Cambridge University Press
978-0-521-66169-0 - High Speed Flow
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Preliminaries

1.1 The Mach Number

The theory of high speed flow is concerned with flows of fluid at speeds high enough that account must be taken of the fluid's compressibility. The theory finds application in many branches of science and technology, from which we may single out, as being of unrivalled importance in the modern world, the applications to high speed flight.

The dimensionless parameter that measures the importance of a fluid's compressibility in high speed flow is the *Mach number*. Suppose that, at a given point in space and time, the speed of the fluid is u and the speed of sound is c. The Mach number M is defined as the ratio u/c. Thus

$$M = \frac{u}{c}. \tag{1.1.1}$$

In general, the value of the Mach number varies with position and time. But in many problems, we may choose a representative flow speed, say U, and a representative sound speed, say c. Then the quantity U/c is a single number measuring the importance of compressibility in the flow, and we may say that the flow is taking place at a Mach number $M = U/c$.

1.2 Flow Regimes

Flows corresponding to different ranges of the Mach number M have very different properties. We shall distinguish five regimes, namely (a) incompressible flow, (b) subsonic flow, (c) transonic flow, (d) supersonic flow, and (e) hypersonic flow.

(a) Incompressible Flow

This regime is defined by $M \ll 1$. The flow speed is low enough for the fluid's compressibility to be negligible. The regime is the basis of the classical subject of hydrodynamics. Simple though it seems to state that $M \ll 1$, there are nevertheless some subtleties in the limit $M \to 0$. For example, the flow corresponding to the limit (U fixed, $c \to \infty$) can differ from that corresponding to ($U \to 0$, c fixed), because the former filters out sound waves, whereas the latter does not. Perhaps (U fixed, $c \to \infty$) should be called the incompressible limit, and ($U \to 0$, c fixed) the low speed limit.

(b) Subsonic Flow

This regime is defined by $0 < M < 1$, subject to the restriction that M is not too close to 0 or 1. The flow speed is high enough for the fluid's compressibility to be important, but low enough for the speed to be comfortably clear of the speed of sound. Since most aircraft fly well below the speed of sound, the regimes of incompressible flow and subsonic flow include most of standard aeronautics. The subsonic regime includes much of acoustics.

(c) Transonic Flow

This regime is defined by M being close to 1 in some important part of the flow. The regime raises difficult and interesting mathematical questions, because the governing partial differential equations are then of mixed type. That is, in some regions the equations are elliptic, in other regions they are hyperbolic, and on the separating lines or surfaces they are parabolic. The transonic regime is of vital importance to an aircraft or a land vehicle that "breaks the sound barrier."

(d) Supersonic Flow

This regime is defined by $M > 1$, subject to the restriction that M is not too close to 1 nor too large. Parts of the mathematical theory of supersonic flow can be obtained from that for subsonic flow by replacing $(1 - M^2)^{\frac{1}{2}}$ and an elliptic equation by $(M^2 - 1)^{\frac{1}{2}}$ and a hyperbolic equation. The theory of the supersonic regime was of importance to the design of the civil supersonic airplane *Concorde.*

(e) Hypersonic Flow

This regime is defined by $M \gg 1$. The flow speed is so high that compressibility is all-important, particularly in producing very high temperatures and ionization. The hypersonic regime is of importance for rockets and for the civil hypersonic aircraft "Orient Express."

In formulae for subsonic flow, two factors that often appear are the Doppler factor $(1 - M^2)^{\frac{1}{2}}$ and a factor that for air takes the value $(1 + \frac{1}{5}M^2)^{\frac{1}{2}}$. In many practical problems, the flow may be regarded as effectively incompressible if the relevant factor is within a few percent of unity. For example, in air at atmospheric pressure and a temperature of 20°C, the speed of sound is 340 m s^{-1}. Thus a speed of 100 m s^{-1} corresponds to $M = 0.294$, for which $(1 - M^2)^{\frac{1}{2}} = 0.956$ and $(1 + \frac{1}{5}M^2)^{\frac{1}{2}} = 1.009$; and a speed of 150 m s^{-1} corresponds to $M = 0.441$, for which $(1 - M^2)^{\frac{1}{2}} = 0.898$ and $(1 + \frac{1}{5}M^2)^{\frac{1}{2}} = 1.019$. Therefore, at sea level, compressibility is important at speeds of 100 m s^{-1}, and in precise work it may be important at somewhat lower speeds. For an aircraft flying at a given speed, the Mach number increases with height, because in the lower atmosphere the speed of sound decreases with height.

