High Speed Flow

C. J. CHAPMAN
University of Keele, England

CAMBRIDGE
UNIVERSITY PRESS

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge CB2 2RU, UK http://www.cup.cam.ac.uk
40 West 20th Street, New York, NY 10011-4211, USA http://www.cup.org
10 Stamford Road, Oakleigh, Melbourne 3166, Australia
Ruiz de Alarcón 13, 28014 Madrid, Spain

© Cambridge University Press 2000

This book is in copyright. Subject to statutory exception
and to the provisions of relevant collective licensing agreements,
no reproduction of any part may take place without
the written permission of Cambridge University Press.

First published 2000

Printed in the United States of America

Typeface Times Roman 10/13 pt. *System* LaTeX 2$_\varepsilon$ [TB]

*A catalog record for this book is available from
the British Library.*

Library of Congress Cataloging in Publication Data
Chapman, C. J. (Christopher John), 1952–
High speed flow / C.J. Chapman
p. cm. – (Cambridge texts in applied mathematics)
Includes bibliographical references and index.
ISBN 0-521-66169-2 (hdbk.). – ISBN 0-521-66647-3 (pbk.)
1. Fluid dynamics. I. Title. II. Series.
QA911.C43 2000
532′.0532 – dc21 99-37544
CIP

ISBN 0 521 66169 2 (hardback)
ISBN 0 521 66647 3 (paperback)

Contents

Preface *page* xi

1 Preliminaries **1**
1.1 The Mach Number 1
1.2 Flow Regimes 1
1.3 Temperature Changes 3
1.4 History 3
1.5 Recent Research 4
1.6 Bibliographic Notes 5

2 Governing Equations **7**
2.1 Conservation of Mass. Jump Condition 7
2.2 Conservation of Momentum. Jump Condition 14
2.3 Conservation of Energy. Jump Condition 16
2.4 Equations in Conservation and Nonconservation Form 18
2.5 Pressure, Viscosity, and Thermal Conductivity 19
2.6 The Navier–Stokes Equations 20
2.7 Three Types of Compressible Flow 25
2.8 Bibliographic Notes 28

3 Thermodynamics **29**
3.1 General Laws and Definitions 29
3.2 Practical Thermodynamic Formulae 37
3.3 Pressure, Volume, and Temperature 40
3.4 Entropy 40
3.5 Internal Energy 42

3.6 Enthalpy 43
3.7 The Helmholtz Function 44
3.8 The Gibbs Function 45
3.9 The Perfect Gas 45
3.10 The Perfect Gas with Constant Specific Heats 50
3.11 Bibliographic Notes 54

4 Smooth Flow of an Ideal Fluid 55
4.1 Lattice of Special Cases 55
4.2 General Flow. Crocco's Equation 57
4.3 Steady Flow 58
4.4 Irrotational Homentropic Flow 61
4.5 Steady Irrotational Homentropic Flow 63
4.6 Bibliographic Notes 64
4.7 Further Results and Exercises 64

5 Characteristic Surfaces and Rays 67
5.1 Nonsmooth Flow 67
5.2 Characteristic Surfaces 68
5.3 The Monge Cone 75
5.4 Ray Equations 77
5.5 Application to High Speed Flow 79
5.6 Bibliographic Notes 83
5.7 Further Results and Exercises 83

6 Shocks 85
6.1 Jump Conditions 85
6.2 Normal Shocks in an Arbitrary Fluid. The Rankine–Hugoniot Relation 87
6.3 Normal Shocks in a Polytropic Gas. The Prandtl Relation 92
6.4 Oblique Shocks. The Shock Polar 98
6.5 Bibliographic Notes 115
6.6 Further Results and Exercises 115

7 Steady One-Dimensional Flow 120
7.1 Flow in a Stream Tube 120
7.2 Flow in Ducts and Nozzles. Choked Flow 125
7.3 The Laval Nozzle 127
7.4 Bibliographic Notes 131
7.5 Further Results and Exercises 131

8 Prandtl–Meyer Expansion 133
8.1 Problems in Two Independent Variables 133
8.2 Prandtl–Meyer Expansion 133
8.3 Flow round a Smooth Bend 141
8.4 Bibliographic Notes 142
8.5 Further Results and Exercises 142

9 Aerofoils 145
9.1 Linear Theory. Subsonic and Supersonic Flow 145
9.2 Nonlinear Theory 152
9.3 Transonic Flow 153
9.4 Bibliographic Notes 160
9.5 Further Results and Exercises 161

10 Characteristics for Steady Two-Dimensional Flow 165
10.1 Governing Equations 165
10.2 The Friedrichs Theory 166
10.3 Ordinary Differential Equations on Characteristics 169
10.4 Riemann Invariants 172
10.5 Flow round a Smooth Bend 173
10.6 Bibliographic Notes 176
10.7 Further Results and Exercises 176

