1 Measurement and units

1.1 Fundamental quantities and units

In science there are five fundamental, or base, quantities – all other quantities are related to these. Mass (symbol \( m \)), length (\( l \)) and time (\( t \)) are three of these quantities. We will use two others later: electric current and temperature. All other quantities are derived quantities.

To measure a physical quantity we compare it with a standard known as the unit of the quantity. Each base quantity has a base unit (Table 1.1).

Table 1.1 Names and symbols for base SI units

<table>
<thead>
<tr>
<th>Physical quantity</th>
<th>Name of SI Base unit</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>metre</td>
<td>m</td>
</tr>
<tr>
<td>Mass</td>
<td>kilogram</td>
<td>kg</td>
</tr>
<tr>
<td>Time</td>
<td>second</td>
<td>s</td>
</tr>
</tbody>
</table>

A quantity is written as its value followed by its unit, e.g. the height of a girl might be 1.40 m. The system we use is called the Système International d’Unités or the SI system.

1.2 Derived quantities

We can multiply or divide base quantities with their units to produce derived quantities with their units. The area of a rectangle with sides 0.6 m and 0.5 m is given by the product of the sides, i.e. \((0.6 \text{ m}) \times (0.5 \text{ m}) = 0.3 \text{ m}^2\). Thus in the SI system the unit of area is the squared metre (m\(^2\)).

The speed of an object is the distance, in metres, travelled each second. The unit of speed is metres per second. This is written m s\(^{-1}\); s\(^{-1}\) means ‘per second’.

1.3 Length

In your practical course you will use several instruments to measure lengths. A rule is one of the most common. With a rule we can usually measure length to the nearest millimetre.

A rule is not suitable for short distances such as the diameter of wires, thin sheets of material or round objects. In these cases we use a micrometer or calipers.

Micrometer

Figure 1.1 shows a micrometer screw gauge. The horizontal scale is marked in millimetres (mm). As the screw rotates once, the micrometer opens 0.5 mm. Each of the 50 divisions on the circular scale is 0.01 mm. To read the micrometer, we add the reading on the horizontal scale to the reading on the circular scale. We check and allow for any zero error when using a micrometer.

Fig.1.1 Micrometer screw gauge

Calipers

We use calipers to measure diameters of rods and balls. We then measure the distance between the caliper points using a rule (see Fig.1.2a).

Fig.1.2 (a) Calipers

(b) Vernier calipers

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We also use vernier calipers and the vernier scale permits greater precision, to 0.01 cm. A vernier scale is a small scale that slides along the main scale (see Fig. 1.2b).

We see that one mark on the vernier scale coincides with a mark on the main scale. In Fig. 1.2b the zero mark on the vernier indicates that the reading is 2.5 cm and a bit. The first mark on the vernier coincides with a mark on the main scale, showing the full reading to be 2.51 cm.

1.4 Area
The area of a square = (side)².
With irregular shapes we can divide the area into small squares, of known size, and estimate the total number of squares.

1.5 Volume
To find the volumes of rectangular solids we measure the lengths of the sides. Then

\[ \text{volume} = \text{length} \times \text{width} \times \text{height} \]
(For irregular solids see Section 1.7.)

We find the volume of liquids using a measuring cylinder and looking carefully at the meniscus. Remember: we read the base of the meniscus for water and the top of the meniscus for mercury.

1.6 Mass
A chemical balance is used to measure mass very accurately but a lever balance is often accurate enough (Fig. 1.5).

The scale on a lever balance is a non-linear scale – the marks are not evenly spaced. Most scales that we use are linear scales – the marks are evenly spaced.

In Unit 4 we look at the important distinction between mass and weight.

1.7 Density
Blocks of material can have the same volume but different masses. A wooden block may have a mass of 48 g while a block of iron of the same volume has a mass of about 420 g. The materials have different densities.

The density (\( \rho \)) of a material is defined as its mass per unit volume:

\[
\rho = \frac{\text{mass (kg)}}{\text{volume (m}^3\text{)}}
\]

The unit of density is kg m\(^{-3}\) (remember that the m\(^{-3}\) means 'per metre cubed'). The density of water is 1 000 kg m\(^{-3}\) or 1 g cm\(^{-3}\). The latter unit is used in measurements involving small masses and volumes.

