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Cambridge University Press  
978-0-521-69182-6 — Surveys in Geometry and Number Theory  
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London Mathematical Society Lecture Note Series. 338

# Surveys in Geometry and Number Theory: Reports on contemporary Russian mathematics

*Edited by*

NICHOLAS YOUNG  
*University of Leeds*



**CAMBRIDGE**  
UNIVERSITY PRESS

Cambridge University Press  
978-0-521-69182-6 — Surveys in Geometry and Number Theory  
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[More Information](#)

CAMBRIDGE  
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom  
One Liberty Plaza, 20th Floor, New York, NY 10006, USA  
477 Williamstown Road, Port Melbourne, VIC 3207, Australia  
314-321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi - 110025, India  
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Information on this title: [www.cambridge.org/9780521691826](http://www.cambridge.org/9780521691826)

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First published 2007

*A catalogue record for this publication is available from the British Library*

ISBN 978-0-521-69182-6 Paperback

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## Preface

This volume is a showcase for the continuing vitality of Russian mathematics in fields related to algebraic geometry. The Eastern European scientific diaspora may have weakened the magnificent edifice of Russian mathematics, but the Russian school had both strength and depth, and there remains a great deal of important scientific activity in the country. Universities continue to attract some of the most able students into mathematics, and their graduates have the knowledge and enthusiasm to be effective participants in the global mathematical endeavour.

There are great difficulties facing new Russian ‘Candidates of Science’ in mathematics. It is rare for a young person to gain a living wage as a lecturer and researcher alone. It requires dedication, self-sacrifice and a willingness to look for other sources of income for a mathematician to become established while remaining in Russia. It is not surprising that many talented mathematicians seek and find employment abroad. Despite these handicaps there are strong research groups that continue to foster new talents.

In the fields of algebraic geometry and algebraic number theory there are healthy groups, particularly those centred around the Steklov Institute in Moscow. To give some examples, in the birational geometry of 3-folds there is a group of four well-established experts who support about 10 research students and postdoctoral fellows. Another group of specialists pioneered the idea of the derived category of coherent sheaves on a variety (up to equivalence) as a geometric invariant of the variety, analogous to K-theory or cohomology theories, and continues to work in this fruitful area. Another strong strand of research is in algebraic and complex versions of quantisation relating to special geometries such as special Lagrangian fibrations in mirror symmetry.

The London Mathematical Society set up the ‘Young Russian Mathematicians’ scheme to help these mathematicians to visit the UK and to provide them with some financial support. Visitors give lectures in this country and write a survey article on the work of their research groups, for which they receive payment. This is the first volume of such survey articles to be published by the Society.

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