
Index

- Adjoint
 (formal) of a differential operator, 116
 of an abstract linear operator, 37, 43
 of conormal derivative operator, 201
 of trace operator, 201
- Annihilator of a subspace, 23
- Arzela–Ascoli theorem, 28
- Associated Legendre function, 336, 340
- Beer, A., 11
- Beltrami operator, 277
- Bessel function, 278
 spherical, 279, 293
- Bessel potential, 75
- Bochner integral, 321
- Boundary integral equation
 and logarithmic capacity, 264, 275
 for Dirichlet problem, 226
 for exterior problem, 236
 for problem with mixed boundary conditions, 231
 for Neumann problem, 229, 242
 side condition for, 262, 274
- Boundary integral operator
 (Schwartz) kernel of, 220, 223
 adjoint of, 218
 arising from a self-adjoint differential operator, 218
 arising from the Laplacian, 248, 249
 definition of, 218, 233
 mapping properties of, 219, 245
 on an open surface, 275
 symbol of, 244, 275
- Calderón, A. P., 309
- Calderón projection, 243
- Capacity, 263
 and exterior Dirichlet problem, 271
 behaviour under dilatation, 272
- logarithmic, 264
 and conformal mapping, 271
 of a line segment, 275
 of an ellipse, 272
 of an open surface, 275
 variational characterisation of, 272
- Cauchy–Riemann equations, 2
- Cauchy–Schwarz inequality, 54
- Chebyshev polynomials, 255
 explicit solutions in terms of, 272, 273
- Codimension, 18
- Coercivity
 abstract definition of, 44
 for differential operators, 118
 change of coordinates, 156
 on $H^1(\Omega)$, 122, 126
 on $H_0^1(\Omega)$, 119
 for elasticity operator, 298, 299
 for hypersingular integral operator, 230
- Compact linear operator, 28
 properties of, 54
 transpose of, 29
- Compact subsets of L_p , 28
- Compatible pair of normed spaces, 317
- Completely continuous linear operator,
see compact linear operator
- Conjugate exponent, 58
- Conjugation, 37
- Conormal derivative, 114
 L_2 estimates, 149
 generalised, 117
 relative to the formal adjoint, 116
- Convergence
 in $\mathcal{D}(\Omega)$, 65
 in $\mathcal{E}(\Omega)$, 65
 in $\mathcal{E}(\Omega)$ but not in $\mathcal{D}(\Omega)$, 109
 in $\mathcal{S}(\mathbb{R}^n)$, 72
- Convex set, 38

348

Convolution, 58
 and approximation, 63, 111
 and differentiation, 62, 109
 and Fourier transform, 73
 associativity of, 108
 support of, 108
 Cooling-off problem, 157
 Costabel, M., 102, 202
 Cutoff function, 83
 d'Alembert, J., 2
 Density
 of $C_{\text{comp}}^{\infty}(\Omega)$ in $L_p(\Omega)$, 63
 of $\mathcal{D}(\Omega)$ in $\mathcal{E}(\Omega)$, 109
 of $\mathcal{D}(\overline{\Omega})$ in $H^s(\Omega)$, 77
 of $\mathcal{D}(\mathbb{R}^n)$ in $\mathcal{S}(\mathbb{R}^n)$, 109
 of $\mathcal{D}(\overline{\Omega})$ in $H^s(\Omega)$, 111
 of $\mathcal{S}(\mathbb{R}^n)$ in $\mathcal{E}(\mathbb{R}^n)$, 109
 of $\mathcal{D}(\overline{\Omega})$ in $W^s(\Omega)$, 91
 of $W^s(\Omega) \cap \mathcal{E}(\Omega)$ in $W^s(\Omega)$, 86
 Dirac delta function(al), 66
 convolution with, 68
 homogeneity of, 187
 Dirichlet form, 246
 Dirichlet problem, 4
 solution operator for, 145
 Dirichlet's principle, 8
 abstract form, 55
 Dirichlet, P., 8
 Distribution, 65, 109
 multiplication with a smooth function, 68
 of form $u(a \cdot x)$, 189
 partial derivative of, 67
 temperate, 72
 with compact support, 67
 Divergence theorem, 97
 Domain
 Lipschitz, *see* Lipschitz domain 89
 of class C^k , 90
 of class $C^{k,\mu}$, 90
 Double-layer potential, *see* surface potential
 du Bois-Reymond, P., 12
 Dual space, 20
 of L_p , 58, 107
 of a Sobolev space, *see* Sobolev spaces,
 duality relations
 realisation of, 27
 Eigenfunction expansions, *see* spectral theory
 Eigenvalue, 45
 Elasticity operator, 297
 Energy inner product, 44
 example from linear elasticity, 298
 Epigraph, 186, 316
 Equicontinuous set of functions
 in $C(X)$, 28
 in L_p , 28

