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P. G. Drazin and N. Riley

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P. G. DRAZIN

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Preface

The origins of this book date to a conversation between the authors a short time before both were due to (formally) retire. Sadly little had been achieved before the more experienced author died. As a consequence any shortcomings of the book must be attributed to the surviving author.

Exact solutions of any system of partial differential equations attract attention. This must be particularly true of the Navier–Stokes equations which, for the best part of 200 years, have been the foundation for the significant and world-wide study of the behaviour of fluids in motion. The subject burgeoned in the twentieth century from stimuli as diverse as international conflict, and a desire to create a better understanding of the environment. In the nineteenth century theoretical advance was slow, and until the approximate or, as we would rather view them, asymptotic theories of Stokes and Prandtl for small and large values of the Reynolds number were devised, only exact solutions, and few at that, were available. In spite of the advances in asymptotic methods during the first half of the twentieth century, and the increasing use of computational methods in its later decades, exact solutions of the Navier–Stokes equations have been pursued. At best these provide an insight into the behaviour of fluids in motion; they may also provide a vehicle for novel mathematical methods or a useful check for a computer code. Some, it must be admitted, provide little of value in either of these senses. The end of the twentieth century appeared to the authors, when computational methods were clearly dominating theoretical research in fluid dynamics, to be an appropriate time to survey the range of exact solutions. We have brought together material from diverse sources, and attempted to present that material in a uniform and coherent manner. The monograph may be viewed as a supplement or complement to the earlier extensive review by R. Berker,

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and subsequent review articles by C. Y. Wang. We believe that the content will be of value to those with an interest in the subject of fluid mechanics, and not least to those who are teaching or learning the subject.

In dedicating the book to the memories of Leslie Howarth, James Lighthill and Keith Stewartson we are mindful of their significant contributions, not only to fluid mechanics but also to the careers of the authors.

*N. Riley
Norwich 2005*