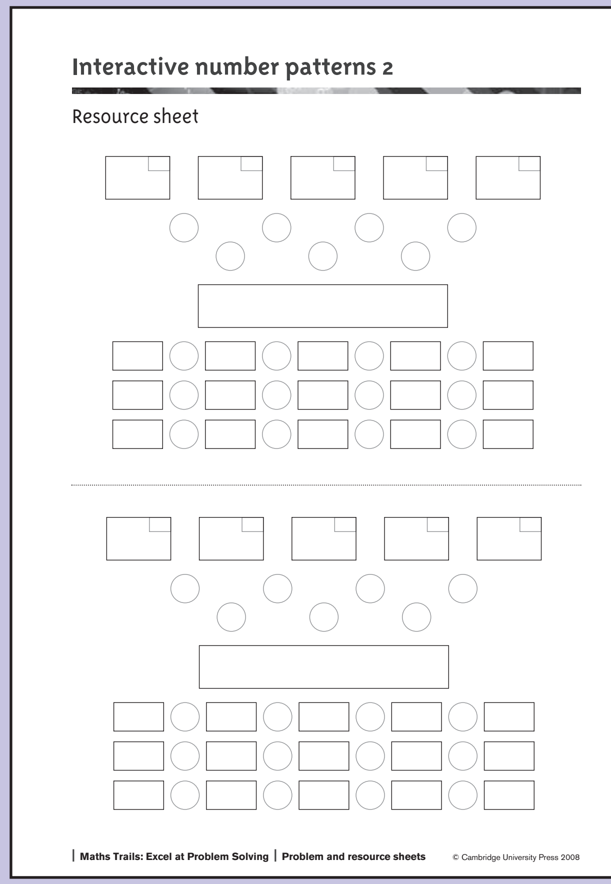
The interactive environment allows pupils to explore quadratic sequences and their algebraic representation. This unit is suitable for pupils for whom 'Interactive number patterns 1' is very straightforward.

Time

Two lessons

Resources

CD-ROM: spreadsheet, resource sheet
NRICH website (optional):
www.nrich.maths.org
September 2007, 'Interactive number patterns 2'


Interactive number patterns 2

Resource sheet

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Introducing the unit

Open 'Patterns (simple)' on the spreadsheet. Ensure that pupils understand:

- the slider at the top changes the red numbers;
- the spinner buttons on the bottom edge of the formula box change the numbers in the formula;
- the spinner buttons on the left of the screen hide or show parts of the display.

Set the formula to $n^2 + 7n + 4$ and the red numbers to read 1 to 5. Check with the group that substituting n as 1 into the expression gives 12, and continue with $n = 2$ through to 5 to produce 22, 34, 48 and 64.

Hide the blue numbers, ask for changes to the formula, and invite pupils to predict the hidden blue values. Include examples where the red n values range up to 20.

Continue this activity but hide the red values instead of the blue. Once the red values are hidden, set the formula back to $n^2 + 7n + 4$ and use the slider at the top to change the first blue number to 202.

- What red value produces the blue of 202? [11]
- How did you work that out? [Pupils may say 'trial and improvement'.]

Show the red values and reset the formula to $n^2 - 10n + 10$. Drag the slider to the extreme left to show blue values of 1, –6, –11, –14, –15.

- When n is 1 the formula gives 1. For what other n value will the formula also give 1? [9]
- Are there any other n values that produce 1? [No, after 9 the function continues ascending indefinitely.]
- Which n value produces 21? [11]
- Is there a second value which also produces 21? [Not within the limits of this spreadsheet, but $n = -1$ makes $n^2 - 10n + 10 = 21$.]

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Make sure that pupils understand fully the process of substituting to find unknown or hidden values before moving to the main activity.

Main part of the unit

The aim of this part of the activity is for pupils to arrive at a method for determining the formula used to generate the terms within a sequence. It is common for pupils to use a method based on the difference between values, and then the difference between those difference values. It is less common for pupils to grasp the validity of this method. This activity is designed to improve pupil understanding of the 'difference method'.

Set the formula to n^2+3n+8 and move to 'Patterns with differences' on the spreadsheet. (Note that changing the quadratic expression and/or the red values on one sheet automatically updates the other.) Many values are displayed on this sheet and it is important that pupils are given time to understand each part.