Index

Equilibrium density, 263
 Euler, L., 2
 Extension operator, 81, 309, 313
 non-existence of, 316
 Seeley, 316
 External conformal radius, 271
 Far-field pattern, 294
 Finite part, 159
 Finite-part extension
 differentiation of, 169
 homogeneity of, 162, 166, 168
 in n dimensions, 166
 of x^{-k-1} , 164
 of x_+^q , 160
 Finite part integral
 change of variables formula, 177, 180
 hypersingular integral operator, 223
 on a surface, 181
 Fourier transform, 70
 inversion theorem, 70
 of a homogeneous distribution, 172, 173,
 189
 of a temperate distribution, 72
 of partial derivatives, 72
 of f.p. u , 174
 Fourier, J.-B.-J., 4
 Fredholm alternative, 14, 37, 43
 for boundary integral equations, 226, 228,
 229, 240
 for coercive operators, 44
 for the mixed boundary value problem, 128
 for the third boundary value problem, 131
 relation to eigensystem, 51
 Fredholm equation of the second kind, 13
 abstract theory, 30, 35
 Fredholm operator, 32
 Fredholm, I., 13
 Fundamental solution, 191, 197
 for elasticity operator, 300
 for Laplacian, 2, 11, 247, 268
 for the Helmholtz operator, 279
 radiating, 282
 integral formula for, 198
 series expansion of, 255, 284
 Funk-Hecke formula, 335
 Gamma function, 169, 188
 Gauss, C. F., 7
 Generalised function, 66
 Green identity
 first, 4, 114, 116, 118, 141
 first, dual version of, 115, 118, 141
 second, 4, 118
 third, 5, 202
 dual version of, 211
 with radiation condition, 235