A feature of high speed flow in all except the incompressible and subsonic regimes is the widespread occurrence of shock waves, that is, surfaces across which the fluid may be regarded as having a discontinuity in pressure. The theory of shock waves is an important part of the subject of high speed flow and occupies an appreciable proportion of this book.

1.3 Temperature Changes

In high speed flow, the variables in the momentum equation are coupled to the thermodynamic variables, because changes in pressure compress or expand the fluid and alter its temperature. Equally, changes in temperature affect the pressure, via the equation of state. Therefore in the study of high speed flow there is no escape from some thermodynamics. In particular, it is necessary at some stage to introduce the concept of entropy.

In this book, the required thermodynamic theory is elementary and is derived from first principles. The main facts used are the definitions and basic properties of specific heats, enthalpy, and the gas constant. Some aspects of high speed flow, for example the theories of hypersonic flow and flow with combustion, require advanced ideas from thermodynamics, and thus they are beyond the scope of this book.

1.4 History

The most striking work on high speed flow in the nineteenth century was the visualisation of shock waves in air. In the 1870s, Mach deduced their positions from the white lines where intersections of shock waves blow the soot off a sooted glass plate, and hence he discovered many of their properties, including the different types of reflection, V-shaped or Y-shaped, that are possible at a wall. The shock waves were generated by electric sparks. In the 1880s, Mach and Salcher photographed the shock wave ahead of a supersonic projectile. They used the schlieren method, invented in 1864 by Toepler, in which changes in refractive index due to changes in density are made visible by a special type of illumination. The photographs were excellent and showed, as well as the shock wave, many details of the flow, such as the "Mach lines" produced by surface roughness and the turbulent wake. Mach and Salcher also observed the diamond-shaped pattern of shock waves that can occur with a supersonic jet. The ratio u/c had been introduced to high speed flow by Doppler in the 1840s, before Mach's scientific work, and was not called the Mach number during Mach's lifetime (1838–1916); it was first called the Mach number by Ackeret in 1929. Mach was primarily an experimentalist and a philosopher, and some of his ideas must sound strange to most fluid dynamicists: For example, he did not believe in atoms and molecules. But Einstein was deeply influenced by Mach and publicized "Mach's principle."

In the twentieth century, the years prior to the Second World War saw steady progress in the theory and practice of high speed flow, especially by German engineers. High speed flow became a particularly important subject during 1939–1945 not only because of the war but also because at that time aircraft speeds were approaching the speed of sound. An intense research effort into the technology of high speed flight and ballistics was stimulated and drew upon the efforts of some powerful mathematicians, including Lighthill and von Neumann. The theory of characteristics reached its modern form during the war years and shortly afterwards. Other mathematical techniques, based on perturbation theory, were steadily extended as they were applied to problems of greater difficulty, and their modern form emerged only some years later with the development of, for example, the theory of matched asymptotic expansions.

A feature of research work on high speed flow since the Second World War has been the increasing use of high speed computers, hand-in-hand with the creation of a new subject, computational fluid dynamics. Among many successes has been the numerical computation of transonic flow fields, as required for the design of transonic aerofoils. The use of high speed computers now pervades all aspects of research into high speed flow and, indeed, other types of flow.

1.5 Recent Research

A large amount of research activity is currently taking place worldwide on problems related to high speed flow. The research takes place in university departments of mathematics, engineering, physics, and chemistry; in government research centers, for example in England at the Defence and Evaluation Research Agency, Farnborough, and in the United States at NASA Ames, Langley, and Lewis; and in corporate research centers, for example in England at Rolls-Royce, Derby and in the United States at research centers connected with Boeing, General Electric, McDonnell Douglas, Pratt and Whitney, United Technologies, and their various merged entities. An inspection of articles in research journals, for example the *Journal of Fluid Mechanics*, shows that, in the several decades up to 1998, particularly active research areas related to high speed flow include (a) shock waves, (b) hypersonic flow, (c) jets, boundary layers, shear layers, and mixing layers, (d) high speed propellers and turbines, (e) aeroacoustics, and (f) combustion. The practical and commercial importance of research in these areas is confirmed by numerous articles in the aerospace magazine *Aviation Week*. Much research work is concerned with combinations of topics taken from areas (a)–(f), for example shock wave/boundary layer interactions, or the aeroacoustics of jets, or combustion in hypersonic flow. The number of such combinations having practical importance is large; and each combination can be related to the three classical fluid dynamical subjects of waves, stability, and turbulence and to the three classical methods of experiment, theory, and numerical computation. In this book we shall not try to survey such an enormous area of work. But throughout the book we give references to a representative sample of recent research papers in areas (a)–(f). We shall resist the temptation to extend the list of research areas even further, for example to magnetohydrodynamics and astrophysics.

