

Contents

<i>Preface</i>	<i>page xi</i>
1 Introduction	1
1.1 Construction and decomposition	1
1.2 Cartesian decompositions	3
1.3 Cartesian factorisations	5
1.4 Primitive, quasiprimitive and innately transitive groups: ‘O’Nan–Scott theories’	7
1.5 The structure of this book	11
Part I Permutation groups: Fundamentals	15
2 Group actions and permutation groups	17
2.1 Group actions, stabilisers, orbits	17
2.2 Isomorphic actions	20
2.3 Coset actions	22
2.4 Primitive and imprimitive groups	24
2.5 Examples of blocks	28
2.6 The fixed points of subgroups of a point stabiliser	31
2.7 Orbitals and orbital graphs	33
3 Minimal normal subgroups of transitive permutation groups	45
3.1 The centraliser of a transitive permutation group	45
3.2 Transitive minimal normal subgroups	48
3.3 The holomorph	50
3.4 Characteristically simple groups	57
3.5 The socles of finite 2-transitive permutation groups	62
3.6 The point stabiliser of a primitive group with a regular normal subgroup	67

Contents	viii
4 Finite direct products of groups	70
4.1 Direct products	71
4.2 Product action and intransitive cartesian decompositions	73
4.3 Subgroups of direct products	76
4.4 Strips	80
4.5 Normalisers in direct products	82
4.6 Scott's Lemma	84
4.7 Normal cartesian decompositions	87
4.8 Uniform automorphisms and factorisations of direct products	94
4.9 Sets of functions as direct products	102
5 Wreath products	104
5.1 Wreath products of groups	104
5.2 Wreath products as permutation groups	109
5.2.1 The imprimitive action	109
5.2.2 The product action	114
5.3 Cartesian decompositions and embedding theorems	117
5.4 Primitivity-type properties of wreath products	126
6 Twisted wreath products	132
6.1 The definition of twisted wreath products	132
6.1.1 The base group	133
6.1.2 An action of P on the base group	134
6.1.3 The twisted wreath product $T \operatorname{twr}_\varphi P$	136
6.2 Internal characterisation of twisted wreath products	138
6.3 Primitive, quasiprimitive and innately transitive twisted wreath products	141
6.4 Finite primitive twisted wreath products	144
6.5 Twisted wreath products with minimal normal FCR-subgroups	149
7 O'Nan–Scott Theory and the maximal subgroups of finite alternating and symmetric groups	151
7.1 Abelian plinth	154
7.2 Simple plinth	154
7.3 Regular plinth	155
7.4 Diagonal plinth	156
7.5 The remaining case	161
7.6 The O'Nan–Scott Theorem	162

ix	Contents	
7.6.1	A comparison of the O’Nan–Scott type subdivisions	166
7.7	Maximal subgroups of finite symmetric and alternating groups	166
Part II Innately transitive groups: Factorisations and cartesian decompositions		173
8	Cartesian factorisations	175
8.1	Cartesian factorisations of transitive permutation groups	175
8.2	Cartesian decompositions and minimal normal subgroups	176
8.3	Cartesian decompositions and affine groups	177
8.4	Cartesian decompositions and cartesian factorisations	181
8.5	Cartesian factorisations of abstract groups	182
8.6	Cartesian factorisations and cartesian decompositions	185
8.7	Factorisations of groups	187
8.8	Cartesian decompositions preserved by finite simple groups	193
9	Transitive cartesian decompositions for innately transitive groups	199
9.1	Abstract cartesian factorisations involving strips	199
9.2	The structure of transitive cartesian factorisations	203
9.3	A classification of transitive cartesian decompositions	206
9.4	Examples of cartesian decompositions	211
9.5	Simple group factorisations and cartesian decompositions	212
9.6	The cartesian decompositions in $CD_S(G)$ and in $CD_1(G)$	213
9.7	The structure of finite innately transitive permutation groups preserving a transitive cartesian decomposition	215
9.7.1	Cartesian decompositions in $CD_{1S}(G)$	216
9.7.2	Cartesian decompositions in $CD_{2\sim}(G)$	219
9.7.3	Cartesian decompositions in $CD_3(G)$	220
9.7.4	The proof of Theorem 9.15	222
9.8	Cartesian decompositions preserved by finite primitive groups	222

Contents	x
10 Intransitive cartesian decompositions	225
10.1 Intransitive cartesian decompositions	225
10.2 Cartesian factorisations involving non-trivial strips	227
10.3 Bounding the number of orbits in an intransitive cartesian factorisation of a finite group	231
10.4 Intransitive homogeneous cartesian decompositions	234
10.5 The component transitive cartesian decompositions for finite groups	237
 Part III Cartesian decompositions: Applications	 243
11 Applications in permutation group theory	245
11.1 The blow-up construction	246
11.2 Normal decompositions and blow-up decompositions	253
11.3 The blow-up construction and the O’Nan–Scott Theorem	256
11.4 The inclusion problem for finite permutation groups	259
12 Applications to graph theory	267
12.1 Graph theoretic definitions	268
12.2 Cartesian graph products of complete graphs	269
12.3 Cartesian decompositions and chamber systems	277
12.4 Arc-transitive automorphism groups of Hamming graphs	278
12.5 Arc-transitive graphs and cartesian decompositions	285
12.6 Direct product graphs and normal cartesian decompositions	287
12.7 2-arc-transitive graphs and permutation groups	290
12.7.1 Abelian plinth	292
12.7.2 Finite simple plinth	293
12.7.3 Finite composite plinth	296
Appendix Factorisations of simple and characteristically simple groups	300
A.1 Some factorisations of finite simple groups	300
A.2 Some factorisations of finite characteristically simple groups	301
<i>References</i>	305
<i>Glossary</i>	314
<i>Index</i>	318