- Green's function, 5
 symmetry of, 16
 Green, G., 4, 8
- Hadamard, J., 159
 Hahn–Banach theorem, 20
 Hankel function, 280
 spherical, 281, 293
 Hardy's inequalities, 111, 112
 Harmonic analysis techniques, 209
 Heat equation, 4
 Helmholtz equation, 276
 radiating solution of, 281
 Hermitian sesquilinear form, 43, 116
 Hilbert space, 38
 best-approximation properties, 38, 39
 dual of, 41, 42
 weak sequential compactness in, 42
 Hilbert, D., 14
 Hölder's inequality, 58
 Homogeneous distribution, 158
 derivatives of, 187
 Homogeneous function, 158
 and change of variables, 175
 derivatives of, 187
 orthogonality condition for, 167
 parity condition for, 168, 169, 175
 Hypograph, 186
 of class C^k , 90
 Lipschitz, *see* Lipschitz hypograph
- Image, 18
 Imbedding
 of $S^*(\mathbb{R}^n)$ in $\mathcal{D}'(\mathbb{R}^n)$, 72
 of $L_{1,\text{loc}}(\Omega)$ in $\mathcal{D}'(\Omega)$, 66
 Index of a Fredholm operator, 33
 compact perturbation, 36
 homotopy, 54
 small perturbation, 54
 Inner product, 38
 Interpolation of normed spaces, 318
 duality properties, 324, 326
 J -method, 322
 K -method, 319
 reiteration theorem, 327
 Inverse point with respect to a sphere, 259, 270
- J -functional, 322
 Jump relations, *see* surface potential
- K -functional, 318
 determined by a positive-definite, self-adjoint operator, 333
 for Sobolev spaces, 329
 Kelvin transform, 259, 270
 Kelvin, Lord, *see* Thomson, W.
- Kernel (null space), 18
 of boundary integral operator, 240
 Helmholtz equation, 288
 Laplacian, 267
 linear elasticity, 304
 Korn's inequality
 first, 298
 second, 299, 305
- Lagrange, J.-L., 2
 Lamé coefficients, 296
 Laplace equation, 1
 Laplace operator, 246
 eigenvalues of, 249
 rotational invariance of, 268
 Laplace, P. S., 3
 Lax–Milgram lemma, 43
 Le Roux, J., 12
 Legendre polynomials, 255
 generating function for, 269
 Laplace representation, 340
 orthogonality property, 339
 recurrence relation for, 269
 Rodrigues formula for, 339
 Liouville, J., 6–8
 Lipschitz dissection, 99
 Lipschitz domain
 definition, 89
 non-examples of, 90
 outward unit normal to, 96
 surface element for, 96
 Lipschitz hypograph, 89
 Locally integrable functions, 64
- Meyers–Serrin theorem, 85
 Modulus of continuity, 60, 110
 Mollifier, *see* convolution
 Multi-index, *see* partial derivative
- Nečas, J., 123, 126, 147
 Nedelec, J. C., 289
 Neumann, C., 10, 12
 Newtonian potential, *see* volume potential
 Nirenberg, L., 133
 Nitsche, J. A., 305
 Noether, F., 33
- Open mapping theorem, 19
 Orthogonal complement, 40
 Orthogonal projection, *see* projection, orthogonal
- Parametrix, 192
 adjoint of, 197, 211
 behaviour of kernel, 195
 mapping property for, 193, 197

- Partial derivative, 61
 weak, 74
- Partition of unity, 83
 and Sobolev norm, 111, 331
- Peetre's inequality, 88, 110
- Pivot space, 44
 interpolation property of, 331
 use of L_2 , 118
- Plancherel's theorem, 73
- Poincaré, H., 10, 13, 145
- Poisson integral formula, 5
- Poisson's equation, 15
- Poisson, S.-D., 3–6
- Positive and bounded below, 43
 boundary integral operator, 262, 264, 267, 275
- Potential
 electrostatic, 3, 5, 263
 gravitational, 3
 surface, *see* surface potential
 vector, 292
 volume, *see* volume potential
- Principal part, 114
 and coercivity, 118
- Principal value, 166, 190
- Projection, 20
 orthogonal, 40, 54
- Quotient norm, 19
- Quotient space, 18
- Radiation condition, 234, 243
 for the Laplacian, 259
 Sommerfeld, 281, 283, 294
- Reflexive Banach space, 22, 37
- Regulariser, 35
- Regularity theory
 for boundary integral equations, 239
 interior, 135, 196
 up to the boundary, 137
- Relatively compact set, 27
- Rellich–Payne–Weinberger identity, 146
- Rellich, F., 87, 147, 286
- Riesz representation theorem, 40
- Riesz, F., 15
- Rigid motion (infinitesimal), 302
- Scalar wave equation, 276
- Self-adjoint operator, *see* adjoint
- Separation of variables, 4, 277
- Sequential compactness, 27
- Sesquilinear form, 42
 arising from a boundary integral operator, 261, 275
 arising from an elliptic differential operator, 114
 arising from the Helmholtz operator, 276
 arising from the Laplacian, 246
- Single-layer potential, *see* surface potential
- Singular integral operator, 190, 312
- Slobodeckii seminorm, 74, 79
 and Fourier transform, 79
- Smoothing operator, 192
- Sobolev imbedding theorem, 86
- Sobolev representation formula, 311
- Sobolev spaces
 compact imbeddings, 87
 definition via Bessel potentials, 76
 definition via weak derivatives, 74
 density theorems, *see* density
 duality relations, 76, 78, 92, 98
 equivalent norms for, 96, 110
 $H^s(\mathbb{R}^n) = W^s(\mathbb{R}^n)$, 80
 $H^s(\Omega) = W^s(\Omega)$, 81, 92
 $\tilde{H}^s(\Omega) = H_0^s(\Omega)$, 95, 112
 $\hat{H}^s(\Omega) = H_{\Omega}^s$, 91
 interpolation properties of, 329–331
 invariance under change of coordinates, 85
 of negative order, 74
 of vector-valued (generalised) functions, 106
 on the boundary of a domain, 98, 99
- Sommerfeld radiation condition, *see* radiation condition, Sommerfeld
- Spectral radius, 55
- Spectral theory
 for coercive self-adjoint operators, 49
 for compact self-adjoint operators, 47, 55
 for self-adjoint elliptic differential operators, 132
- Spectrum of a linear operator, 45
- Spherical harmonics, 250, 252
 addition theorem for, 335
 and boundary integral operators, 252
 and the Helmholtz equation, 279
 classical, 338
 eigenfunctions of the Beltrami operator, 278
 for the circle, 336
 orthogonal basis for, 337
 orthogonal projection onto, 334
 orthogonality in $L_2(\mathbb{S}^{n-1})$, 265
 series expansion in, 257
- Stein, E. M., 309
- Steklov–Poincaré operators, 145
 estimates for a $C^{r+1,1}$ domain, 146
 estimates for a Lipschitz domain, 155
 representations in terms of boundary integral operators, 244
- Strain tensor, 296
- Stress tensor, 296
- Stress–strain relation, 296
- Strictly positive-definite operator, 44