1.6 Bibliographic Notes

Some articles on the history and practioners of high speed flow are: "Contributions of Ernst Mach to fluid dynamics," Reichenbach (1983); "Jakob Ackeret and the history of the Mach number," Rott (1985); "Compressible flow in the thirties," Busemann (1971); "Recalling the Vth Volta congress: High speeds in aviation," Ferrari (1996); and "Keith Stewartson: His life and work," Stuart (1986). The study of shock waves by the white lines they produce on a sooted plate is described in Mach (1878), and photographs of shock waves appear in Mach & Salcher (1887). A list of selected early papers relating to high speed flow, from Earnshaw (1860) to Frankl (1945), is given in Table 13.2.1 of Chapter 13. The table lists the founders of the subject, and a perusal of the titles of the papers, in the references, will give a preliminary idea of the way the subject developed. Other contributors to research in high speed flow, now better known for their work in other fields, were Chandrasekhar, Gamow, Robinson, Shepherdson, Tukey, and Weyl.

Much of the research work on high speed flow performed in the war years 1939–1945 and shortly afterwards appeared in systematic form in the monograph "Supersonic flow and shock waves," Courant & Friedrichs (1948) and in two reference works, all with extensive bibliographies. The first reference work was published in 1953 as the two volumes of *Modern Developments in Fluid Dynamics: High Speed Flow*, edited by Howarth. The second was published during the period 1955–1964 as the twelve volumes of *High Speed Aerodynamics and Jet Propulsion*. Each volume had its own editors, and the series as a whole was edited at different times by Summerfield, Charyk, and Donaldson.

Another reference work is the *Encyclopedia of Physics*, edited by Flügge. The volumes containing articles on high speed flow were published in 1959 as *Fluid Dynamics I* and in 1960 as *Fluid Dynamics III*, forming Volume 8 Part I and Volume 9 of the encyclopedia. The volumes were coedited by Truesdell.

The three reference works are each the work of many authors, and together with Courant & Friedrichs (1948) they present the theory of high speed flow in the form used by later writers and researchers. The works still appear modern. A selection of articles, from Bickley (1953) to Moore (1964), is given in Table 13.3.1. The individual volumes are listed on the first page of the references. Their contents indicate the great increase in knowledge of high speed flow that had been obtained in the period 1939–1964.

The requirements of teaching and research in high speed flow, after the war, led to the production of textbooks and monographs, particularly in the 1950s. A selection of these, from Sauer (1947) to Laney (1998), is given in Table 13.4.1.

Many surveys and reviews of research on high speed flow performed since the mid-1960s appear in the *Annual Review of Fluid Mechanics*. The first volume, published in 1969, contained "Shock waves and radiation" by Zel'dovich & Raizer, and the 1999 volume contains "Computational fluid dynamics of whole-body aircraft" by Agarwal. A selection of these reviews, together with some from other sources, is listed in Table 13.5.1. The first article, a survey of previous work on high speed flow, is by Lighthill (1949), and the next two items, on transonic flow, are by Germain & Bader (1952) and Guderley (1953). Fifteen items are listed for the period 1993–1999.

The papers on high speed flow that have appeared in the *Journal of Fluid Mechanics* in the period 1990–1998 are listed in the tables in Section 13.6. Many papers on high speed flow may be found in Collected Papers of Sir James Lighthill (1997), edited by Hussaini, and many excellent photographs are in *An Album of Fluid Motion*, assembled by Van Dyke (1982).

2 Governing Equations

2.1 Conservation of Mass. Jump Condition

In this chapter we apply conservation laws to arbitrary volumes of fluid, to obtain integral forms of the equations of motion. We consider carefully the conditions under which the integrals can be differentiated to give differential equations of motion, and we show that, at a surface of nonsmoothness, where differentiation is not possible, the integrals nevertheless determine jump conditions relating conditions on opposite sides of the surface. We then discuss the most useful forms of the equations of motion as required in problems of high speed flow.