Determination of the density of solids
We measure the mass and volume of a sample and calculate the density. We use a lever balance to find the mass of the sample.
We find the volume of rectangular solids by measuring the lengths of the sides and finding the product of the length, width and height.

We immerse irregular objects, such as stones, in water. The volume of water that they displace equals the volume of the object. For a large object we use a ‘eureka’ or displacement can. We measure the volume of water that overflows (Fig. 1.6).

We can add a small object directly to water in a measuring cylinder. The rise in the reading is equal to the volume of the object. Care must be taken to submerge the object carefully without splashing to prevent droplets from sticking to the sides of the container.

1.8 Time
You will use a stopwatch or stop clock to measure time intervals. Your reaction time causes inaccuracy in your timings. You should repeat timings and average your results. Many schools use stop clocks that have an error of ± 0.5 s.

Simple pendulum
A simple pendulum is a small mass hanging on a thin string. It oscillates from side to side. The time period of a simple pendulum is the time taken for one oscillation. (A complete oscillation is from one side, across to the other side and back to the original point.)

The amplitude of the oscillator is the maximum distance of the bob from its rest position.

You will investigate the factors that determine the time period. You should time 20 or more vibrations, more than once, to improve the accuracy. You should consider changing the mass of the bob, the amplitude of the oscillation and the length of the string. You must change just one factor (or variable) at a time.

You will find that the time period depends only on the length of the string (as long as the amplitude is not too large i.e. <10° from vertical).

1.9 Relative density
It is useful to compare the density of a substance with the density of water.

Relative density (ρ_r) is defined as follows:

\[
\text{relative density of a substance} = \frac{\text{density of the substance}}{\text{density of water}}
\]

Relative density is a ratio and has no unit.
We can also use the following expression:

\[
\text{Relative density} = \frac{\text{mass of a volume of the substance}}{\text{mass of the same volume of water}}
\]

1.10 Quality control
In order to protect the consumer, quality control standards are developed and enforced, e.g.

1. Regular checks are made of weighing machines.
2. Electrical appliances must meet strict safety standards. Safety standards are also applied to the construction and wiring of buildings.
3. The strength of milk or beer can be checked by measurements of their densities.
4. The amount of possibly harmful chemicals in processed food is monitored.
2 Measurement and mathematics

2.1 Measurement and significant figures

During your physics course you will record results of experiments. Look at Fig. 2.1. You can see that the length of side AB is somewhere between 2.0 cm and 2.1 cm and you might estimate the length to be 2.08 cm. However, the ’8’ is not very certain. The number of figures that you write gives an indication of your confidence in the result. It would not be sensible in this example to write 2.085 cm. It is best written 2.1 cm.

Fig. 2.1 Approximate measurement

Lengths measured with a metre rule can often only be written to the nearest millimetre. So we may write, for example, 24.2 cm (or 0.242 m). In each case here we have three significant figures. When we calculate a value from our results the answer should be written to the same numbers of significant figures as the original results.

If the two sides of a rectangle are measured as 24.2 cm and 18.3 cm, then

Area of rectangle = 24.2 cm × 18.3 cm

= 442.86 cm² (by calculator)

= 443 cm² (three significant figures)

In any calculation, the reading with the smallest number of significant figures determines the number of significant figures of the final answer.

2.2 Reading scales

Many readings you make in physics are taken from a scale on an instrument. Some were described in Unit 1; thermometers (temperature), ammeters (electric current) and spring balances (force) are other examples.

When reading a scale you make an estimate when the pointer is not actually on a mark on the scale. Fig. 2.2 is part of the scale of an ammeter used to measure current in amperes (A). This would be recorded as 1.35 A.

Results can often be taken to three significant figures.

2.3 Accuracy of results

For an experiment to be useful we must obtain accurate results. In any reading there is some uncertainty, which we can reduce as follows:

1 We can take the same reading more than once and calculate an average value.
2 We can measure a large number of a quantity and calculate the value for one. For example, if we have to find the thickness of a sheet of paper we can measure the thickness of 300 sheets. We then divide our result by 300 to find the thickness of one sheet.
3 We can select an instrument that is appropriate to the reading. If a current of about 0.4 A is being measured, we use an ammeter with a range of 0 to 1 A, not 0 to 5 A.
4 We take particular care to avoid ‘parallax’ (see Unit 17). We always read scales from directly over the mark (Fig. 2.3).