Cambridge University Press

978-0-521-66332-8 - Strongly Elliptic Systems and Boundary Integral Equations

William McLean

Index

[More information](#)

Index

351

- Strongly elliptic differential operator, 119
 change of coordinates, 156
 linear elasticity, 297
 with constant coefficients, 193
- Sturm–Liouville problem, 6
- Sturm, C. F., 6
- Successive approximations, method of, 11
- Support
 essential, 66
 of a distribution, 66
 of a function, 61
- Surface area of unit sphere, 247, 268
- Surface potential
 double layer, 10, 202
 traces of, 221
 duality relations, 212, 213
 for self-adjoint differential operator, 212, 218
 jump relations for, 3, 11, 186, 203, 215
 mapping properties of, 203, 205, 210
 single layer, 3, 201
 traces and conormal derivatives of, 218
- Tangential differential operator, 147
- Taylor expansion, 61
- Test function, 65
- Thomson, W., 6, 8
- Totally bounded set, 27
- Trace operator, 100
 and $H_0^s(\Omega)$, 105
 and surface potentials, 209
 for $C^{k-1,1}$ domains, 102
 for Lipschitz domains, 102
 from $W_p^k(\mathbb{R}^n)$ to $W_p^{k-1/p}(\mathbb{R}^{n-1})$, 112
 one-sided, 141
 right inverse for, 101
- Traction, 296
 alternative formula for, 308
 as conormal derivative, 297
- Transmission property, 142, 143
 for surface potentials, 183, 186, 210
- Transpose of a linear operator, 22
 inverse of, 53
- Uniformly directionally differentiable surface, 221
- Uniqueness theorem
 for the Helmholtz equation, 288
 for the Laplace equation, 260
 mixed boundary conditions, 250
 Neumann problem, 266
- Volterra, V., 13
- Volume potential, 2, 191
 behaviour at the boundary, 216
- Wave number, 276
- Weak convergence, 42, 55
- Weber, H., 9
- Weierstraß, K., 10

Cambridge University Press

978-0-521-66332-8 - Strongly Elliptic Systems and Boundary Integral Equations

William McLean

Index

[More information](#)