Let a volume fixed in space be denoted V, and let its fixed surface be S. The volume contains fluid that at position $\mathbf{x}$ and time t has density $\rho(\mathbf{x}, t)$ and velocity $\mathbf{u}(\mathbf{x}, t)$. The equation of conservation of mass, in integral form, asserts that the rate of change of the mass of fluid in V equals the rate at which mass flows into V through S. Let an element of surface area be dS, always taken to be positive, and let the unit normal on S pointing out of V be $\mathbf{n}$, so that the vector element $d\mathbf{S}$ of surface area is $d\mathbf{S} = \mathbf{n}\, dS$, also pointing out of V, and the volume of fluid per unit time leaving V through dS is $\mathbf{u} \cdot d\mathbf{S}$. The definitions are illustrated in Figure 2.1.1. The mass of fluid per unit time leaving V through dS is $\rho\mathbf{u} \cdot d\mathbf{S}$, and the mass of fluid in an element of volume dV is $\rho\, dV$. Thus the equation of conservation of mass is

$$\frac{d}{dt}\int_V \rho\, dV = -\int_S \rho\,\mathbf{u} \cdot d\mathbf{S}. \qquad (2.1.1)$$

The generalization of (2.1.1) to allow for mass sources, as required in the theory of aeroacoustics or combustion for example, is readily obtained by adding to the right-hand side a source integral over V but will not be needed in this book.

If ρ and $\mathbf{u}$ are continuously differentiable functions of $\mathbf{x}$ and t, then in the surface integral on the right of (2.1.1) we may apply the divergence theorem,

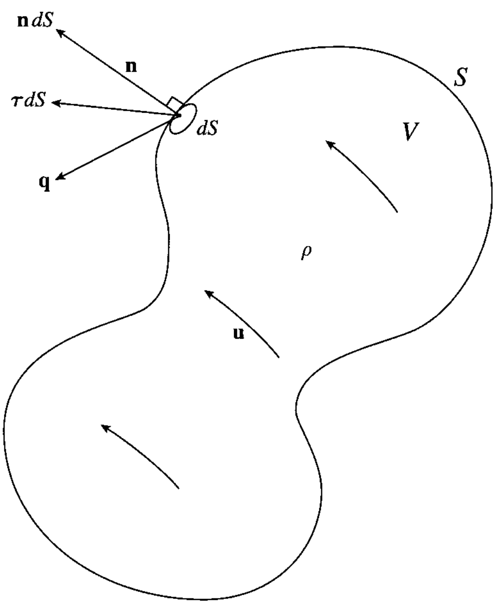

Fig. 2.1.1. Fluid of density ρ and velocity $\mathbf{u}$ in a volume V, fixed in space, bounded by a fixed surface S. At the element of surface area dS, the outward normal is $\mathbf{n}$, the stress is $\boldsymbol{\tau}$, and the heat flux is $\mathbf{q}$. The vector element of area is $\mathbf{n}\,dS$, and the force at dS on the fluid in V is $\boldsymbol{\tau}\,dS$.

and in the volume integral on the left we may take the differentiation with respect to t under the integration sign. Thus

$$\int_V \left\{ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) \right\} dV = 0. \tag{2.1.2}$$

Since (2.1.2) holds for an arbitrary volume V, the integrand is zero. Therefore

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0. \tag{2.1.3}$$

Expanded, (2.1.3) is

$$\frac{\partial \rho}{\partial t} + \mathbf{u} \cdot \nabla \rho + \rho \nabla \cdot \mathbf{u} = 0. \tag{2.1.4}$$

The convective derivative D/Dt is defined by

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla. \tag{2.1.5}$$

Thus (2.1.4) is

$$\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{u} = 0. \tag{2.1.6}$$

Equation (2.1.3), and its equivalent forms (2.1.4) and (2.1.6), are differential versions of the equation of conservation of mass.

Now suppose that ρ and $\mathbf{u}$ are no longer continuously differentiable functions of $\mathbf{x}$ and t, but may be discontinuous, for example because of the presence of a shock or a vortex sheet. Then the integral version (2.1.1) of the equation of conservation of mass remains valid, but not the step from (2.1.1) to (2.1.2) at the surface of discontinuity. Therefore the differential versions of the equation of conservation of mass need to be supplemented by a further equation.

Figure 2.1.2 shows a fixed volume V containing a moving surface of discontinuity. The density and velocity of the fluid in the two parts of V are $\rho_1, \mathbf{u}_1$, and $\rho_2, \mathbf{u}_2$. The differential versions of the equation of conservation of mass apply separately to $\rho_1, \mathbf{u}_1$, and to $\rho_2, \mathbf{u}_2$, and the supplementary condition we need must relate $\rho_1, \mathbf{u}_1$ to $\rho_2, \mathbf{u}_2$ on opposite sides of the moving surface. The limiting values of $\rho_1, \mathbf{u}_1$ and $\rho_2, \mathbf{u}_2$ as the surface is approached are denoted $\rho^-, \mathbf{u}^-$ and $\rho^+, \mathbf{u}^+$.