Fig. 2.2 Ammeter scale

Fig. 2.3 Reading a scale
2.4 Large and small numbers

When we have very large and small numbers there are useful alternative ways to write them.

Standard form

In standard form we write numbers in two parts as follows:

\[
\begin{align*}
100 & = 10 \times 10 = 10^1 = 1.0 \times 10^1 \\
240 & = 2.4 \times 100 = 2.4 \times 10^2 \\
3600000 & = 3.6 \times 1000000 = 3.6 \times 10^6 \\
0.3 & = 3 \times \frac{1}{10} = 3.0 \times 10^{-1} \\
0.0024 & = 2.4 \times \frac{1}{1000} = 2.4 \times 10^{-3}
\end{align*}
\]

You should also be able to multiply and divide numbers in standard form:

\[
\begin{align*}
(3 \times 10^3) \times (2 \times 10^4) & = 6 \times 10^7 \\
(4 \times 10^3) \times (6 \times 10^4) & = 2.4 \times 10^8 \\
(6 \times 10^3) / (3 \times 10^2) & = 2 \times 10^5 \\
(6 \times 10^3) / (3 \times 10^5) & = 2 \times 10^{-3}
\end{align*}
\]

Prefixes

We also use prefixes for convenience. The following examples show their meaning.

\[
\begin{align*}
1 \text{ MJ} & = 1 \text{ mega}joule = 1 \text{ million joules} = 10^6 \text{ J} \\
1 \text{ km} & = 1 \text{ kilometre} = 1 \text{ thousand metres} = 10^3 \text{ m} \\
1 \text{ cm} & = 1 \text{ centi}metre = 1 \text{ hundredth metre} = 10^{-2} \text{ m} \\
1 \text{ mm} & = 1 \text{ milli}metre = 1 \text{ thousandth metre} = 10^{-3} \text{ m} \\
1 \text{ µA} & = 1 \text{ microampere} = 1 \text{ millionth ampere} = 10^{-6} \text{ A}
\end{align*}
\]

Changing units from one to the other

\[
\begin{align*}
1 \text{ m}^3 & = 1 \text{ m} \times 1 \text{ m} \times 1 \text{ m} \\
& = 100 \text{ cm} \times 100 \text{ cm} \times 100 \text{ cm} \\
& = 10^6 \\
\text{i.e. } 1 \text{ m}^3 & = 10^6 \text{ cm}^3 \\
\text{also } 1 \text{ m}^3 & = (100 \times 100) \text{ cm}^3 \\
1 \text{ m}^3 & = 100000 \text{ cm}^3
\end{align*}
\]

Sometimes these conversions are necessary because we would not record the volume of a medicine bottle in m³ but in the more convenient form of cm³.

Density may be expressed in kg m⁻³, but in a situation involving small quantities it may be more practical to express it in g cm⁻³.

In converting 1 g cm⁻³ to kg m⁻³, the following reasoning is used:

\[
\begin{align*}
1 \text{ kg} & = 1000 \text{ g} \\
1 \text{ m}^3 & = 10^6 \text{ cm}^3 \\
1 \text{ g} & \text{ is divided by } 1000 \text{ to covert to kg} \\
\text{and } 1 \text{ cm}^3 & \text{ is multiplied by } 10^6 \text{ to convert to m}^3 \\
i.e. 1 \text{ g cm}^{-3} & = \frac{1}{1000} \times \frac{10^6 \text{ cm}^3}{1} \\
& = 1000 \text{ kg m}^{-3}
\end{align*}
\]

2.5 Graphs

A common way to present results is to draw a graph. The graph often provides us with extra information and helps our understanding.

When plotting a graph the axes are labelled with the quantities involved, their symbols and the units of the quantities. We choose a sensible and convenient scale so that the results occupy most of the graph. For instance, if a set of results of length has a range from 1.2 cm to 8.4 cm, the scale would be 0 to 10 cm, not 0 to 100 cm.

