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052166313X - Introduction to the Physics of the Earth's Interior, Second Edition

Jean-Paul Poirier

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Introduction to the Physics of the Earth's Interior describes the structure, composition and temperature of the deep Earth in one comprehensive volume.

The book begins with a succinct review of the fundamentals of continuum mechanics and thermodynamics of solids, and presents the theory of lattice vibration in solids. The author then introduces the various equations of state, moving on to a discussion of melting laws and transport properties. The book closes with a discussion of current seismological, thermal and compositional models of the Earth. No special knowledge of geophysics or mineral physics is required, but a background in elementary physics is helpful. The new edition of this successful textbook has been enlarged and fully updated, taking into account the considerable experimental and theoretical progress recently made in understanding the physics of deep-Earth materials and the inner structure of the Earth.

Like the first edition, this will be a useful textbook for graduate and advanced undergraduate students in geophysics and mineralogy. It will also be of great value to researchers in Earth sciences, physics and materials sciences.

Jean-Paul Poirier is Professor of Geophysics at the Institut de Physique du Globe de Paris, and a corresponding member of the Académie des Sciences. He is the author of over one-hundred-and-thirty articles and six books on geophysics and mineral physics, including *Creep of Crystals* (Cambridge University Press, 1985) and *Crystalline Plasticity and Solid-state flow of Metamorphic Rocks* with A. Nicolas (Wiley, 1976).

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INTRODUCTION TO THE
PHYSICS OF THE EARTH'S
INTERIOR
SECOND EDITION

JEAN-PAUL POIRIER
Institut de Physique du Globe de Paris



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UNIVERSITY PRESS

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PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge CB2 2RU, UK <http://www.cup.cam.ac.uk>
40 West 20th Street, New York, NY 10011-4211, USA <http://www.cup.org>
10 Stamford Road, Oakleigh, Melbourne 3166, Australia
Ruiz de Alarcón 13, 28014 Madrid, Spain

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First published 2000

Printed in the United Kingdom at the University Press, Cambridge

Typeset in Times 10/13pt [vN]

A catalogue record for this book is available from the British Library

Library of Congress Cataloging in Publication data

Poirier, Jean Paul.

Introduction to the physics of the Earth's interior / Jean-Paul Poirier. – 2nd ed.
p. cm.

Includes bibliographical references and index.

ISBN 0 521 66313 X (hardbound). – ISBN 0 521 66392 X (pbk.)

1. Earth – Core. 2. Earth – Mantle. 3. Geophysics. I. Title.

QE509.2.P65 2000

551.1'1 – dc21 99-30070 CIP

ISBN 0 521 66313 X hardback

ISBN 0 521 66392 X paperback

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Preface to the first edition

Not so long ago, Geophysics was a part of Meteorology and there was no such thing as Physics of the Earth's interior. Then came Seismology and, with it, the realization that the elastic waves excited by earthquakes, refracted and reflected within the Earth, could be used to probe its depths and gather information on the elastic structure and eventually the physics and chemistry of inaccessible regions down to the center of the Earth.

The basic ingredients are the travel times of various phases, on seismograms recorded at stations all over the globe. Inversion of a considerable amount of data yields a seismological earth model, that is, essentially a set of values of the longitudinal and transverse elastic-wave velocities for all depths. It is well known that the velocities depend on the elastic moduli and the density of the medium in which the waves propagate; the elastic moduli and the density, in turn, depend on the crystal structure and chemical composition of the constitutive minerals, and on pressure and temperature. To extract from velocity profiles self-consistent information on the Earth's interior such as pressure, temperature, and composition as a function of depth, one needs to know, or at least estimate, the values of the physical parameters of the high-pressure and high-temperature phases of the candidate minerals, and relate them, in the framework of thermodynamics, to the Earth's parameters.

Physics of the Earth's interior has expanded from there to become a recognized discipline within solid earth geophysics, and an important part of the current geophysical literature can be found under such key words as "equation of state", "Grüneisen parameter", "adiabaticity", "melting curve", "electrical conductivity", and so on.

