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978-0-521-70015-3 - Dependence Logic: A New Approach to Independence Friendly Logic

Jouko Vaananen

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Dependence Logic

Dependence is a common phenomenon, wherever one looks: ecological systems, astronomy, human history, stock markets – but what is the logic of dependence? This book is the first to carry out a systematic logical study of this important concept, giving on the way a precise mathematical treatment of Hintikka's independence friendly logic. Dependence logic adds the concept of dependence to first order logic. Here the syntax and semantics of dependence logic are studied, dependence logic is given an alternative game theoretic semantics, and sharp results about its complexity are proven. This is a textbook suitable for a special course in logic in mathematics, philosophy, and computer science departments, and contains over 200 exercises, many of which have a full solution at the end of the book. It is also accessible to readers with a basic knowledge of logic, who are interested in new phenomena in logic.

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and

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Contents

<i>Preface</i>	<i>page</i>	ix
1 Introduction		1
2 Preliminaries		5
2.1 Relations		5
2.2 Vocabularies and structures		5
2.3 Terms and formulas		6
2.4 Truth and satisfaction		7
3 Dependence logic		10
3.1 Examples and a mathematical model for teams		11
3.2 Formulas as types of teams		16
3.3 Logical equivalence and duality		29
3.4 First order formulas		37
3.5 The flattening technique		42
3.6 Dependence/independence friendly logic		44
4 Examples		48
4.1 Even cardinality		48
4.2 Cardinality		51
4.3 Completeness		53
4.4 Well-foundedness		55
4.5 Connectedness		56
4.6 Natural numbers		57
4.7 Real numbers		59
4.8 Set theory		60

Cambridge University Press

978-0-521-70015-3 - Dependence Logic: A New Approach to Independence Friendly Logic

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Frontmatter

[More information](#)

viii

Contents

5	Game theoretic semantics	63
5.1	Semantic game of first order logic	63
5.2	Perfect information game for dependence logic	69
5.3	Imperfect information game for dependence logic	80
6	Model theory	86
6.1	From \mathcal{D} to Σ_1^1	86
6.2	Applications of Σ_1^1	90
6.3	From Σ_1^1 to \mathcal{D}	94
6.4	Truth definitions	100
6.5	Model existence game	110
6.6	Ehrenfeucht–Fraïssé game for dependence logic	121
7	Complexity	134
7.1	Decision and other problems	134
7.2	Some set theory	135
7.3	Σ_2 -completeness in set theory	140
8	Team logic	144
8.1	Preorder of determination	144
8.2	Dependence and independence	148
8.3	Formulas of team logic	150
8.4	From team logic to L^2	158
8.5	From L^2 to team logic	161
8.6	Ehrenfeucht–Fraïssé game for team logic	163
	<i>Appendix: Solutions to selected exercises, by Ville Nurmi</i>	169
	<i>References</i>	220
	<i>Index</i>	223

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Preface

This book is based on lectures I gave at the Department of Mathematics and Statistics, University of Helsinki, during the academic year 2005–2006. I am indebted to the students who followed the course, in particular to Åsa Hirvonen, Meeri Kesälä, Ville Nurmi, Eero Raaste, and Ryan Siders. Thanks also go to Ville Nurmi for suggesting numerous corrections to the text, compiling the solutions to the exercises in the course, and for allowing me to include the solutions in this book. I am very grateful to Wilfrid Hodges for many useful discussions on dependence. I thank the Newton Institute (Cambridge, UK) for inviting me for the five weeks, during which time the final manuscript was prepared. The preparation of the manuscript was partially supported by grant 40734 of the Academy of Finland. I wish to thank Peter Thompson of Cambridge University Press for all the arrangements concerning publishing, and I am deeply grateful to Juliette Kennedy for her generous help in all stages of writing this book.