11 Shock Reflections and Intersections 180
11.1 Common Shock Patterns 180
11.2 The von Neumann–Henderson Theory 185
11.3 Regular Reflection 187
11.4 Sonic Flow behind the Incident Shock 192
11.5 The Detachment Line for Regular Reflection 192
11.6 Mach Waves 193
11.7 The Sonic Line for Regular Reflection 194
11.8 Mach Reflection 194
11.9 The Number of Mach-Reflection Configurations 195
11.10 The von Neumann Point and Line 196
11.11 Henderson Points, Lines, and Regions. The von Neumann Region 197
11.12 The Normal Reflected-Shock Line for Mach Reflection 198
11.13 Angles of Incidence and Reflection 199
11.14 Curved Shocks, Effective Nozzle Flow, and Hysteresis 200
11.15 Possible and Impossible Configurations 201

11.16 Bibliographic Notes 204
11.17 Further Results and Exercises 204

12 The Hodograph Method 206
12.1 Arbitrary Fluid 206
12.2 Polytropic Gas 209
12.3 Ringleb's Flow 211
12.4 Geometrical Theory. The Legendre Transformation 214
12.5 Bibliographic Notes 220
12.6 Further Results and Exercises 220

13 Guide to High Speed Flow 222
13.1 Introduction 222
13.2 Research on High Speed Flow, 1860–1945 222
13.3 Reference Works on High Speed Flow, 1953–1964 222
13.4 Texts and Monographs on High Speed Flow, 1947–1998 223
13.5 Surveys and Reviews of High Speed Flow, 1949–1999 225
13.6 Research on High Speed Flow, 1990–1998 225

References 229

Index 247

1

Preliminaries

1.1 The Mach Number

The theory of high speed flow is concerned with flows of fluid at speeds high enough that account must be taken of the fluid's compressibility. The theory finds application in many branches of science and technology, from which we may single out, as being of unrivalled importance in the modern world, the applications to high speed flight.

The dimensionless parameter that measures the importance of a fluid's compressibility in high speed flow is the *Mach number.* Suppose that, at a given point in space and time, the speed of the fluid is u and the speed of sound is c. The Mach number M is defined as the ratio u/c. Thus

$$M = \frac{u}{c}. \tag{1.1.1}$$

In general, the value of the Mach number varies with position and time. But in many problems, we may choose a representative flow speed, say U, and a representative sound speed, say c. Then the quantity U/c is a single number measuring the importance of compressibility in the flow, and we may say that the flow is taking place at a Mach number $M = U/c$.

1.2 Flow Regimes

Flows corresponding to different ranges of the Mach number M have very different properties. We shall distinguish five regimes, namely (a) incompressible flow, (b) subsonic flow, (c) transonic flow, (d) supersonic flow, and (e) hypersonic flow.

(a) Incompressible Flow

This regime is defined by $M \ll 1$. The flow speed is low enough for the fluid's compressibility to be negligible. The regime is the basis of the classical subject

of hydrodynamics. Simple though it seems to state that $M \ll 1$, there are nevertheless some subtleties in the limit $M \to 0$. For example, the flow corresponding to the limit (U fixed, $c \to \infty$) can differ from that corresponding to ($U \to 0$, c fixed), because the former filters out sound waves, whereas the latter does not. Perhaps (U fixed, $c \to \infty$) should be called the incompressible limit, and ($U \to 0$, c fixed) the low speed limit.

(b) Subsonic Flow

This regime is defined by $0 < M < 1$, subject to the restriction that M is not too close to 0 or 1. The flow speed is high enough for the fluid's compressibility to be important, but low enough for the speed to be comfortably clear of the speed of sound. Since most aircraft fly well below the speed of sound, the regimes of incompressible flow and subsonic flow include most of standard aeronautics. The subsonic regime includes much of acoustics.

(c) Transonic Flow

This regime is defined by M being close to 1 in some important part of the flow. The regime raises difficult and interesting mathematical questions, because the governing partial differential equations are then of mixed type. That is, in some regions the equations are elliptic, in other regions they are hyperbolic, and on the separating lines or surfaces they are parabolic. The transonic regime is of vital importance to an aircraft or a land vehicle that "breaks the sound barrier."

(d) Supersonic Flow

This regime is defined by $M > 1$, subject to the restriction that M is not too close to 1 nor too large. Parts of the mathematical theory of supersonic flow can be obtained from that for subsonic flow by replacing $(1 - M^2)^{\frac{1}{2}}$ and an elliptic equation by $(M^2 - 1)^{\frac{1}{2}}$ and a hyperbolic equation. The theory of the supersonic regime was of importance to the design of the civil supersonic airplane *Concorde.*

(e) Hypersonic Flow

This regime is defined by $M \gg 1$. The flow speed is so high that compressibility is all-important, particularly in producing very high temperatures and ionization. The hypersonic regime is of importance for rockets and for the civil hypersonic aircraft "Orient Express."