The problem, however, is that, although most geophysics textbooks devote a few paragraphs, or even a few chapters, to the basic concepts of the physics of solids and its applications, there still is no self-contained book

that offers the background information needed by the graduate student or the non-specialist geophysicist to understand an increasing portion of the literature as well as to assess the weight of physical arguments from various parties in current controversies about the structure, composition, or temperature of the deep Earth.

The present book has the, admittedly unreasonable, ambition to fulfill this role. Starting as a primer, and giving at length all the important demonstrations, it should lead the reader, step by step, to the most recent developments in the literature. The book is primarily intended for graduate or senior undergraduate students in physical earth sciences but it is hoped that it can also be useful to geophysicists interested in getting acquainted with the mineral physics foundations of the phenomena they study.

In the first part, the necessary background in thermodynamics of solids is succinctly given in the framework of linear relations between intensive and extensive quantities. Elementary solid-state theory of vibrations in solids serves as a basis to introduce Debye's theory of specific heat and anharmonicity. Many definitions of Grüneisen's parameter are given and compared.

The background is used to explain the origin of the various equations of state (Murnaghan, Birch–Murnaghan, etc.). Velocity–density systematics and Birch's law lead to seismic equations of state. Shock-wave equations of state are also briefly considered. Tables of recent values of thermodynamic and elastic parameters of the most important mantle minerals are given. The effect of pressure on melting is introduced in the framework of anharmonicity, and various melting laws (Lindemann, Kraut–Kennedy, etc.) are given and discussed. Transport properties of materials – diffusion and viscosity of solids and of liquid metals, electrical and thermal conductivity of solids – are important in understanding the workings of the Earth; a chapter is devoted to them.

The last chapter deals with the application of the previous ones to the determination of seismological, thermal, and compositional Earth models.

An abundant bibliography, including the original papers and the most recent contributions, experimental or theoretical, should help the reader to go further than the limited scope of the book.

It is a pleasure to thank all those who helped make this book come into being: First of all, Bob Liebermann, who persuaded me to write it and suggested improvements in the manuscript; Joël Dyon, who did a splendid job on the artwork; Claude Allègre, Vincent Courtillot, François Guyot,

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Jean-Louis Le Mouël, and Jean-Paul Montagner, who read all or parts of the manuscript and provided invaluable comments and suggestions; and last but not least, Carol, for everything.

1991

Jean-Paul Poirier

Preface to the second edition

Almost ten years ago, I wrote in the introduction to the first edition of this book: 'It will also probably become clear that the simplicity of the inner Earth is only apparent; with the progress of laboratory experimental techniques as well as observational seismology, geochemistry and geomagnetism, we may perhaps expect that someday "Physics of the Inner Earth" will make as little sense as "Physics of the Crust"'. We are not there yet, but we have made significant steps in this direction in the last ten years. No geophysicist now would entertain the idea that the Earth is composed of homogeneous onion shells. The analysis of data provided by more and better seismographic nets has, not surprisingly, revealed the heterogeneous structure of the depths of the Earth and made clear that the apparent simplicity of the lower mantle was essentially due to its remoteness. We also know more about the core.

Mineral physics has become an essential part of geophysics and the progress of experimental high-pressure and high-temperature techniques has provided new results, solved old problems and created new ones. Samples of high-pressure phases prepared in laser-heated diamond-anvil cells or large-volume presses are now currently studied by X-ray diffraction, using synchrotron beams, and by transmission electron microscopy. In ten years, we have thus considerably increased our knowledge of the deep minerals, including iron at core pressures. We know more about their thermoelastic properties, their phase transitions and their melting curves. Concurrently, quantum mechanical *ab-initio* computer methods have made such progress as to be able to reproduce the values of physical quantities in the temperature- and pressure-ranges that can be experimentally reached, and therefore predict with confidence their values at deep-Earth conditions.

In this new edition, I have therefore expanded the chapters on equations

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of state, on melting, and the last chapter on Earth models. Close to two-hundred-and-fifty new references have been added.

I thank Dr Brian Watts of CUP, my copy editor, for a most thorough review of the manuscript.

1999

Jean-Paul Poirier