

High Speed Flow

High Speed Flow is a textbook suitable for undergraduates, postgraduates, and research workers in fluid dynamics. It covers such topics as subsonic and supersonic flight, shock waves, high-speed aerofoils, and thermodynamics. Starting from first principles, the book gives complete and elementary derivations of all results and takes the reader to research level in the subject.

The book contains many exercises and an extensive bibliography, providing access to the entire literature of the subject from 1860 to the present day, and including over two hundred items published since 1990. It contains the most extensive set of formulae on oblique shock waves ever assembled.

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Cambridge University Press
978-0-521-66169-0 - High Speed Flow
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CAMBRIDGE
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Cambridge University Press
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CAMBRIDGE UNIVERSITY PRESS
 Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo, Delhi

Cambridge University Press
 The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
 Information on this title: www.cambridge.org/9780521661690

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First published 2000

A catalogue record for this publication is available from the British Library

Library of Congress Cataloguing in Publication data

Chapman, C. J. (Christopher John), 1952–
 High speed flow / C.J. Chapman
 p. cm. – (Cambridge texts in applied mathematics)
 Includes bibliographical references and index.
 ISBN 0-521-66169-2 (hbk.). – ISBN 0-521-66647-3 (pbk.)
 1. Fluid dynamics. I. Title. II. Series.
 QA911.C43 2000
 532'.0532 – dc21 99-37544
 CIP

ISBN 978-0-521-66169-0 hardback
 ISBN 978-0-521-66647-3 paperback

Transferred to digital printing 2009

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Contents

<i>Preface</i>	<i>page xi</i>
1 Preliminaries	1
1.1 The Mach Number	1
1.2 Flow Regimes	1
1.3 Temperature Changes	3
1.4 History	3
1.5 Recent Research	4
1.6 Bibliographic Notes	5
2 Governing Equations	7
2.1 Conservation of Mass. Jump Condition	7
2.2 Conservation of Momentum. Jump Condition	14
2.3 Conservation of Energy. Jump Condition	16
2.4 Equations in Conservation and Nonconservation Form	18
2.5 Pressure, Viscosity, and Thermal Conductivity	19
2.6 The Navier–Stokes Equations	20
2.7 Three Types of Compressible Flow	25
2.8 Bibliographic Notes	28
3 Thermodynamics	29
3.1 General Laws and Definitions	29
3.2 Practical Thermodynamic Formulae	37
3.3 Pressure, Volume, and Temperature	40
3.4 Entropy	40
3.5 Internal Energy	42

3.6	Enthalpy	43
3.7	The Helmholtz Function	44
3.8	The Gibbs Function	45
3.9	The Perfect Gas	45
3.10	The Perfect Gas with Constant Specific Heats	50
3.11	Bibliographic Notes	54
4	Smooth Flow of an Ideal Fluid	55
4.1	Lattice of Special Cases	55
4.2	General Flow. Crocco's Equation	57
4.3	Steady Flow	58
4.4	Irrotational Homentropic Flow	61
4.5	Steady Irrotational Homentropic Flow	63
4.6	Bibliographic Notes	64
4.7	Further Results and Exercises	64
5	Characteristic Surfaces and Rays	67
5.1	Nonsmooth Flow	67
5.2	Characteristic Surfaces	68
5.3	The Monge Cone	75
5.4	Ray Equations	77
5.5	Application to High Speed Flow	79
5.6	Bibliographic Notes	83
5.7	Further Results and Exercises	83
6	Shocks	85
6.1	Jump Conditions	85
6.2	Normal Shocks in an Arbitrary Fluid. The Rankine–Hugoniot Relation	87
6.3	Normal Shocks in a Polytropic Gas. The Prandtl Relation	92
6.4	Oblique Shocks. The Shock Polar	98
6.5	Bibliographic Notes	115
6.6	Further Results and Exercises	115
7	Steady One-Dimensional Flow	120
7.1	Flow in a Stream Tube	120
7.2	Flow in Ducts and Nozzles. Choked Flow	125
7.3	The Laval Nozzle	127
7.4	Bibliographic Notes	131
7.5	Further Results and Exercises	131