We use a small cross, or a dot with a small circle around it, to indicate the plotted points. We plot results as accurately as possible – we do not ‘round off’ to make them easier to plot.

Our results are often approximately in a straight line. We draw a line of best fit. This line goes as close as possible to as many points as possible. The points should also be ‘balanced’ about the line, with equal numbers above and below the line (if they are not exactly on the line). Sometimes the shape of a graph will be a smooth curve.

The line of best fit has the effect of averaging experimental inaccuracies.

Common graphs

If a graph is a straight line passing through the origin, this means that the two quantities are proportional to each other (Fig. 2.4).
A graph can also be a straight line but **not** pass through the origin – the quantities are then **linearly related** to each other but are **not** proportional to each other (Fig. 2.5).

**Fig. 2.5 Straight-line graph not passing through origin**

**Gradient and intercepts of a graph**

Two important quantities of a straight-line graph are its gradient and its intercepts on the axes.

In Figs. 2.5 and 2.6 the intercept on the Y-axis is 4. If we draw the graph line until it hits the X-axis, the intercept is a **negative value** -8 (Fig. 2.6).

This triangle should be wider than half the width of the graph paper and the height should be greater than half the height of the page.

**Fig. 2.6 Intercepts on a graph**

\[
\text{gradient} = \frac{\text{the change in the value of } y}{\text{the change in the value of } x} = \frac{y_2 - y_1}{x_2 - x_1}
\]

In our example the gradient is:

\[
\frac{10 - 4}{12 - 0} = \frac{6}{12} = \frac{1}{2}
\]

**Questions on Unit 2**

1. Calculate the values of the following:
   a. \((6 \times 10^3) \times (5 \times 10^3)\)
   b. \((4 \times 10^{-2}) \times (3 \times 10^2)\)
   c. \((6 \times 10^3) / (3 \times 10^9)\)
   d. \((4 \times 10^{-6}) / (2 \times 10^{-3})\)

2. A student carries out an experiment to find the density of wood in a wooden block. The mass of the block is recorded as 163 g and its volume as 240 cm³. Which of the following values should the student write for the density of the material?
   a. 0.7 g cm⁻³
   b. 0.68 g cm⁻³
   c. 0.679 g cm⁻³
   d. 0.6791 g cm⁻³

3. Examine the graph drawn below.

   a. What is the gradient of this graph?
   b. What is the value of \(T\) when \(m\) is 8.0?
   c. What is the value of \(m\) when \(T\) is 0.0?
   d. Does this graph indicate that \(m\) and \(T\) are proportional to each other? Explain your answer.
3.1 Vectors and scalars
Some quantities need to have both their size and direction stated to fully describe the quantity. These quantities are known as vectors. Examples of vectors are displacement, velocity, acceleration and force.

Other quantities, which only have size (but no direction), are known as scalars. Examples of scalars are distance, speed and mass.

In Unit 5 we discuss the addition of vectors and scalars.

3.2 Distance and displacement
When we calculate the total distance travelled by an object we take no account of the directions in which it travels. Distance is a scalar quantity.

The displacement is defined as the distance moved in a particular direction. Displacement is a vector quantity.

Figure 3.1 represents a man walking on a football field. He starts at P and walks 50 m due east – a displacement. He then walks 50 m due north. The total distance he has travelled is 100 m but his displacement is about 71 m north-east of his starting point.

![Fig. 3.1 Distance and displacement](image)

3.3 Speed
Speed is defined as the distance moved per second. Speed is a scalar quantity.

If the speed does not vary (if it is constant or uniform) then

\[
\text{speed} = \frac{\text{distance travelled}}{\text{time taken}}
\]

Distance is measured in metres and time in seconds. The unit of speed is metres per second, m s\(^{-1}\).

Remember, in the unit m s\(^{-1}\) the –1 means ‘divided by’ or ‘per’. The symbol for speed is \(v\).

3.4 Velocity
Velocity is defined as the displacement per second. Thus the velocity is the distance moved per second in a particular direction. Velocity is a vector quantity.

If the velocity is constant,

\[
\text{velocity} = \frac{\text{displacement}}{\text{time taken}}
\]

The unit of velocity is metres per second, m s\(^{-1}\). The symbol for velocity is also \(v\). For a velocity to be constant, both the speed and direction must be constant.

