

Cambridge University Press

978-0-521-66375-5 - Strongly Elliptic Systems and Boundary Integral Equations

William McLean

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