ONE

A PERSPECTIVE

A discussion on the Earth's rotation is conveniently separated into three parts: (i) precession and nutation, (ii) polar motion and (iii) changes in length-of-day (l.o.d.). Precession and nutation describes the rotational motion of the Earth in space and is a consequence of the lunar and solar gravitational attraction on the Earth's equatorial bulge. Polar motion, or wobble, is the motion of the rotation axis with respect to the Earth's crust. Changes in the l.o.d. are a measure of a variable speed of rotation about the instantaneous pole. We are primarily concerned here with the last two components of the motion.

The standard treatment of precession and nutation for a rigid Earth is that by Woolard (1953), but a more comprehensive treatment is by Kinoshita (1977). Observational evidence is discussed by Federov (1963). Further discussions are found in the symposium proceedings edited by Federov, Smith & Bender (1977). The main discrepancies between the observed and theoretical nutations are consequences of the presence of the liquid core. The problem of the precession and nutation of a shell with a liquid-filled spheroidal cavity continues to draw the attention of mathematicians and geophysicists (see, for example, Roberts & Stewartson 1965; Busse 1968; Toomre 1966, 1974). It is touched upon briefly in chapter 3.

Perturbations in the rotation from the rigid body state are caused by motions and deformations of the Earth by a variety of forces. Chapter 2 discusses some general aspects of the deformations of the solid part of the Earth. The theory of the rotation of the deformable Earth is outlined in chapter 3, while chapter 4 consists of some general observations on the form and characteristics of the geophysical excitation functions that drive the variable rotation. The source of information for the variable rotation are threefold, depending on the time scale considered. The primary source is the record of about 150 yr of observations collected by positional astronomers since the introduction of telescopes (chapter 5). Historical records, back to 1000 BC and earlier, contain observations of eclipses, occultations, conjunctions and other configurations of the celestial bodies. These are valuable for studying secular changes in the l.o.d. The deciphering of these records requires a background in Classical, Islamic and Oriental languages, literature and history in addition to a knowledge of positional astronomy (chapter 10). Evidence of the motion of the pole of rotation during the geological past is contained in the paleomagnetic records, while evidence for a changing l.o.d. is found in the fossil records of certain invertebrates. These data are discussed in chapter 11.

Figure 1.1 illustrates the polar motion amplitude spectrum based on astronomical observations taken since the beginning of this century. The spectrum is characterized by two main peaks, one centred at 12 months, the other at about 14 months. The former is a forced wobble driven by seasonal redistribution of mass within and between the atmosphere, oceans, and ground and surface waters (chapter 7). The amplitude of this oscillation is about 0.″10, or about 3 m on the Earth's surface. The second peak is the famous Chandler wobble, also referred to as the free Eulerian precession. It represents a free oscillation of the Earth and is associated with three main problems: (i) Can its period be quantitatively explained? (ii) How is it maintained against dissipation since any free oscillation in physics will eventually be damped? (iii) Where is the rotational energy dissipated? The search for the answers to these questions involves excursions into the physics of the core, mantle, oceans and atmosphere (chapter 8). Seventy years of polar motion data have not been sufficient to resolve in a satisfactory manner the existence of oscillations on a decade time scale (chapter 9). More certain seems to be the observation that the pole is drifting in a direction towards Greenland at a rate of about 0.″002–0.″003 yr^{-1}, and this is possibly related to an exchange of mass between the world's oceans and the ice caps, and to post-glacial rebound. These drifts would have been much larger at the time when melting was first initiated. On the geological time scale, paleomagnetic evidence suggests that the rotational pole has wandered over the Earth's surface relative to the continents. Whether it is the pole that has moved or the