In formulae for subsonic flow, two factors that often appear are the Doppler factor $(1 - M^2)^{\frac{1}{2}}$ and a factor that for air takes the value $(1 + \frac{1}{5}M^2)^{\frac{1}{2}}$. In many

practical problems, the flow may be regarded as effectively incompressible if the relevant factor is within a few percent of unity. For example, in air at atmospheric pressure and a temperature of 20°C, the speed of sound is $340\,\mathrm{m\,s^{-1}}$. Thus a speed of 100 m s^{-1} corresponds to $M = 0.294$, for which $(1 - M^2)^{\frac{1}{2}} = 0.956$ and $(1 + \frac{1}{5}M^2)^{\frac{1}{2}} = 1.009$; and a speed of 150 m s^{-1} corresponds to $M = 0.441$, for which $(1 - M^2)^{\frac{1}{2}} = 0.898$ and $(1 + \frac{1}{5}M^2)^{\frac{1}{2}} = 1.019$. Therefore, at sea level, compressibility is important at speeds of 100 m s^{-1}, and in precise work it may be important at somewhat lower speeds. For an aircraft flying at a given speed, the Mach number increases with height, because in the lower atmosphere the speed of sound decreases with height.

A feature of high speed flow in all except the incompressible and subsonic regimes is the widespread occurrence of shock waves, that is, surfaces across which the fluid may be regarded as having a discontinuity in pressure. The theory of shock waves is an important part of the subject of high speed flow and occupies an appreciable proportion of this book.

1.3 Temperature Changes

In high speed flow, the variables in the momentum equation are coupled to the thermodynamic variables, because changes in pressure compress or expand the fluid and alter its temperature. Equally, changes in temperature affect the pressure, via the equation of state. Therefore in the study of high speed flow there is no escape from some thermodynamics. In particular, it is necessary at some stage to introduce the concept of entropy.

In this book, the required thermodynamic theory is elementary and is derived from first principles. The main facts used are the definitions and basic properties of specific heats, enthalpy, and the gas constant. Some aspects of high speed flow, for example the theories of hypersonic flow and flow with combustion, require advanced ideas from thermodynamics, and thus they are beyond the scope of this book.

1.4 History

The most striking work on high speed flow in the nineteenth century was the visualisation of shock waves in air. In the 1870s, Mach deduced their positions from the white lines where intersections of shock waves blow the soot off a sooted glass plate, and hence he discovered many of their properties, including the different types of reflection, V-shaped or Y-shaped, that are possible at a wall. The shock waves were generated by electric sparks. In the 1880s, Mach and Salcher photographed the shock wave ahead of a supersonic projectile. They used the schlieren method, invented in 1864 by Toepler, in which changes

in refractive index due to changes in density are made visible by a special type of illumination. The photographs were excellent and showed, as well as the shock wave, many details of the flow, such as the "Mach lines" produced by surface roughness and the turbulent wake. Mach and Salcher also observed the diamond-shaped pattern of shock waves that can occur with a supersonic jet. The ratio u/c had been introduced to high speed flow by Doppler in the 1840s, before Mach's scientific work, and was not called the Mach number during Mach's lifetime (1838–1916); it was first called the Mach number by Ackeret in 1929. Mach was primarily an experimentalist and a philosopher, and some of his ideas must sound strange to most fluid dynamicists: For example, he did not believe in atoms and molecules. But Einstein was deeply influenced by Mach and publicized "Mach's principle."

In the twentieth century, the years prior to the Second World War saw steady progress in the theory and practice of high speed flow, especially by German engineers. High speed flow became a particularly important subject during 1939–1945 not only because of the war but also because at that time aircraft speeds were approaching the speed of sound. An intense research effort into the technology of high speed flight and ballistics was stimulated and drew upon the efforts of some powerful mathematicians, including Lighthill and von Neumann. The theory of characteristics reached its modern form during the war years and shortly afterwards. Other mathematical techniques, based on perturbation theory, were steadily extended as they were applied to problems of greater difficulty, and their modern form emerged only some years later with the development of, for example, the theory of matched asymptotic expansions.

A feature of research work on high speed flow since the Second World War has been the increasing use of high speed computers, hand-in-hand with the creation of a new subject, computational fluid dynamics. Among many successes has been the numerical computation of transonic flow fields, as required for the design of transonic aerofoils. The use of high speed computers now pervades all aspects of research into high speed flow and, indeed, other types of flow.

