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Elliptic Cohomology

Geometry, Applications, and Higher Chromatic Analogues

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Contents

Preface

A workshop entitled "Elliptic Cohomology and Chromatic Phenomena" was held at the Isaac Newton Institute for Mathematical Sciences in Cambridge, UK, on 9–20 December, 2002. The workshop attracted over 75 participants from thirteen nations. The event, an EU Workshop, was the final one in INI's program New Contexts for Stable Homotopy Theory held in the fall of that year. During the first week nineteen talks described a wide range of perspectives on elliptic genera and elliptic cohomology, including homotopy theory, vertex operator algebras, 2-vector spaces, and open string theories. The second week featured ten talks with a more specifically homotopy theoretic focus, but encompassing the higher chromatic variants of elliptic cohomology.

This was the first conference on elliptic cohomology since the one organized by Peter Landweber at the Institute for Advanced Study in Princeton in 1986. The proceedings of that conference were published in [10]. The breadth of that volume is an indication of the multifaceted nature of the subject. From the start it has provided a meeting point for algebraic topology, number theory, and theoretical physics, playing in the present era a role analogous to the role of K-theory in the second half of the last century. Landweber's introduction to that volume, together with Serge Ochanine's contribution [13] to it, provide good introduction to the origins of this subject.

The starting point was the study of genera of spin manifolds. A genus is a multiplicative bordism invariant, with values in some commutative ring. A classical result asserts that signature provides the universal example of a rational genus of oriented manifolds with the property that it is multiplicative for oriented fiber bundles with connected compact Lie structure groups. Ochinine showed that if a spin genus has this property then it is "elliptic." In this description, Ochanine used the observation due to Quillen (understood also by Novikov in the rational case) that a genus for almost complex manifolds with values in a ring R is the same as a formal group law over R. Ochanine called a genus "elliptic" if it factors through the genus representing a formal group law used by Euler in 1756 to describe the addition law for certain elliptic integrals. Such a genus automatically factors uniquely through the map from unitary to spin bordism. The value ring can be regarded as a ring of modular forms.

Ochanine conjectured that the converse is true as well, and Edward Witten provided intuitive physics rationale for this conjecture in an article [23] in the IAS conference proceedings, interpreting the elliptic genus as the character-valued index of a certain hypothetical signature operator on the free loop space of the manifold.

The conjecture was proven by Raoul Bott and Cliff Taubes [3], and later by a different method by Kefeng Liu [12].

The traditional genera occur as indexes of differential operators, and hence extend to families versions. The topological manifestation of this phenomenon is that the genus is the effect on homotopy groups of a map of ring specta. Landweber [9] had established a general criterion guaranteeing such a geometric realization, and Landweber, Bob Stong, and Doug Ravenel [11] showed that after suitable localization the Ochanine genus corresponds to a map of ring spectra $MU \to Ell$. The target spectrum represented the first example of an elliptic cohomology theory.

One of the principal attractions of elliptic cohomology is the possibility that it admits a geometric description, in analogy with the description of K-theory via vector bundles. This dream was beautifully enunciated by Graeme Segal in his early survey [17], and expanded on in the his 1988 Oxford University lecture notes, now published as [18]. The idea, very crudely, is to regard a vector bundle with connection as associating a vector space to each point and an isomorphism to each path; the next step would then be to instead associate a Hilbert space to each loop and an operator to each bordism between loops. This is a field theory, and the hope has been that the physics of field theories would provide the ideas necessary to construct this mathematical object, which would, in turn, provide a framework for further physics. These hopes are reviewed in the paper by Segal in this volume. The torch is currently being carried by Stephan Stolz and Peter Teichner [19].

A representative of the string physics arising from the considerations which originally led Witten to his genera is provided by Emmanuel Diaconescu, Dan Freed, and Greg Moore, in their contribution.

Other geometric constructions have been attempted. Po Hu and Igor Kriz [6] pursue the conformal field theory approach, while Nils Baas, Bjorn Dundas, and John Rognes [2] start from the philosophy that elliptic cohomology should be a form of the K-theory of the complex K-theory spectrum, and obtain a functor by considering the theory of 2-vector spaces. (This theory was in fact created by Mikhail Kapranov and Vladimir Voevodsky [8] with this application in mind.) Any geometric source of modular forms provides a hint for geometric sources of elliptic cohomology. One such source is vertex operator algebras. It has been globalized by Vassily Gorbunov, Fyodor

Malikov, and Vadim Schechtman, [5], who use them identify a geometric object associated with a $BU\langle 6\rangle$ structure. A useful introduction to vertex operator algebras, focusing on their modular properties, is provided by Geoffrey Mason's article in this volume.

Another approach to a geometric interpretation of elliptic cohomology starts from Witten's interpretation of the elliptic genus and develops the notion that the corresponding cohomology theory should be a form of equivariant K-theory of the free loop space. The article of Nitu Kitchloo and Jack Morava pursues this idea.

Further evidence of the geometric depth of elliptic genera is provided by the theorems of Lev Borisov, Anatoly Libgober, and Burt Totaro. Igor Krichever and Gerald Höhn had considered certain two-variable variants of the Ochanine genus, taking values in Jacobi forms and exhibiting rigidity for complex manifolds. It turns out that the same genus enjoys quite a different universal property. It is unchanged when one passes from one resolution of singularities of a given complex projective variety to another. One must either specify the process of passing from one to another (by a certain form of complex surgery, "classical flops") or restrict to certain types of resolution of singularities. Burt Totaro reviews these theorems in his contribution.