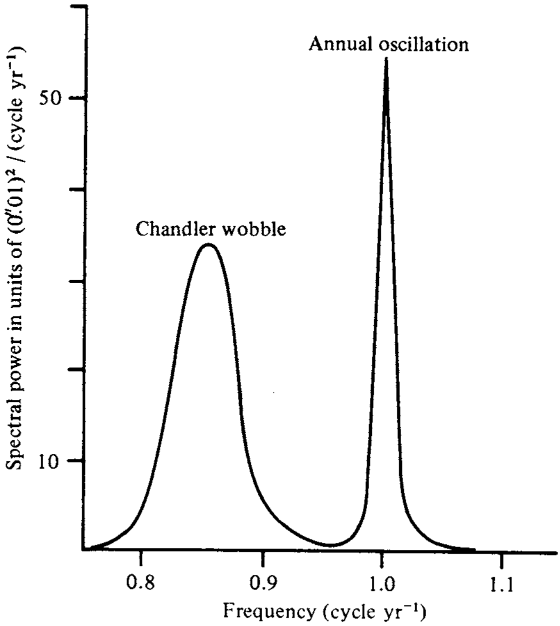

Figure 1.1. Wobble amplitude spectrum based on astronomical observations taken since the beginning of the twentieth century.

continents, or both, remains indeterminate, but the evidence is that there is no compelling reason to invoke polar wander (chapter 11).

Figure 1.2 illustrates the amplitude spectrum of the l.o.d. changes. The telescope observations and the historical records indicate a pronounced secular change: the l.o.d. is increasing at a rate of about 0.001–0.002 s every century. This is mainly a consequence of the work done by the Moon in raising the ocean tides, resulting in a transfer of angular momentum from the Earth's spin to the lunar motion and in a gradual increase in the Earth–Moon distance (chapter 10). Despite the smallness of this acceleration, its consequences – when integrated over geological time – are impressive: if this mechanism operated throughout the past, the Moon would have been very close to the Earth about 1.5×10^9 yr ago and the l.o.d. would have been only about 5 h. Due to the exchange of mass between ice caps and oceans and the readjustment of the mantle due to this redistribution fluctuations on a time scale of 10^4 yr must be expected at times of glaciation and

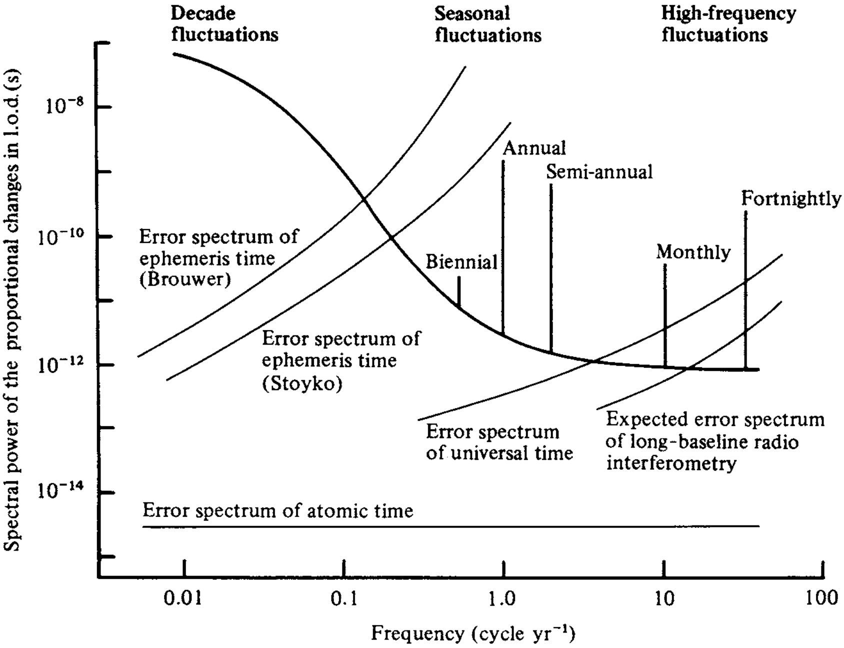

Figure 1.2. Spectrum of l.o.d. changes based on astronomical observations taken since the early nineteenth century. The observational error spectra are discussed in chapter 5.