To obtain the required condition, we use an argument based on the construction of a small cylinder. Consider a fixed cylinder of small radius and still smaller depth, so oriented that its end faces are parallel to a surface of discontinuity that may be moving through it. A unit normal to the surface of discontinuity, pointing away from the fluid with density ρ^- and velocity $\mathbf{u}^-$, and towards the fluid with density ρ^+ and velocity $\mathbf{u}^+$, is denoted $\mathbf{n}$. Thus $\mathbf{n}$ is normal to the end faces of the cylinder. The velocity of the moving surface of discontinuity is $\mathbf{V}$. We allow $\mathbf{V}$ to point in any direction, although the final formulae depend on $\mathbf{V}$ only through $\mathbf{n} \cdot \mathbf{V}$. For any quantity associated with the fluid, the jump in value across the moving surface, from the negative side to the positive side, is denoted by square brackets, so that, for example,

$$[\rho] = [\rho]_-^+ = \rho^+ - \rho^-. \tag{2.1.7}$$

The volume of the cylinder is V, and the area of each end face is A. The definitions are illustrated in Figure 2.1.3.

We now evaluate, for the cylinder, each side of the integral version of the equation of conservation of mass, (2.1.1). Since fluid of density ρ^+ is being replaced with fluid of density ρ^- by a surface moving at a speed normal to itself of $\mathbf{n} \cdot \mathbf{V}$, and $\mathbf{n} \cdot \mathbf{V}$ measures the volume swept out per unit time by unit area of the surface of discontinuity, it follows that the change per unit time of the mass

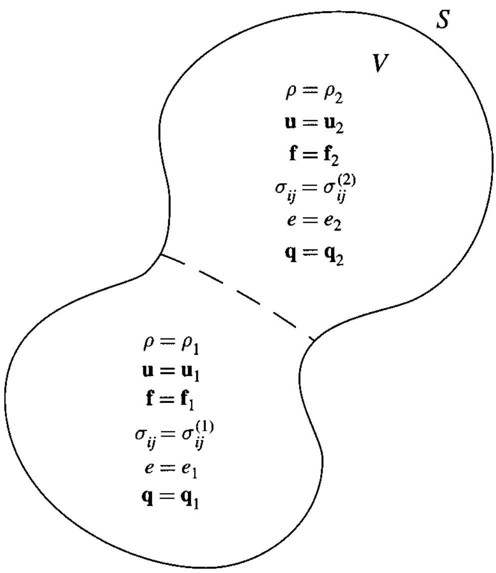

Fig. 2.1.2. Volume V, fixed in space, bounded by a fixed surface S and containing a moving surface of discontinuity (— —). Fluid density ρ, fluid velocity $\mathbf{u}$, body force per unit mass $\mathbf{f}$, stress tensor σ_{ij}, internal energy per unit mass e, heat flux $\mathbf{q}$; opposite sides of the surface of discontinuity are denoted 1, 2.

of fluid in the cylinder is given by

$$\begin{aligned} \frac{d}{dt}\int_V \rho\, dV &= (\rho^- - \rho^+)(\mathbf{n}\cdot\mathbf{V})A \\ &= -[\rho](\mathbf{n}\cdot\mathbf{V})A. \end{aligned} \tag{2.1.8}$$

The depth of the cylinder is assumed small enough compared with its radius that the mass flow through the curved surface is negligible compared with that through the end faces. Hence

$$\begin{aligned} \int_S \rho\mathbf{u}\cdot d\mathbf{S} &= \{\rho^+(\mathbf{u}^+\cdot\mathbf{n}) - \rho^-(\mathbf{u}^-\cdot\mathbf{n})\}A \\ &= [\rho\mathbf{u}\cdot\mathbf{n}]A. \end{aligned} \tag{2.1.9}$$

Therefore conservation of mass gives

$$[\rho](\mathbf{n}\cdot\mathbf{V}) = [\rho\mathbf{u}\cdot\mathbf{n}]. \tag{2.1.10}$$

Thus

$$[\rho(\mathbf{u}-\mathbf{V})\cdot\mathbf{n}] = 0. \tag{2.1.11}$$