1.5 Recent Research

A large amount of research activity is currently taking place worldwide on problems related to high speed flow. The research takes place in university departments of mathematics, engineering, physics, and chemistry; in government research centers, for example in England at the Defence and Evaluation Research Agency, Farnborough, and in the United States at NASA Ames, Langley, and Lewis; and in corporate research centers, for example in England at Rolls-Royce, Derby and in the United States at research centers connected

with Boeing, General Electric, McDonnell Douglas, Pratt and Whitney, United Technologies, and their various merged entities. An inspection of articles in research journals, for example the *Journal of Fluid Mechanics*, shows that, in the several decades up to 1998, particularly active research areas related to high speed flow include (a) shock waves, (b) hypersonic flow, (c) jets, boundary layers, shear layers, and mixing layers, (d) high speed propellers and turbines, (e) aeroacoustics, and (f) combustion. The practical and commercial importance of research in these areas is confirmed by numerous articles in the aerospace magazine *Aviation Week*. Much research work is concerned with combinations of topics taken from areas (a)–(f), for example shock wave/boundary layer interactions, or the aeroacoustics of jets, or combustion in hypersonic flow. The number of such combinations having practical importance is large; and each combination can be related to the three classical fluid dynamical subjects of waves, stability, and turbulence and to the three classical methods of experiment, theory, and numerical computation. In this book we shall not try to survey such an enormous area of work. But throughout the book we give references to a representative sample of recent research papers in areas (a)–(f). We shall resist the temptation to extend the list of research areas even further, for example to magnetohydrodynamics and astrophysics.

1.6 Bibliographic Notes

Some articles on the history and practioners of high speed flow are: "Contributions of Ernst Mach to fluid dynamics," Reichenbach (1983); "Jakob Ackeret and the history of the Mach number," Rott (1985); "Compressible flow in the thirties," Busemann (1971); "Recalling the Vth Volta congress: High speeds in aviation," Ferrari (1996); and "Keith Stewartson: His life and work," Stuart (1986). The study of shock waves by the white lines they produce on a sooted plate is described in Mach (1878), and photographs of shock waves appear in Mach & Salcher (1887). A list of selected early papers relating to high speed flow, from Earnshaw (1860) to Frankl (1945), is given in Table 13.2.1 of Chapter 13. The table lists the founders of the subject, and a perusal of the titles of the papers, in the references, will give a preliminary idea of the way the subject developed. Other contributors to research in high speed flow, now better known for their work in other fields, were Chandrasekhar, Gamow, Robinson, Shepherdson, Tukey, and Weyl.

Much of the research work on high speed flow performed in the war years 1939–1945 and shortly afterwards appeared in systematic form in the monograph "Supersonic flow and shock waves," Courant & Friedrichs (1948) and in two reference works, all with extensive bibliographies. The first reference work was published in 1953 as the two volumes of *Modern Developments in Fluid Dynamics: High Speed Flow*, edited by Howarth. The second was published during the period 1955–1964 as the twelve

volumes of *High Speed Aerodynamics and Jet Propulsion*. Each volume had its own editors, and the series as a whole was edited at different times by Summerfield, Charyk, and Donaldson.

Another reference work is the *Encyclopedia of Physics*, edited by Flügge. The volumes containing articles on high speed flow were published in 1959 as *Fluid Dynamics I* and in 1960 as *Fluid Dynamics III*, forming Volume 8 Part I and Volume 9 of the encyclopedia. The volumes were coedited by Truesdell.

The three reference works are each the work of many authors, and together with Courant & Friedrichs (1948) they present the theory of high speed flow in the form used by later writers and researchers. The works still appear modern. A selection of articles, from Bickley (1953) to Moore (1964), is given in Table 13.3.1. The individual volumes are listed on the first page of the references. Their contents indicate the great increase in knowledge of high speed flow that had been obtained in the period 1939–1964.

The requirements of teaching and research in high speed flow, after the war, led to the production of textbooks and monographs, particularly in the 1950s. A selection of these, from Sauer (1947) to Laney (1998), is given in Table 13.4.1.

Many surveys and reviews of research on high speed flow performed since the mid-1960s appear in the *Annual Review of Fluid Mechanics*. The first volume, published in 1969, contained "Shock waves and radiation" by Zel'dovich & Raizer, and the 1999 volume contains "Computational fluid dynamics of whole-body aircraft" by Agarwal. A selection of these reviews, together with some from other sources, is listed in Table 13.5.1. The first article, a survey of previous work on high speed flow, is by Lighthill (1949), and the next two items, on transonic flow, are by Germain & Bader (1952) and Guderley (1953). Fifteen items are listed for the period 1993–1999.

The papers on high speed flow that have appeared in the *Journal of Fluid Mechanics* in the period 1990–1998 are listed in the tables in Section 13.6. Many papers on high speed flow may be found in Collected Papers of Sir James Lighthill (1997), edited by Hussaini, and many excellent photographs are in *An Album of Fluid Motion*, assembled by Van Dyke (1982).