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978-0-521-69964-8 - Ranks of Elliptic Curves and Random Matrix Theory

Edited by J. B. Conrey, D. W. Farmer, F. Mezzadri and N. C. Snaith

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