deglaciation. Decade fluctuations are clearly evident in the l.o.d. records; changes in the l.o.d. of some 4–5 ms within 10–30 yr have occurred on several occasions since the record became reliable in the middle of the nineteenth century. Only the core is sufficiently mobile and has sufficient inertia to explain these fluctuations, the core motions being almost certainly transmitted to the mantle by electromagnetic forces (chapter 9). The decade fluctuations also exhibit trends that are similar to changes in a variety of global climatic indicators and lead to the interesting speculation that climatic changes on the decade time scale may be associated with internal processes. Seasonal changes in l.o.d. including annual, semi-annual and biennial periods, are almost entirely a consequence of a variable strength of the zonal wind circulation (chapter 7). This circulation also appears to be responsible for much of the higher frequency fluctuations in the l.o.d. Periodic tidal deformations (chapter 6) cause fluctuations near 27 d and 14 d. The precise determination of these tidal perturbations may ultimately shed some light on the anelastic behaviour of the Earth's mantle at frequencies beyond the seismic band (chapter 3).

TWO

SOME PHYSICAL PROPERTIES OF THE EARTH

Many aspects of the Earth's irregular rotation are consequences of deformations of the solid part of the Earth, and the quantitative evaluation of the excitation functions requires a knowledge of the planet's physical properties. The relevant properties include its shape and gravity, the variation of density and elastic parameters with depth, a measure of anelasticity and viscosity, and electrical conductivity. Information on these properties comes from a variety of sources, including seismology, geodesy and magnetism. These are discussed in most geophysics textbooks (see, for example, Stacey 1977; Kaula *et al.* 1980), and we discuss here only those aspects that are relevant to the subject of the Earth's rotation. Magnetic and electromagnetic data are discussed in chapter 9.

2.1 Elastic deformation

2.1.1 *Equations of motion*

The theory of the elastic deformations of the Earth is a classic subject in geophysics and is discussed in numerous books and papers (see, for example, Backus 1967; Takeuchi 1967; Jobert 1973*a*). The relevant equations describing the deformations in an element of volume are:

(i) A relation equating the rate of change of linear momentum of an element of volume with applied body forces **F** and surface forces, or stress, **T**

$$\rho\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}=\mathbf{F}+\nabla\cdot\mathbf{T},$$

where **v** is the velocity of the element and ρ its density.

(ii) Relations describing the deformation of the element. These consist of a relation between displacements d_i and strain e_{ij},

$$e_{ij} = \tfrac{1}{2}(\partial d_i/\partial x_j + \partial d_j/\partial x_i), \qquad i, j = 1, 2, 3,$$

and a relation between stress and strain. We are mainly concerned with a linear law or Hookean elasticity

$$T_{ij} = \lambda \Delta \delta_{ij} + 2\mu e_{ij}.$$

Here λ and μ are the Lamé constants; $\lambda = K - \frac{4}{3}\mu$, where K is the incompressibility or bulk modulus, and μ the rigidity. $\Delta = \sum_k e_{kk}$ is the cubic dilatation.

(iii) An equation of continuity

$$\partial\rho/\partial t + \nabla \cdot (\rho \mathbf{v}) = 0.$$

The initial state of the Earth can usually be considered as one of hydrostatic equilibrium, and in most problems one is concerned with small perturbations from this state. Furthermore, for these studies the Earth can be considered to be spherically symmetrical. Then the linearized form of the perturbation equations of conservation of linear momentum is, for small deformations,

$$\rho \partial^2 \mathbf{d}/\partial t^2 = \nabla \cdot \mathbf{T} - \nabla(\rho g \mathbf{d} \cdot \mathbf{e}_r) - \rho \nabla U + g \nabla \cdot (\rho \mathbf{d})\mathbf{e}_r. \quad (2.1.1)$$