There are three rows below the formula box. These display the values of the quadratic, linear and constant terms from the quadratic formula respectively. Change the terms in the formula box to help pupils grasp what each row shows. Verify that the three values added together produce the blue function values in the boxes at the top of the screen. Draw attention to the two rows of differences above the formula box and verify that the data showing is the correct difference for the blue function values. Change the red values using the slider and have pupils verify the new values displayed within the sheet.

- What changes when we alter the red values and what stays the same? [The first difference changes; the second difference remains unaltered.]
- Why? [The constant term contributes nothing to the first difference; the linear term contributes to the first difference but not the second; the coefficient (multiple) of the quadratic term is the only coefficient influencing the second difference.]

Return to 'Patterns (simple)'. Change the formula and ask pupils to write what the other sheet would show if they could see it. (There is a template for this on the resource sheet which enables pupils to record their solutions prior to discussion.)

Return to 'Patterns with differences' to compare pupil answers with the display and discuss.

Repeat this exercise as a group or ask pupils to work in pairs, with one of them altering the formula for the other to explain, until they are familiar with the role of differences.

Use 'Patterns with differences' and set the formula to $2n^2$ with no linear or constant term. Discuss the effect of the 2. Change the 2 to 3, 4 and then 5, discussing the effect until the group is confident that for a quadratic sequence the second difference will always be constant at double the coefficient of the quadratic term. Ask them to explain why this is true. Allow plenty of time for discussion as this is a central objective for this activity.

Ask pupils to work in groups to determine rules for finding a formula for any given sequence.

Plenary

Pupils discuss and evaluate some of their methods, leading to a whole-group challenge to determine hidden formulae for given sequences.

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Consecutive sums

Ready-made spreadsheets to explore mathematical ideas

Prerequisite knowledge

- Simplification of algebraic expressions

Why do this unit?

The sum of consecutive integers occurs frequently in mathematics. This activity uses an interactive spreadsheet to assist visualisation, leading to a key formula, and tests conjectures about sums of consecutive numbers.

Time

Two lessons

Resources

CD-ROM: spreadsheet, problem sheets 1 and 2
NRICH website (optional): www.nrich.maths.org, May 2004, 'Sequences and series'; December 2001, 'Proof sorter – sum of an AP'; May 1997, 'Consecutive sums'

Consecutive sums

Problem sheet 1

Adding consecutive numbers

First do it the long way. Add up 1 + 2 + 3 + 4 + 5. What's the answer?

Now see the pattern another way.

The sequence can be written a second time in reverse order.

1 + 2 + 3 + 4 + 5
5 + 4 + 3 + 2 + 1

Notice that each number in the lower line makes a sum of 6 with the number directly above it.

Why do all these individual pairs have the same total?

In all, there are 5 of these pairs, all of which make 6, so the total of all the numbers in both lines must be 5 lots of 6 (that's 30).

Both lines have the same total so they must be 15 each (30 divided by 2).
So now we know that the numbers 1 to 5 have a sum of 15.

What about the sum of the numbers 1 to 10?
Write them down in line and again in a second line but backwards.
What will each number in the lower line make with the number directly above it this time?
How many pairs will there be?
So what is the total of all the numbers together in both lines?
And then you'll know the total in each line.
You have found the answer to 1 + 2 + … + 10 but without having to add them all!

Try finding the sum of 1 to 20 using this method.

What is the sum of the numbers 1 to 50? to 100? to 1000?

Could you write a formula explaining how to sum 1 to n?

Can you see how to use this method to add 10 + 11 + . . . + 20?

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Introducing the unit

The introductory activity aims to give pupils an efficient means of adding a long sequence of consecutive numbers.

Open 'Quick adding' on the spreadsheet and explain to pupils that this sheet shows the calculation $1+2+3+\ldots+n$. Tell pupils that they are going to find a quick way of doing the calculation, even when the numbers are large and there are lots of them.

Use the spinner buttons by the top table to show the results for $n=6$, 7, 8, 9 and 10. Adding the consecutive numbers 1 to 6 gives a total of 21, 1 to 7 gives 28 and so on.

- Can you see a connection between the last number in the sequence and the total (sum) we arrive at? [Each time the total increases by the value of the last number – 6 makes 21, 7 makes 28, 8 makes 36, 9 makes 45 and 10 makes 55.]
- What do you notice about 21, 28, 36, 45 and 55? [They are triangle numbers.]

