

CONTENTS

Preface to the second edition	<i>page</i> xv
Preface	xvii
0 Background material	1
0.1 Basic concepts and terminology	1
0.2 Transformations	3
0.3 Basic convexity	6
0.4 The Hausdorff metric	9
0.5 Measure and integration	10
0.6 The support function	16
0.7 Star sets and the radial function	18
0.8 Polar duality	20
0.9 Differentiability properties	22
1 Parallel X-rays of planar convex bodies	28
1.1 What is an X-ray?	28
1.2 X-rays and Steiner symmetrals of planar convex bodies	29
Open problems	52
Notes	52
1.1. Computerized tomography	52
1.2. Parallel X-rays and Steiner symmetrals of convex bodies	54
1.3. Exact reconstruction	55
1.4. Well-posedness and stability	56
1.5. Reconstruction of convex bodies from possibly noisy data	57
1.6. Geometric probing	58
1.7. Jakob Steiner (1796–1863)	59

2 Parallel X-rays in n dimensions	60
2.1 Parallel X-rays and k -symmetrals	61
2.2 X-rays of convex bodies in \mathbb{E}^n	65
2.3 X-rays of bounded measurable sets	69
Open problems	86
Notes	87
2.1. <i>Parallel X-rays and k-symmetrals of convex bodies</i>	87
2.2. <i>Switching components and discrete tomography</i>	88
2.3. <i>Parallel X-rays and k-symmetrals of measurable sets</i>	91
2.4. <i>Blaschke shaking</i>	92
2.5. <i>Reconstruction of polygons and polyhedra from possibly noisy X-rays</i>	93
2.6. <i>Ridge functions and the additivity conjecture</i>	94
2.7. <i>X-rays of bounded density functions</i>	94
2.8. <i>Johann Radon (1887–1956)</i>	95
3 Projections and projection functions	97
3.1 Homothetic and similar projections	98
3.2 The width function and central symmetrals	106
3.3 Projection functions and the Blaschke body	110
Open problems	125
Notes	126
3.1. <i>Homothetic and similar projections</i>	126
3.2. <i>Bodies with congruent or affinely equivalent projections</i>	128
3.3. <i>Sets of constant width and brightness</i>	129
3.4. <i>Blaschke bodies and Blaschke sums</i>	130
3.5. <i>Determination by one projection function</i>	131
3.6. <i>Determination by more than one projection function</i>	132
3.7. <i>Determination by directed projection functions, etc</i>	134
3.8. <i>Reconstruction</i>	135
3.9. <i>Mean projection bodies</i>	137
3.10. <i>Projections of convex polytopes</i>	137
3.11. <i>Critical projections</i>	138
3.12. <i>Almost-spherical or almost-ellipsoidal projections, and related results</i>	138
3.13. <i>Aleksander Danilovich Aleksandrov (1912–1999)</i>	139
4 Projection bodies and volume inequalities	141
4.1 Projection bodies and related concepts	142
4.2 Smaller bodies with larger projections	154
4.3 Stability	164

	Contents	xi
4.4 Reconstruction from brightness functions	171	
Open problems	180	
Notes	180	
4.1. <i>Projection bodies and zonoids</i>	180	
4.2. <i>The Fourier transform approach I: The brightness function and projection bodies</i>	182	
4.3. <i>The Minkowski map and Minkowski linear combinations of projection bodies</i>	183	
4.4. <i>Generalized zonoids</i>	184	
4.5. <i>Bodies whose projections are zonoids</i>	185	
4.6. <i>Projection bodies of order i</i>	185	
4.7. <i>The L^p-Brunn–Minkowski theory and L^p-projection bodies</i>	186	
4.8. <i>Characterizations in terms of mixed volumes</i>	186	
4.9. <i>Results related to Aleksandrov's projection theorem</i>	187	
4.10. <i>Smaller bodies with larger projections</i>	187	
4.11. <i>Stability results</i>	189	
4.12. <i>Reconstruction from brightness functions</i>	190	
4.13. <i>Hermann Minkowski (1864–1909)</i>	192	
5 Point X-rays	194	
5.1 Point X-rays and chordal symmetrals	195	
5.2 The X-ray of order i	201	
5.3 Point X-rays of planar convex bodies	206	
5.4 X-rays in the projective plane	221	
Open problems	225	
Notes	226	
5.1. <i>Point X-rays and chordal symmetrals</i>	226	
5.2. <i>Point X-rays of planar convex bodies</i>	226	
5.3. <i>Reconstruction</i>	227	
5.4. <i>Well-posedness</i>	228	
5.5. <i>Point X-rays in higher dimensions</i>	228	
5.6. <i>Discrete point X-rays</i>	228	
5.7. <i>Point projections</i>	229	
5.8. <i>Wilhelm Süss (1895–1958) and the Japanese school</i>	229	
6 Chord functions and equichordal problems	232	
6.1 i -chord functions and i -chordal symmetrals	233	
6.2 Chord functions of star sets	237	
6.3 Equichordal problems	254	
Open problems	264	
Notes	264	
6.1. <i>Chord functions, i-chordal symmetrals, and ith radial sums</i>	264	