<i>Contents</i>		ix
8 Prandtl–Meyer Expansion		133
8.1 Problems in Two Independent Variables		133
8.2 Prandtl–Meyer Expansion		133
8.3 Flow round a Smooth Bend		141
8.4 Bibliographic Notes		142
8.5 Further Results and Exercises		142
9 Aerofoils		145
9.1 Linear Theory. Subsonic and Supersonic Flow		145
9.2 Nonlinear Theory		152
9.3 Transonic Flow		153
9.4 Bibliographic Notes		160
9.5 Further Results and Exercises		161
10 Characteristics for Steady Two-Dimensional Flow		165
10.1 Governing Equations		165
10.2 The Friedrichs Theory		166
10.3 Ordinary Differential Equations on Characteristics		169
10.4 Riemann Invariants		172
10.5 Flow round a Smooth Bend		173
10.6 Bibliographic Notes		176
10.7 Further Results and Exercises		176
11 Shock Reflections and Intersections		180
11.1 Common Shock Patterns		180
11.2 The von Neumann–Henderson Theory		185
11.3 Regular Reflection		187
11.4 Sonic Flow behind the Incident Shock		192
11.5 The Detachment Line for Regular Reflection		192
11.6 Mach Waves		193
11.7 The Sonic Line for Regular Reflection		194
11.8 Mach Reflection		194
11.9 The Number of Mach-Reflection Configurations		195
11.10 The von Neumann Point and Line		196
11.11 Henderson Points, Lines, and Regions. The von Neumann Region		197
11.12 The Normal Reflected-Shock Line for Mach Reflection		198
11.13 Angles of Incidence and Reflection		199
11.14 Curved Shocks, Effective Nozzle Flow, and Hysteresis		200
11.15 Possible and Impossible Configurations		201

11.16 Bibliographic Notes	204
11.17 Further Results and Exercises	204
12 The Hodograph Method	206
12.1 Arbitrary Fluid	206
12.2 Polytropic Gas	209
12.3 Ringleb's Flow	211
12.4 Geometrical Theory. The Legendre Transformation	214
12.5 Bibliographic Notes	220
12.6 Further Results and Exercises	220
13 Guide to High Speed Flow	222
13.1 Introduction	222
13.2 Research on High Speed Flow, 1860–1945	222
13.3 Reference Works on High Speed Flow, 1953–1964	222
13.4 Texts and Monographs on High Speed Flow, 1947–1998	223
13.5 Surveys and Reviews of High Speed Flow, 1949–1999	225
13.6 Research on High Speed Flow, 1990–1998	225
<i>References</i>	229
<i>Index</i>	247

Preface

This book is based on a course of lectures I gave for several years to students reading Part III of the Mathematical Tripos at the University of Cambridge. The course was well received, and since giving the course I have had requests for copies of the lecture notes. The students on the course were familiar with the basics of fluid dynamics, but they had little prior knowledge of the effects of compressibility and required a course that, though elementary, would prepare them for the literature of high speed flow in the larger texts and in research journals, for example in the *Journal of Fluid Mechanics*.

In writing the book I have kept closely to the topics included in the original lecture course, which consisted of sixteen one-hour lectures. Accordingly, the book is suitable for undergraduate or beginning graduate students and could be studied subject-by-subject or in its entirety. The book derives most results from first principles, so that the only formal prerequisite from fluid dynamics is familiarity with the equations of motion (i.e., the equations of conservation of mass, momentum, and energy). All results from thermodynamics are derived from first principles. The emphasis is on topics that occur as simple components in the description of general flows; these topics include the Rankine–Hugoniot relation, Prandtl–Meyer expansion, Prandtl’s relation, Riemann invariants, Mach reflection, von Neumann reflection, Maxwell’s relations, Legendre transformations, the Laval nozzle, Hamilton’s ray equations, the Monge cone, Crocco’s equation, Ringleb’s flow, Helmholtz free energy, and Gibbs free energy. The book contains a large number of exercises and problems for the reader, which would also be suitable for a lecturer in setting problem sheets or examinations.

After deliberation, I decided to include in the book a set of references large enough to provide an entry point to the entire literature of high speed flow, but to place all references in special sections at the end of each chapter and in a special chapter at the end of the book. In particular, I have included references to all

the papers on high speed flow published in the *Journal of Fluid Mechanics* in the period 1990–1998. These papers, somewhat over two hundred in number, lend themselves readily to classification into four main groups, indicated by the tables in Section 13.6, and give a fair indication of the scope of current research in high speed flow. After studying this elementary book, the reader may be pleasantly surprised to find that an understanding of nearly all these papers is within his or her grasp.

I thank many colleagues, in the United Kingdom and throughout the world, for their generosity in providing comments on individual chapters of the book.