$\mathbf{T}$ now represents the non-hydrostatic stress tensor. Density ρ and gravity g refer to the deformed state, and $\mathbf{e}$ is a unit vector with radial component $\mathbf{e}_r$ and tangential component $\mathbf{e}_t$. The potential U is the sum of two parts: U_1, the potential of the external force $\mathbf{F}$, and U_2, the non-hydrostatic potential of self-attraction after deformation. Inside the body $U = U_1 + U_2$ is subject to Poisson's equation,

$$\nabla^2 U = -4\pi G \nabla \cdot (\rho \mathbf{d}). \quad (2.1.2)$$

Partial solutions to these equations can be found if U is harmonic and of frequency σ, i.e. if

$$U = \sum_n U_n'(r) S_n \, e^{j\sigma t}, \quad (2.1.3)$$

where S_n is a surface harmonic of degree n. A. E. H. Love showed that in this case the solution of (2.1.1) and (2.1.2) can be written in the form

$$\mathbf{d} = \sum_n [V_n(r) S_n \mathbf{e}_r + W_n(r) \nabla S_n \mathbf{e}_t] \, e^{j\sigma t}, \quad (2.1.4)$$

where $V_n(r)$ and $W_n(r)$ are unknown functions defining the radial and tangential deformations. For realistic Earth models the solution is further facilitated by transforming (2.1.1) with (2.1.2) and (2.1.4) into six first-order differential equations of a form, first given by Alterman, Jarosch & Pekeris (1959),

$$\mathrm{d}y_\alpha/\mathrm{d}r = a_{\alpha\beta}y_\beta, \qquad \alpha, \beta = 1, \ldots 6 \tag{2.1.5}$$

The six parameters, y_α, represent the radial factors in the following quantities:

$\alpha = 1$; radial displacement, $y_1 = V_n(r)$

$\alpha = 2$; radial stress

$\alpha = 3$; tangential displacement, $y_3 = W_n(r)$

$\alpha = 4$; tangential stress

$\alpha = 5$; potential perturbation, $y_5 = U_n'(r)$

$\alpha = 6$; perturbation in potential gradient, $Y_6 = \partial U_n'/\partial r - 4\pi G\rho V_n$.

The $a_{\alpha\beta}$ are functions of the Lamé constants, $\lambda(r)$ and $\mu(r)$, and of $\rho(r)$, $g(r)$, the harmonic degree n, and the frequency σ of the deformation. The equations (2.1.5) are solved with boundary conditions relevant to the particular problem discussed. They include (i) regularity at the origin, (ii) that stresses vanish across free surfaces, (iii) continuity of deformation and stress across internal surfaces of discontinuity, and (iv) that internal and external gravitational potentials and their respective gradients must be equal at free surfaces and across surfaces of discontinuity (see, for example, Chinnery 1975).

A number of geophysical problems can be resolved with the above formulation. These include free oscillations, tidal deformations, response to loading of the Earth by surface or internal loads, and rotational deformations. Free oscillations represent solutions of (2.1.5) if there is no external force acting on the body. The so-modified equations give non-trivial solutions for only certain values of σ; the eigenvibration or free-oscillation frequency. Measurements at the surface of the Earth give the frequencies and relative amplitudes of these oscillations when they are excited by large earthquakes. The problem is discussed by Alterman *et al.* (1959), Gilbert (1972), and others. In the problem of the tidal deformations, the equations (2.1.5) are solved for a planet subject to a known external potential that does not load the surface. Stresses across the free surface must vanish. The observed quantities are the deformation of this surface $r = R$ and the change in the gravitational potential at $r = R$. This problem has been discussed for realistic Earth models by Takeuchi (1950), Pekeris & Accad (1972) and others. Peltier (1974) has extended the theory to a Maxwell rheology. Rotational deformation problems are very similar to the tidal problems, the tidal potential being replaced by the potential of the centrifugal force. When the Earth is loaded at its free surface, the potential $U_1(r)$ represents the gravitational potential of the load. The boundary conditions differ from those of the tide problem in that the stress is now continuous across the loaded surface $r = R$. Longman (1963) and Farrell (1972) discuss the problem. A related question is for internal loading. The Earth's gravity field indicates that important lateral density anomalies occur within the mantle. These will stress the Earth relative to the hydrostatic equilibrium state. Kaula (1963) has discussed the problem of estimating the stress and density in the Earth corresponding to a known external gravitational potential.

