

Cambridge University Press & Assessment
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Edited by Michael H. Albert, Richard J. Nowakowski
Frontmatter
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This fascinating look at combinatorial games, that is, games not involving chance or hidden information, offers updates on standard games such as Go and Hex, on impartial games such as Chomp and Wythoff's Nim, and on aspects of games with infinitesimal values, plus analyses of the complexity of some games and puzzles and surveys on algorithmic game theory, on playing to lose, and on coping with cycles. The volume is rounded out with an up-to-date bibliography by Aviezri S. Fraenkel and, for readers eager to get their hands dirty, a list of unsolved problems by Richard K. Guy and Richard J. Nowakowski.

Highlights include some of Aaron N. Siegel's groundbreaking work on loopy games, the unveiling by Eric J. Friedman and Adam S. Landsberg of the use of renormalization to give very intriguing results about Chomp, and Teigo Nakamura's "Counting liberties in capturing races of Go."

Like its predecessors, this book should be on the shelf of all serious games enthusiasts.

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Games of No Chance 3

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Preface

The June 2005 Combinatorial Game Theory Workshop was held at the Banff International Research Station (BIRS) and organized by Elwyn Berlekamp, Martin Mueller, Richard J. Nowakowski, and David Wolfe. It attracted researchers from Asia, Europe, and North America. The highlights were many and the results already have had a great impact in the field.

Aaron N. Siegel had his hand in quite a few of the highlights. Some of his groundbreaking work is presented in “Loopy games,” with applications given in “Coping with cycles” and “Backsliding Toads and Frogs.” His work with J. P. Grossman, “Reductions of partizan games,” showed that the *reduced canonical form* of a game exists. This approximation to the canonical form has already proved very useful; it has been used, for example, in the Mesdal ensemble’s analysis of Partizan Splittles as well as in the analysis of other games.

In “Advances in losing,” Thane E. Plambeck surveys the state of the art for last-player-to-move-loses games. Even more advances in this area were made during the conference, primarily by Plambeck and Siegel. Indeed, the approach of forming a monoid of game positions in order to discover the structure of winning and losing positions in a *misère* game has subsequently matured sufficiently that the BIRS Games Workshop in January 2008, had *misère* games as one of its main themes.

Eric J. Friedman and Adam S. Landsberg unveiled the use of renormalization, a technique from physics, to give very intriguing results about Chomp. This caught the participants off guard and engendered much discussion. It is surprising that a technique known for giving approximations gave such definite results. Like the previous two topics, there is much to be said and explored here.

Another talk that excited many of the participants was Teigo Nakamura’s “Counting liberties in Go capturing races.” By popular demand, that talk was extended by an hour. *Chilling by 2* is the main idea, but its application is the first ever to appear in analysis of real games, and it appears in a very surprising context.

This volume includes many other interesting papers. To help the reader drill down to a particular topic, the articles are grouped into a number of separate areas of interest.

- *Surveys*, which is self-explanatory;
- *Standards*, which refers to well-known partizan games such as Go, Hex, and others;
- *Complexity* of some games and some puzzles;
- *Impartial games* such as Chomp and Wythoff's Nim;
- *Theory of the small*, that is, aspects of games with infinitesimal values;
- *Columns*, to wit: Aviezri S. Fraenkel's updated bibliography of combinatorial games, and an expanded and reorganized "Unsolved problems in combinatorial games," by Richard K. Guy and Richard J. Nowakowski, for those eager to get their hands dirty.

As editors, we would like to thank the organizers and participants for helping making the workshop a great experience.

As participants and organizers, we would like to recognize and thank the BIRS organization and staff: the former for giving us the chance to hold the workshop in such a wonderful setting and the latter for ensuring that the participants and organizers only had to worry about the scientific aspect of the workshop.

Lastly, a big thanks to Silvio Levy for the final preparation of this document.

Michael H. Albert
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