Changing m s\(^{-1}\) to km h\(^{-1}\)

\[
20 \text{ m s}^{-1} = (20 \times \frac{3600}{1}) \text{ km h}^{-1}
\]

we divide by 1 000 to change metres to seconds and we multiply by 3 600 to change ‘per second’ to ‘per hour’.

(There are 3600 s in 1 h.)

3.5 Acceleration
Acceleration is defined as the rate of change of velocity, i.e. the change of velocity per second.

If the acceleration is constant,

\[
\text{acceleration} = \frac{\text{change of velocity}}{\text{time taken}}
\]

The unit of acceleration is metres per second squared, m s\(^{-2}\). This means a certain change of velocity, in m s\(^{-1}\), each second.

Acceleration is a vector quantity and its symbol is \(a\).
3.6 Representing motion using graphs

We use graphs to represent and analyse motion. The most useful to us are graphs of displacement against time, and velocity against time.

**Constant velocity**

If a car is moving at a constant velocity of 15 m s\(^{-1}\), then each second it will travel 15 metres, in the same direction.

Two graphs can represent this motion:

1. **Displacement/time graph** – the gradient of the graph (Fig. 3.2) represents the car’s motion. The gradient of the line is \(\frac{60}{4} = 15\). The gradient of a displacement/time graph always represents the velocity.

![Fig. 3.2 Displacement/time graph](image)

2. **Velocity/time graph** – the gradient of the graph represents the acceleration. In the example (Fig. 3.3) the velocity is constant, so the gradient and acceleration are zero.

![Fig. 3.3 Velocity/time graph: constant velocity](image)

The area under a velocity/time graph represents the distance travelled. So in the 4 s, the distance travelled is 60 m.

Velocity/time graphs are more useful than distance/time graphs. All four quantities are represented on a velocity/time graph: velocity, time, distance and acceleration.

**Uniform acceleration**

If a car starts from rest and has an acceleration of 5 m s\(^{-1}\), each second its velocity increases by 5 m s\(^{-1}\). We represent this in a velocity/time graph (Fig. 3.4).

![Fig. 3.4 Velocity/time graph: uniform acceleration](image)

We can use the graph to calculate the distance travelled in 6 s. Remember that the distance travelled is represented by the area under the graph. The triangle has an area of

\[
\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 6 \times 30 = 90
\]

The distance travelled in 6 s is 90 m.

**Example**

A bus starts from rest and accelerates at 3 m s\(^{-1}\) for 5 s. It travels at a constant speed for 10 s and then quickly comes to a halt in a further 3 s.

Draw a velocity/time graph to represent this motion and calculate the distance between the two stops.

**Answer**

The distance travelled is equal to the area under the graph. We calculate this area by considering the three stages separately.

1. During the acceleration the area under the graph is the area of the triangle, \(\frac{1}{2} \times 5 \times 15 = 37.5 \text{ m}^2\).
2. During the constant speed the area of the rectangle is \(10 \times 15 = 150 \text{ m}^2\).
3. During the deceleration the area is again the area of a triangle:

\[
\frac{1}{2} \times 3 \times 15 = 22.5
\]

The total area is 37.5 + 150 + 22.5 = 210

The distance travelled is 210 m.
3.7 Equations of uniformly accelerated motion

When we have objects moving with a constant acceleration we can also use certain equations to solve problems. In these equations we use the following symbols:

- initial velocity \( u \)
- final velocity \( v \)
- acceleration \( a \)
- displacement \( x \)
- time \( t \)

Acceleration = change in velocity / time taken

\[ a = \frac{v - u}{t} \]

Rearranging,

\[ at = v - u \]

and \[ v = u + at \] (1)

Also,

velocity = displacement / time taken

displacement = velocity \( \times \) time taken

This is only true if the velocity is constant. If the velocity is changing then

\[ \text{displacement} = \frac{\text{average velocity} \times \text{time taken}}{2} \] (2)

We now substitute into equation (2).