After early work of Jens Franke [4], Hopkins, Haynes Miller, Paul Goerss, and Charles Rezk have constructed an étale sheaf of commutative ring spectra over the elliptic modular stack. This construction has not yet been fully documented, though it has been the subject of courses ([16]) and a week-long workshop in Mainz, Germany, in October, 2003. The paper of Bertrand Toën and Gabriele Vezzosi puts this construction into the general context of derived moduli spaces. The rigidity of this construction allows one to form a homotopy inverse limit, which is the spectrum *tmf* of "topological modular forms."

One of the hallmarks of a geometrically defined cohomology theory is the presence of equivariant versions of the theory. An analytic circle-equivariant theory associated to an elliptic curve E over $\mathbb{C}$, taking values in sheaves of algebras over E, was constructed in 1994 by Ian Grojnowski. This construction has had a large impact on the subject, and his announcement appears here for the first time along with a historical introduction. Quillen [15] described the connection between even multiplicative cohomology theories and formal groups. It appears that lifting the cohomology theory to a circle equivariant form is related to extending the formal group to an algebraic group. This idea is explored in John Greenlees's contribution.

A variant of the problem of an equivariant extension is the possibility of an orbifold theory. Physicists had studied an orbifold elliptic genus, and this story is brought into contact with the homotopy theoretic enrichment of the Witten genus due to Ando, Hopkins, and Neil Strickland [1] in the paper of Matthew Ando and Christopher French.

Elliptic spectra represent rather computable cohomology theories, which carry deep information about stable homotopy in their coefficients and in their operations. Following the approach set out by Frank Adams, these operations are most conveniently studied by means of the self homology Ell_*Ell. Relations between this object and numerical polynomials (familiar from Adams's study of K_*K) are described in Keith Johnson's article.

The second part of the workshop focused on higher analogues of elliptic cohomology. From the chromatic viewpoint on stable homotopy theory, elliptic cohomology is the third in a sequence of types of theories, starting with rational cohomology and K-theory; and, locally at a prime at least, this sequence can be continued. One still wishes to realize these still higher "height" theories by some geometric data. A futuristic venture, representing joint work with the late Charles Thomas, is described by Jorge Devoto in this volume. Here the objective is to construct a variant of elliptic cohomology using moduli of K3-surfaces, and geometrically realize this theory by means of a quaternionic field theory. A step towards a different extension of the notion of elliptic cohomology is proposed in Ravenel's article, in which he observes that the Jacobians of certain curves over $\mathbb{F}_p$ have one-dimensional formal factors of high height.

The theory of operations of these higher height theories is the topic of the review article here by Mark Hovey. More detailed computations and homotopy theoretic constructions carried out by Goerss, Hans-Werner Henn, Mark Mahowald, and Rezk in the case of elliptic cohomology are reviewed and in part extended to higher height by Henn in his paper. Nori Minami explores a variety of analogies and connections between the various localizations of stable homotopy theory associated with these higher height theories.

We dedicate this volume to the memory of our friend Professor Charles Thomas of the University of Cambridge, who died suddenly on December 16, 2005. Charles was constant in his belief in the geometric promise of elliptic cohomology. His book *Elliptic Cohomology* [20] and his review article [21] were important contributions to this development. His generosity and his warmth are missed by the whole community. We are pleased to publish here a paper derived from the lecture he gave at the workshop.

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October 25, 2006

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Charles Thomas, 1938–2005

Charles B. Thomas, Professor of Algebraic Topology in the University of Cambridge died on 16 December 2005. Although some ill health had come his way, his death came as a shock, particularly to those who wished him well at his retirement dinner but nine days earlier. He was born on 17 August 1938 and educated at the Benedictines' Douai School near Reading. After two years' service in the Royal Air Force he entered Trinity College, Cambridge, in 1958. An initial intention to read physics was soon converted into a life-long commitment to the study of mathematics. He worked for his PhD initially at Trinity College under D.B.A. Epstein and then under A. Dold at Heidelberg (where he met his wife, Maria). He was a Research Associate at Cornell University (1965–1967), a Lecturer at the University of Hull (1967–1969) and a Lecturer at University College London (1969–1979). He returned to Cambridge as a University Lecturer and Fellow of Robinson College in 1979.

Charles Thomas' research concerned the interplay between algebra (in particular the intricacies of finite group theory), algebraic topology, number theory, the topology of manifolds and various structures in differential geometry. He studied finite group actions on spheres and homotopy spheres and worked on many aspects of the spherical space form problem. His study of 3-manifold groups and Poincaré complexes culminated in his book Elliptic structures on 3-manifolds. Work on characteristic classes, classifying spaces and the cohomology of finite groups led to another book; problems on the cohomology of a wide range of types of group were a continuing interest to him. In two papers in the 1970s Charles was one of the first to note the significance of contact structures on smooth manifolds. He returned to this topic with renewed vigour some twenty years later when contact and symplectic structures became fashionable. Aspects of his research led to his writing books on elliptic cohomology, on differential manifolds (written with D. Barden) and on representation theory. He edited the volumes of the proceedings of several meetings he had organised and was particularly proud to be co-editor of a selection of the works of J.F. Adams, whom he particularly admired.

Several of Charles' former PhD students now propagate his high standards to students of their own. Charles valued academic excellence. A list of his mathematical achievements gives only a limited impression of his intellect. He spoke something of most of the main European languages and was fluent in French and German. He translated into English four mathematics books from German and one from French. He could converse on any academic subject but had a passion for medieval European and Byzantine history. Whilst giving first priority to research, Charles did his share of university administration. Recently, he was Chairman of the Faculty Board and editor of the Mathematical Proceedings of the Cambridge Philosophical Society. His careful lecturing was illuminated by his immaculate handwriting on the blackboard and on duplicated notes; an occasional stutter allowed the student a moment to catch up. He planned in his retirement to continue his work partly in Cambridge, partly in California at Santa Cruz. His wife, two sons, a daughter and a very new grandson survive him.

Raymond Lickorish

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