Set n to 6 and point out the arrangement of numbers in the sheet. 1 to 6 is above 6 to 1. Draw attention to the sum of each column, 7, as the upper number is added to the number below it.

- Why is this total the same all the way along? [Values ascend in the upper line and descend in the lower line by the same amount.]
- What is the sum of each pair? [One more than the largest number, $n+1$.] How many columns are there? [n]
- So what is the total of both rows all together? [$n(n+1)$]
- And what must the total for one line be? [$\frac{1}{2}n(n+1)$]

Ask pupils to check this for enough specific n values until they have a strong understanding of the result and can justify it and use it.

We have looked at consecutive numbers which start at 1. What happens if we do not start at 1? Problem sheet 1 may be helpful at this stage. Choose a few examples to clarify what is being discussed and invite pupils to suggest methods

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for finding the sum. Some pupils may suggest a method similar to the method when starting at 1 (reverse and add in pairs, multiply and halve). Other pupils may suggest a subtraction method (the sum of 16 to 100 is equal to the sum of 1 to 100 minus the sum 1 to 15).

Main part of the unit

Show the problem ‘Consecutive sums’ on problem sheet 2. Explain that the class is going to work in pairs to find sums using consecutive numbers that need not start from 1.

As pupils work in groups on the problem some questions to stimulate discussions might include:

- Do you notice which numbers can be formed from the sum of two consecutive numbers? [All the odd numbers – because with two consecutive numbers one must be odd and one even.]
- Why can’t you make an even number from two consecutive numbers? [As above.]
- Can you think of a quick way of finding the two consecutive numbers? [Roughly half the odd number $\frac{1}{2}(n+1)$.]
- What numbers can be represented by three consecutive numbers?
- What about four or five consecutive numbers? [Related to multiples.]
- Is there a method for producing more using the answers you already have? [There are many methods but an example might be: ‘If I have $15 = 4 + 5 + 6$ then I can go up from 15 in steps of 3 by adding one to each number like this: $18 = 5 + 6 + 7$’ – this justifies the multiples suggested in the question above.]
- Can you predict what consecutive sums are possible given any number? [Yes, by looking at its factors.]

Investigate sums that are not possible.

- Are there any numbers you haven’t been able to find consecutive sums for? [Pupils may offer quite a few but through discussion, reduce this to powers of 2.]

Plenary

‘Sums’ and ‘Sums zoom 20%’ on the spreadsheet automatically calculate sums of consecutive numbers and are available to support discussions in the plenary.

There will be lots to share from the main activity but you may wish to focus on one idea that has been noted but not justified. For example, prove that pure powers of 2 can never be made as the sum of consecutive numbers.

Follow this by asking pupils to justify the conclusion to each other and to produce a proof in their own words on paper.

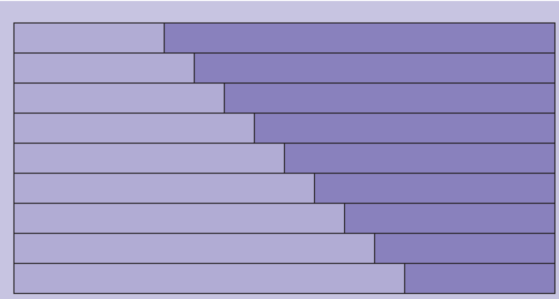
Solution notes

Plenary

Visually: This particular image happens to have nine strips but it is offered as the general case, so the number of strips (number of consecutive numbers) could be odd or even.

If the number of strips is odd, the longest and shortest strips will either both be even or both odd. This means their sum (the width of the rectangle) must be even. In either case the rectangle in the image has its height odd and its width even.

If instead the number of strips is even, the longest and shortest strips will be odd and even (or even and odd) respectively – so their sum must be odd. In this case the rectangle in the image would have its height even and its width odd.



However, if we need to arrange a power of 2 into a rectangle, both dimensions must be even, and this cannot be done with an odd or an even number of strips, so cannot be done at all.

Algebraically: $\frac{1}{2}n(n+1)$ always contains an odd factor because either n or $n + 1$ must be odd, and the other one even. The even factor can be divided by the 2 but the overall result must always contain an odd factor, and pure powers of 2 will only have even factors, so they will never match the sum for a set of consecutive numbers.