Index of Notation

Functional Analysis

A^*	adjoint of A , 37
A_j	induced map on cosets modulo $\ker A$, 18
A^t	transpose of A , 22
$\text{dist}(u, W)$	distance from point u to set W , 21
$\text{im } A$	image (range) of linear operator A , 18
(g, u)	same as $\langle \bar{g}, u \rangle$, 37
$(\cdot, \cdot)_A$	energy inner product for A , 44
$\ \cdot\ _A$	energy norm for A , 44
$\ker A$	kernel (null space) of linear operator A , 18
$\mathcal{L}(X, Y)$	space of bounded linear operators from X to Y , 18
\oplus	direct sum, 20
(g, u)	value of functional $g \in X^*$ at $u \in X$, 20
$u \perp v$	u is orthogonal to v , 39
$u \perp W$	u is orthogonal to the set W , 39
\sim	equivalence of norms, 17
$\text{spec}(A)$	spectrum of A , 45
W^a	subspace of X^* that annihilates $W \subseteq X$, 23
aV	subspace of X that annihilates $V \subseteq X^*$, 23
$u_j \rightharpoonup u$	u_j converges weakly to u , 42
$(X_0, X_1)_{\theta, q}$	interpolation space, 318
X^*	dual space of X , 20

Theory of Distributions

$C_{\text{comp}}^\infty(\Omega)$	space of C^∞ functions with compact support in Ω , 61
$C_{\text{comp}}^r(\Omega)$	space of C^r functions with compact support in Ω , 61

$C^\infty(\Omega)$	space of infinitely differentiable functions on Ω , 61
$C_K^\infty(\Omega)$	space of functions in $C^\infty(\Omega)$ having support in K , 61
$C^r(\Omega)$	space of r times continuously differentiable functions on Ω , 61
$C_K^r(\Omega)$	space of functions in $C^r(\Omega)$ having support in K , 61
$\mathcal{D}(\Omega)$	$C_{\text{comp}}^\infty(\Omega)$ with sequential convergence defined, 65
$\mathcal{D}(\overline{\Omega})$	space of restrictions to Ω of functions in $\mathcal{D}(\mathbb{R}^n)$, 77
δ	same as δ_0 , 66
δ_x	Dirac delta function(al) at x , 66
$\mathcal{D}_K(\Omega)$	$C_K^\infty(\Omega)$ with sequential convergence defined, 65
$\mathcal{D}^*(\Omega)$	space of Schwartz distributions on Ω , 65
$\mathcal{E}^*(\Omega)$	space of distributions with compact support in Ω , 66
$\mathcal{E}(\Omega)$	$C^\infty(\Omega)$ with sequential convergence defined, 65
f.p. u	finite-part extension of u , 166
p.v. u	principal value of u , 166
$H_a(\phi)$	finite-part integral of $x^a \phi(x)$ over the half line $x > 0$, 160
(u, v)	abbreviation for $(u, v)_\Omega$ when $\Omega = \mathbb{R}^n$, 68
$(u, v)_\Omega$	same as $\langle \bar{u}, v \rangle_\Omega$, 68, 107
$L_{1,\text{loc}}(\Omega)$	space of locally integrable functions on Ω , 64
M_t	dilatation operator, 158
\otimes	tensor product of functions or distributions, 104
$\langle u, v \rangle$	abbreviation for $\langle u, v \rangle_\Omega$ when $\Omega = \mathbb{R}^n$, 66
$\langle u, v \rangle_\Omega$	integral (generalised, if necessary) of $u \cdot v$ over Ω , 58, 66, 106
$\Pi_a^\pm(\xi)$	Fourier transform of f.p. x_\pm^a , 169
$R_a \phi$	finite-part integral of $\rho^{a+n-1} \phi(x)$ over $\rho > 0$, 166
$\mathcal{S}(\mathbb{R}^n)$	Schwartz class of rapidly decreasing C^∞ functions on \mathbb{R}^n , 72
x_+^a	x^a if $x > 0$, but 0 if $x < 0$, 159
f.p. x_+^a	finite-part extension of x_+^a , 160
x_-^a	$ x ^a$ if $x < 0$, but 0 if $x > 0$, 163
f.p. x_-^a	finite-part extension of x_-^a , 163
f.p. x^{-k-1}	finite-part extension of x^{-k-1} for an integer $k \geq 0$, 164
$(x \pm i0)^a$	164

