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978-0-521-69973-0 - Inequalities: A Journey into Linear Analysis

D. J. H. Garling

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INEQUALITIES: A JOURNEY INTO LINEAR ANALYSIS

Contains a wealth of inequalities used in linear analysis, and explains in detail how they are used. The book begins with Cauchy's inequality and ends with Grothendieck's inequality, in between one finds the Loomis–Whitney inequality, maximal inequalities, inequalities of Hardy and of Hilbert, hypercontractive and logarithmic Sobolev inequalities, Beckner's inequality, and many, many more. The inequalities are used to obtain properties of function spaces, linear operators between them, and of special classes of operators such as absolutely summing operators.

This textbook complements and fills out standard treatments, providing many diverse applications: for example, the Lebesgue decomposition theorem and the Lebesgue density theorem, the Hilbert transform and other singular integral operators, the martingale convergence theorem, eigenvalue distributions, Lidskii's trace formula, Mercer's theorem and Littlewood's $4/3$ theorem.

It will broaden the knowledge of postgraduate and research students, and should also appeal to their teachers, and all who work in linear analysis.

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