2.1.2 *Love numbers*

In the tidal, loading, and rotation problems with which we are concerned here, the response to an applied potential of the form (2.1.3) is assumed to be linear: the deformations are also harmonic with the same degree n as U_n. Then with

$$U_1 = \sum_n U_{1,n} = \sum_n U'_{1,n}(r) S_n e^{j\sigma t},$$

$$\begin{pmatrix} V_n(r) \\ W_n(r) \\ U'_{2,n}(r) \end{pmatrix} = U'_{1,n}(r) \begin{pmatrix} h_n(r)/g \\ l_n(r)/g \\ k_n(r) \end{pmatrix}.$$

In terms of the y_α of (2.1.5)

$$\left.\begin{aligned} y_1(r) &= h_n(r) U'_{1,n}(r)/g(r), \\ y_3(r) &= l_n(r) U'_{1,n}(r)/g(r), \\ y_5(r) &= [1 + k_n(r)] U'_{1,n}(r). \end{aligned}\right\} \tag{2.1.6}$$

On the surface $r=R$,

$$\left.\begin{aligned} \mathbf{d}_r(R) &= \mathbf{e}_r y_1(R) S_n \mathrm{e}^{j\sigma t} = [h_n(R)/g] U_{1,n}(R)\mathbf{e}_r, \\ \mathbf{d}_t(R) &= \mathbf{e}_t y_3(R)(\nabla S_n)\mathrm{e}^{j\sigma t} = [l_n(R)/g]\nabla U_{1,n}(R)\mathbf{e}_t, \\ \Delta U(R) &= y_5(R) S_n \mathrm{e}^{j\sigma t} = [1+k_n(R)]U_{1,n}(R). \end{aligned}\right\} \quad (2.1.7)$$

The constants $h_n(R)$, $k_n(R)$ and $l_n(R)$, or simply h_n, k_n, l_n are referred to as Love numbers of degree n. The h and k were first introduced by Love in 1909 and the l was introduced by T. Shida in 1912. This notation is usually reserved for Love numbers that define the deformation of a radially symmetric and elastic Earth in the absence of loading. If the potential does load the Earth, we denote the appropriate Love numbers by h', k', l'. These are often referred to as load Love numbers or load deformation coefficients. Similar parameters, h''_n, k''_n, l''_n, can be introduced to define the elastic deformations due to tangential stresses applied at the Earth's surface. These Love and load numbers are not all independent, Molodensky (1977) having demonstrated that for any radially symmetrical model and for any degree n, there exist three relations between them. They are

$$\left.\begin{aligned} k'_n &= k_n - h_n \\ h''_n &= 3n[(n+1)/(2n+1)](l-l') \\ 1+k''_n &= 3n[(n+1)/(2n+1)]l. \end{aligned}\right\} \quad (2.1.8)$$

Amongst the early studies of the Earth's tidal deformations, the work of Kelvin is most important. In his well-known estimate that the Earth's mean rigidity is greater than that of steel, Kelvin treated the Earth as homogeneous and incompressible. Inherent in Kelvin's treatment are two parameters that describe the deformation at the surface of the body in response to a harmonic potential of degree 2, and that relate to the density and rigidity. These correspond to the Love numbers h_2 and k_2 for the Kelvin Earth model, the homogeneous incompressible sphere. The three Love numbers for this model are

$$h_2(R) = \frac{5/2}{1+\tilde{\mu}}, \qquad k_2(R) = \frac{3/2}{1+\tilde{\mu}}, \qquad l_2(R) = \frac{3/4}{1+\tilde{\mu}}, \quad (2.1.9a)$$