Sobolev Spaces

H_F^s	space of distributions in $H^s(\mathbb{R}^n)$ with support in F , 76
$H^s(\mathbb{R}^n)$	Sobolev space on \mathbb{R}^n (definition via Bessel potential), 76
$H^s(\Gamma)$	Sobolev space on Γ , 98
$H^s(\Omega)$	space of restrictions to Ω of distributions in $H^s(\mathbb{R}^n)$, 77
$H^s(\Omega)^m$	space of H^s functions on Ω with values in \mathbb{C}^m , 107

$H_0^s(\Omega)$	closure of $\mathcal{D}(\Omega)$ in $H^s(\Omega)$, 77
$\tilde{H}^s(\Omega)$	closure of $\mathcal{D}(\Omega)$ in $H^s(\mathbb{R}^n)$, 77
$\tilde{H}^s(\Omega)^m$	space of \tilde{H}^s functions on Ω with values in \mathbb{C}^m , 107
\mathcal{J}^s	Bessel potential of order s , 75
$ \cdot _{\mu,p,\Omega}$	Slobodeckii seminorm, 74
$ \cdot _{\mu,\Omega}$	abbreviation for $ \cdot _{\mu,2,\Omega}$, 79
$W^s(\Omega)$	abbreviation for $W_2^s(\Omega)$, 75
$W_p^s(\Omega)$	Sobolev space of order $s \geq 0$ based on $L_p(\Omega)$, 73, 74
$W_p^s(\Omega)^m$	space of W_p^s functions on Ω with values in \mathbb{C}^m , 107

Differential and Integral Operators

$\Delta_{\mathbb{S}^{n-1}}$	Beltrami operator on the unit sphere, 277
\mathcal{B}_j	j th component of generalised flux or traction, 114
\mathcal{B}_ν	conormal derivative, 114
\mathcal{B}_ν^\pm	conormal derivative from Ω^\pm , 141
$\tilde{\mathcal{B}}_\nu^*$	adjoint of $\tilde{\mathcal{B}}_\nu$, 201
$\tilde{\mathcal{B}}_j$	dual version of \mathcal{B}_j , 115
$\tilde{\mathcal{B}}_\nu$	dual version of \mathcal{B}_ν , 115
DL	double-layer potential, 10, 202
$\tilde{\text{DL}}$	dual version of DL, 211
$E_{jk}(u)$	strain tensor, 296
$G(x, y)$	fundamental solution or parametrix, 2, 191
\mathcal{G}	volume potential, 191
$G(x - y)$	$G(x, y)$ in translation-invariant case, 193
$G_j(x, x - y)$	j th term in homogeneous expansion of $G(x, y)$, 195
$[\mathcal{B}_\nu u]_\Gamma$	jump in $\mathcal{B}_\nu u$ across Γ , 142
$[\tilde{\mathcal{B}}_\nu u]_\Gamma$	jump in $\tilde{\mathcal{B}}_\nu u$ across Γ , 142
$[u]_\Gamma$	jump in u across Γ , 142
Δ	Laplace operator, 1
\mathcal{P}	second-order differential operator, usually strongly elliptic, 113
\mathcal{P}_0	principal part of \mathcal{P} , 114
$\Phi = \Phi_\Omega$	sesquilinear form arising from \mathcal{P} for the domain Ω , 114
Φ^\pm	abbreviation for Φ_{Ω^\pm} , 141
R	boundary integral operator, conormal derivative of DL, 218
\mathcal{M}	operator arising in radiation conditions, 234
S	boundary integral operator, trace of SL, 7, 218
Σ_{jk}	stress tensor, 296