6.2. <i>Chord functions of star sets</i>	265
6.3. <i>Equichordal problems</i>	265
6.4. <i>Wilhelm Blaschke (1885–1962)</i>	267
7 Sections, section functions, and point X-rays	269
7.1 Homothetic and similar sections	270
7.2 Section functions and point X-rays	276
7.3 Point X-rays of measurable sets	286
Open problems	288
Notes	289
7.1. <i>Homothetic and similar sections</i>	289
7.2. <i>Bodies with congruent or affinely equivalent sections</i>	290
7.3. <i>Sets of constant section</i>	290
7.4. <i>Determination by section functions</i>	290
7.5. <i>Determination by half-volumes</i>	291
7.6. <i>Point X-rays of measurable sets</i>	293
7.7. <i>Sections of convex polytopes</i>	294
7.8. <i>Critical sections</i>	295
7.9. <i>Almost-spherical or almost-ellipsoidal sections</i>	296
7.10. <i>A characterization of star-shaped sets</i>	297
7.11. <i>Sections by other sets of planes</i>	297
7.12. <i>Integral geometry</i>	299
7.13. <i>Mean section bodies</i>	300
7.14. <i>Stereology</i>	301
7.15. <i>Local stereology</i>	301
7.16. <i>Paul Funk (1886–1969)</i>	303
8 Intersection bodies and volume inequalities	304
8.1 Intersection bodies of star bodies	305
8.2 Larger bodies with smaller sections	324
8.3 Cross-section bodies	334
Open problems	336
Notes	337
8.1. <i>Intersection bodies</i>	337
8.2. <i>The Fourier transform approach II: The section function and intersection bodies</i>	340
8.3. <i>The map I</i>	340
8.4. <i>Generalized intersection bodies</i>	340
8.5. <i>Bodies whose central sections are intersection bodies</i>	341
8.6. <i>Intersection bodies of order i</i>	341
8.7. <i>k-intersection bodies and related notions</i>	341
8.8. <i>Characterizations in terms of dual mixed volumes</i>	342

	Contents	xiii
8.9. <i>Larger bodies with smaller sections I: The Busemann–Petty problem</i>	343	
8.10. <i>Larger bodies with smaller sections II: Generalizations and variants of the Busemann–Petty problem</i>	345	
8.11. <i>Stability results</i>	346	
8.12. <i>Cross-section bodies</i>	346	
8.13. <i>Problems involving both projections and sections</i>	348	
8.14. <i>Herbert Busemann (1905–1994)</i>	348	
9 Estimates from projection and section functions	350	
9.1 Centroid bodies	351	
9.2 Some affine isoperimetric inequalities	355	
9.3 Volume estimates from projection functions	362	
9.4 Volume estimates from section functions	370	
Open problems	375	
Notes	376	
9.1. <i>Centroid bodies and polar projection bodies</i>	376	
9.2. <i>The floating body problem</i>	376	
9.3. <i>Affine surface area, the covariogram, and convolution and sectional bodies</i>	377	
9.4. <i>Affine isoperimetric inequalities</i>	379	
9.5. <i>The L^p-Brunn–Minkowski theory: centroid bodies, ellipsoids, and inequalities</i>	380	
9.6. <i>Volume estimates from projection functions</i>	382	
9.7. <i>Volume estimates from section functions</i>	384	
9.8. <i>The slicing problem</i>	385	
9.9. <i>Central limit theorems for convex bodies</i>	387	
9.10. <i>Estimates concerning both projections and sections</i>	388	
9.11. <i>Estimates for inradius and circumradius</i>	389	
9.12. <i>Hugo Hadwiger (1908–1981)</i>	389	

Appendices

A Mixed volumes and dual mixed volumes	391
A.1 An example	392
A.2 Area measures	395
A.3 Mixed volumes and mixed area measures	397
A.4 Reconstruction from surface area measures	401
A.5 Quermassintegrals and intrinsic volumes	403
A.6 Projection formulas	406
A.7 Dual mixed volumes	409

B Inequalities	413
B.1 Inequalities involving means and sums	413
B.2 The Brunn–Minkowski inequality	415
B.3 The Aleksandrov–Fenchel inequality	419
B.4 The dual Aleksandrov–Fenchel inequality	420
B.5 Other inequalities	422
C Integral transforms	424
C.1 X-ray transforms	424
C.2 The cosine and spherical Radon transforms	427
Open problem	436
References	437
Notation	471
Author index	476
Subject index	482