

# Contents

---

<b>Preface</b>	<b>xi</b>
Acknowledgements	xiii
<b>Introduction</b>	<b>1</b>
Polyhedra in architecture	1
Polyhedra in art	2
Polyhedra in ornament	3
Polyhedra in nature	4
Polyhedra in cartography	7
Polyhedra in philosophy and literature	9
About this book	9
The inclusion of proofs	10
Approaches to the book	11
Basic concepts	12
Making models	14
<b>1. Indivisible, Inexpressible and Unavoidable</b>	<b>17</b>
Castles of eternity	17
Egyptian geometry	19
Babylonian geometry	23
Chinese geometry	24
A common origin for oriental mathematics	28
Greek mathematics and the discovery of incommensurability	29
The nature of space	33
Democritus' dilemma	35
Liu Hui on the volume of a pyramid	38
Eudoxus' method of exhaustion	41
Hilbert's third problem	44
<b>2. Rules and Regularity</b>	<b>51</b>
The Platonic solids	51
The mathematical paradigm	58
Abstraction	58
Primitive objects and unproved theorems	59

The problem of existence	61
Constructing the Platonic solids	66
The discovery of the regular polyhedra	70
What is regularity?	74
Bending the rules	79
The Archimedean solids	79
Polyhedra with regular faces	86
<b>3. Decline and Rebirth of Polyhedral Geometry</b>	<b>95</b>
The Alexandrians	95
Mathematics and astronomy	97
Heron of Alexandria	98
Pappus of Alexandria	99
Plato’s heritage	100
The decline of geometry	101
The rise of Islam	102
Thabit ibn Qurra	103
Abu’l-Wafa	104
Europe rediscovers the classics	104
Optics	105
Campanus’ sphere	106
Collecting and spreading the classics	107
The restoration of the Elements	108
A new way of seeing	109
Perspective	111
Early perspective artists	112
Leon Battista Alberti	114
Paolo Uccello	115
Polyhedra in woodcrafts	116
Piero della Francesca	118
Luca Pacioli	122
Albrecht Dürer	126
Wenzel Jamnitzer	128
Perspective and astronomy	132
Polyhedra revived	136
<b>4. Fantasy, Harmony and Uniformity</b>	<b>139</b>
A biographical sketch	139
A mystery unravelled	142
The structure of the universe	148

Fitting things together	149
Rhombic polyhedra	151
The Archimedean solids	156
Star polygons and star polyhedra	168
Semisolid polyhedra	173
Uniform polyhedra	175
<b>5. Surfaces, Solids and Spheres</b>	<b>181</b>
Plane angles, solid angles, and their measurement	183
Descartes' theorem	187
The announcement of Euler's formula	189
The naming of parts	191
Consequences of Euler's formula	193
Euler's proof	197
Legendre's proof	198
Cauchy's proof	200
Exceptions which prove the rule	202
What is a polyhedron?	205
Von Staudt's proof	210
Complementary viewpoints	213
The Gauss–Bonnet theorem	215
<b>6. Equality, Rigidity and Flexibility</b>	<b>219</b>
Disputed foundations	220
Stereo-isomerism and congruence	225
Cauchy's rigidity theorem	228
Cauchy's early career	233
Steinitz' lemma	235
Rotating rings and flexible frameworks	237
Are all polyhedra rigid?	240
The Connelly sphere	243
Further developments	244
When are polyhedra equal?	247
<b>7. Stars, Stellations and Skeletons</b>	<b>249</b>
Generalised polygons	249
Poinsot's star polyhedra	251
Poinsot's conjecture	256
Cayley's formula	257
Cauchy's enumeration of star polyhedra	259

Face-stellation	263
Stellations of the icosahedron	267
Bertrand’s enumeration of star polyhedra	281
Regular skeletons	282
<b>8. Symmetry, Shape and Structure</b>	<b>289</b>
What do we mean by symmetry?	289
Rotation symmetry	291
Systems of rotational symmetry	292
How many systems of rotational symmetry are there?	297
Reflection symmetry	300
Prismatic symmetry types	301
Compound symmetry and the $S_{2n}$ symmetry type	305
Cubic symmetry types	308
Icosahedral symmetry types	311
Determining the correct symmetry type	312
Groups of symmetries	314
Crystallography and the development of symmetry	318
<b>9. Counting, Colouring and Computing</b>	<b>327</b>
Colouring the Platonic solids	328
How many colourings are there?	330
A counting theorem	331
Applications of the counting theorem	334
Proper colourings	337
How many colours are necessary?	347
The four-colour problem	348
What is proof?	354
<b>10. Combination, Transformation and Decoration</b>	<b>359</b>
Making symmetrical compounds	359
Symmetry breaking and symmetry completion	361
Are there any regular compounds?	365
Regularity and symmetry	366
Transitivity	367
Polyhedral metamorphosis	373
The space of vertex-transitive convex polyhedra	376
Totally transitive polyhedra	385
Symmetrical colourings	394
Colour symmetries	397

Cambridge University Press

978-0-521-66405-9 - Polyhedra: “One of the Most Charming Chapters of Geometry”

Peter R. Cromwell

Table of Contents

[More information](#)

ix

Perfect colourings 400

The solution of fifth degree equations 402

**Appendix I 406****Appendix II 408****Sources of Quotations 411****Bibliography 416****Name Index 439****Subject Index 443**