356

Index of Notation

SL	single-layer potential, 3, 202
$\widetilde{\text{SL}}$	dual version of SL, 211
T	boundary integral operator, sum of one-sided traces of DL, 11, 218
\widetilde{T}	dual version of T , 218
\mathcal{U}	solution operator for the Dirichlet problem, 145
\mathcal{V}	solution operator for the adjoint Dirichlet problem, 145

Other Symbols

$\alpha!$	factorial of the multi-index α , 61
$ \alpha $	order of the partial derivative determined by α , 61
$\partial^\alpha u$	partial derivative of u determined by the multi-index α , 61
y^α	monomial determined by the multi-index α , 61
\mathbb{C}^+	complex upper half plane $\text{Im } z > 0$, 183
\mathbb{C}^-	complex lower half plane $\text{Im } z < 0$, 183
Cap_Γ	capacity of Γ , 263
$u * v$	convolution of u and v , 58
$\Delta_{l,h}$	difference quotient in l th variable with step size h , 62
$d\sigma$	element of surface area on Γ , 1, 97
$\hat{u} = \mathcal{F}u$	Fourier transform of u , 70
$u = \mathcal{F}^*\hat{u}$	inverse Fourier transform of \hat{u} , 70
Γ	(common) boundary of $\Omega = \Omega^-$ and of Ω^+ , 1, 89, 141
γ	trace operator for Ω , 100, 102
γ^*	adjoint of γ , 201
γ^\pm	trace operator for Ω^\pm , 1, 141
Γ_D	portion of Γ with Dirichlet boundary condition, 128
Γ_N	portion of Γ with Neumann boundary condition, 128
$h_m^{(1)}, h_m^{(2)}$	spherical Hankel functions, 281
J_μ	Bessel function of the first kind, 278
j_m	spherical Bessel function of the first kind, 279
$L_p(\Omega)$	Lebesgue space of p th-power-integrable functions on Ω , 58
$M(n, m)$	dimension of $\mathcal{P}_m(\mathbb{R}^n)$, 250
$N(n, m)$	dimension of $\mathcal{H}_m(\mathbb{R}^n)$, 250
$ x $	Euclidean norm in \mathbb{R}^n or unitary norm in \mathbb{C}^m , 1
ν	outward unit normal to $\Omega = \Omega^-$, 1, 97, 141
Ω	domain in \mathbb{R}^n , 1
Ω^\pm	interior ($-$) and exterior ($+$) domains, 1, 141
p^*	conjugate exponent to p , 58

Other Symbols

357

$P_m(n, t)$	generalised Legendre polynomial of degree m for the dimension n , 255
$\mathcal{P}_m(\mathbb{R}^n)$	space of homogeneous polynomials of degree m , 250
\mathbb{R}_+^n	upper half space $x_n > 0$, 183
\mathbb{R}_-^n	lower half space $x_n < 0$, 183
$\mathcal{H}_m(\mathbb{R}^n)$	space of solid spherical harmonics of degree m , 250
$\mathcal{H}_m(\mathbb{S}^{n-1})$	space of surface spherical harmonics of degree m , 252
u^*	transpose of the complex conjugate of the vector u , 107
$u \cdot v$	dot product of vectors u and v , 107
$u^{(k)}(x; y)$	k th Fréchet derivative of u , 61
u^\sharp	Kelvin transform of u , 259
Υ_n	surface area of \mathbb{S}^{n-1} , 247
$\Upsilon^\pm(x)$	221
x^\sharp	inverse point of x with respect to a sphere, 258
Y_μ	Bessel function of the second kind, 278
y_m	spherical Bessel function of the second kind, 279
$\Gamma(a)$	gamma function, 169
$H_\mu^{(1)}, H_\mu^{(2)}$	Hankel functions, 280
\int^\pm	special contour integral, 183