As \[ v = u + at \]

\[ x = \frac{(u + v) \times t}{2} \]

Thus \[ x = ut + \frac{1}{2} at^2 \] (3)

Also, substituting \[ t = \frac{v - u}{a} \] in equation (2),\n
\[ x = \frac{(u + v)}{2} \times \left( \frac{v - u}{a} \right) \]

Rearranging, \[ 2ax = v^2 - u^2 \]

and \[ v^2 = u^2 + 2ax \] (4)

Equations (1), (2), (3) and (4) are called the equations of motion. We can only apply them when the acceleration is constant throughout the motion.

In applying the equations of motion, the student should read the questions carefully, write down all data given and decide which variable has to be found. It is then easy to match up which of the four examples of motion he/she can use.

Examples of use of equations of motion

1. A racing car has a constant acceleration, from rest, of 4.0 m s\(^{-2}\). How far does it travel in the first 10 s?

   The initial velocity is zero, \( u = 0 \), \( a = 4.0 \text{ m s}^{-2} \), \( t = 10 \text{ s} \), \( x = ? \text{ m} \).

   We use the third equation of motion:

   \[ x = ut + \frac{1}{2} at^2 \]

   \[ x = 0 \times 10 + \frac{1}{2} \times 4 \times 10^2 \]

   \[ x = 2 \times 100 = 200 \text{ m} \]

2. What deceleration is needed to bring a car, with a velocity of 20 m s\(^{-1}\), to rest in 80 m?

   The final velocity is zero, \( v = 0 \), \( u = 20 \text{ m s}^{-1} \), \( s = 80 \text{ m} \), \( a = ? \text{ m s}^{-2} \).

   \[ v^2 = u^2 + 2ax \]

   \[ 0^2 = (20^2 + 2 \times a \times 80) \]

   \[ = 400 + 160a \]

   \[ -400 = 160a \]

   \[ a = -2.5 \text{ m s}^{-2} \]

A deceleration is a negative acceleration.

3.8 Measuring velocities and accelerations

In the laboratory we use a ticker-tape timer to measure velocities and accelerations (Fig. 3.5). It has a strip of metal that vibrates 60 times a second. A dot is printed at each 1/60th second on the paper that runs underneath.

![Fig. 3.5 Ticker-tape timer](ticker-tape-timer.png)
The tape can be pulled at a constant speed through the timer. The tape is then cut up into sections of ‘10-dot’ lengths, each 1/6th second. If the speed is constant then the sections of the tape obtained are the same length. The length of each of the strips is the distance travelled in 1/6th second.

The speed, distance per second, is calculated by multiplying by 6, the distance travelled in 1/6th second.

A graph can also be made from the tapes (Fig.3.6).

Calculating acceleration from the tapes

If the tape is pulled so that it accelerates through the timer, the spacing of the dots increases. We cut 10-dot lengths and make a new graph (Fig. 3.7). The lengths of the tapes increase as the speed increases. The rate of increase is constant if the acceleration is constant.

We calculate the acceleration by measuring the change of speed in a certain time. The initial velocity in Fig. 3.7 was 15 cm s\(^{-1}\) and after one second the velocity was 165 cm s\(^{-1}\).

\[
\text{acceleration} = \frac{\text{change in velocity}}{\text{time taken}} = \frac{165 - 15}{1} = 150 \text{ cm s}^{-2}
\]

Fig. 3.6 Graph from ticker-tape timer

Fig. 3.7 Calculating acceleration

3.9 Falling objects

We can drop a large stone and a piece of chalk from a height of 3 or 4 m and they will land simultaneously. However, a stone will reach the ground faster than a piece of paper. If the stone and paper are allowed to fall in a vacuum, they fall together.

When objects fall in air, the resistance of the air has a greater effect on objects with larger surface areas, such as paper, than on heavy ones, such as the stone. Fluid friction (or viscous drag) always opposes the fall of objects through fluids, i.e. liquids and gases.

Acceleration due to gravity

If the effects of air resistance are eliminated or are negligible, then all objects fall with the same acceleration. This is called the acceleration due to gravity, \(g\).

The acceleration due to gravity has slightly different values in different parts of the world but the average value is 9.81 m s\(^{-2}\). We often take \(g\) as 10 m s\(^{-2}\), as a convenient approximation.

An object falling vertically gains speed, 10 m s\(^{-1}\) in each second.