

Index

- acceleration amplitude, 7, 10
- accommodation function, 196, 199
- activation energies, 95–96
- Aeolian sand ripples, 168–174
- aftermath piles, 121–125
- ageing
 - anomalous, 163
 - shape-dependent, 99–100, 102–103
- amplification, 134, 138, 146
- amplitude cycling, 90–93
- amplitude equations, 207
- angle of maximal stability, 12, 63, 69, 171, 174
- angle of repose
 - athermal requirement for, 52, 63
 - Bagnold angle and, 17, 65–66
 - crossover length and, 131
 - formation history dependence, 12, 66, 74
 - pressure profiles and, 248–249
 - ripples, 170, 174
 - roughness and, 74, 130–131
 - sandpile bistability and collapse, 63–71
 - SOC model and, 115–116
- angoricity, 3–4, 215
- anisotropic elasticity, 267
- annealed cooling, 97–100
- annealed disorder, 119
- annealed values, Edwards' flatness, 111
- anomalous ageing, 163
- anomalous roughening, 74–75, 165–168
- anticorrelations, 36, 40, 106, 108–109, 112
- apertures, flow through, 4–5
- apparent mass, overloaded silos, 246–248, 271
- 'arching', 245
- asymptotic density, 86–87, 92, 111
- asymptotic packing fraction, 96–97
- asymptotic roughness, 119–129
- asymptotic smoothing, 149, 162–163, 165, 167
- athermal system consequences, 19, 52, 63
- autocorrelation function, 31, 33
- avalanche footprints, 72–78
 - triangular avalanches, 71–78
 - uphill avalanches, 71–78, 121, 124
- avalanche models
 - cellular automaton models, 14–15, 116–120
 - coupled continuum equations, 148–167
 - coupled map lattice model, 133–147
 - directed percolation model, 70
 - self-organised criticality and, 12–13, 115–116
- avalanche size distribution, 14–15, 120–124, 126–127, 137–143
 - Type I and Type II, 115–120
- avalanches
 - angle of repose and, 1, 63, 72
 - bistability of sandpiles and, 1, 69–75
 - intermittent and continuous, 148–150, 156, 163, 167
 - large avalanche surface effects, 118–131, 142
 - long duration and continuous, 149–150, 156–162
 - mass time series, 122–124
 - percolative transport and, 39, 70
 - in rotating cylinders, 132–135
 - snow and rock, 115, 148
 - system-spanning, 128
- Bagnold angle, 17, 63, 65–66, 69
- Bagnold number, 2, 176
- barrier-height-to-temperature ratio, 67
- base extension, bridges, 55, 58
- BCRE (Bouchaud, Cates, Ravi Prakash, and Edwards) model, 157, 166
- Bennett model, 20
- Bessell functions, 231, 266
- biaxial tests, 273
- bistability
 - angle of repose and, 1, 12, 66
 - in ripples, 171, 174
 - in sandpiles, 63, 65–66
 - in tilted sandpiles, 69–78
- blocked configurations, 83, 85
- Boltzmann equations
 - applicability to granular gases, 177–178, 184–185, 188, 196–197
 - applicability to the jammed state, 209, 212–213
- Born–Huang formula, 253, 256
- boundary conditions, 196–200, 207, 266, 269

- boundary layers
 - avalanche motion and, 13, 147, 163
 - fast dynamics within, 107
 - grain-inertia simulations, 23
 - ordering depth and, 101, 113
- boundary stress, 258
- branch vectors, 256, 258
- 'Brazil nut' phenomenon, 47–50
- bridge formation, 30, 38, 52–62
- bridges
 - characteristic descriptors, 55
 - collapse and grain anticorrelation, 36, 40, 107
 - defined, 53
 - linear, 53–56, 58–61
 - shape and vibrational intensity, 30
- Brownian motion, 2, 6–7, 52–53, 67–68
- Burgers' equation, 16
- Burnett equations, 183, 187, 189, 195
- Burnett order, 182–183, 195, 200, 204
- CA models, *see* cellular automaton models
- Cahn–Hilliard model, 206
- captors, stress, 257
- carbon paper technique, 237–238
- cascade reorientation, 86–90
- Case A model, rotating cylinders, 151–156
- Case B model, rotating cylinders, 156–162
- Case C model, rotating cylinders, 162–167
- CE expansion, *see* Chapman–Enskog expansion
- cellular automaton (CA) models, 5, 13–15
 - grain-inertia regime, 23
 - with long-range interactions, 105–106
 - nonequilibrium regimes, 102–103
 - with orientational rearrangement, 94–96, 118–119
- ripples and dunes, 168
- sandpile collapse and, 70–78, 94–96, 118–131
- self-organised criticality and, 11–13, 116–117
- Type I avalanches, 116–117
- Type II avalanches, 118–131
- channels, flow through, 4–5, 23
- Chapman–Enskog (CE) expansion, 185–189, 202, 204, 206, 208
 - limitations of, 177, 182, 197
- chute flows, 208
- cluster aggregation model, 112–113
- cluster reorganisation, 28, 70
- clustering as a hydrodynamic effect, 178–181
- clusters
 - coupling with mobile grains, 148
 - deformation and shaking intensity, 43
 - instability and continuous avalanching, 161–162
 - intercluster and intracluster relaxation, 169, 174
 - merger as 'coarsening', 178, 180
 - as 'stuck' grains, 151
- CML (coupled map lattice) model, 15, 118, 134–141, 143–146
- coarse-graining
 - effective medium theory, 259–261
 - microscopic theory of, 226–230
 - stress field data, 253–261
- coarse-graining function, 253
- 'coarsening' process, 107, 178, 180
- coefficients of restitution, 22, 181, 184–185
- cohesion and segregation, 51
- collapse mechanisms, 181–182, *see also* bridges; sandpiles
- collision forces, *see* Bagnold number
- collision laws, granular gases, 184, 196–197
- colloids, 16, 97, 112, 229–230
- column grain size, 137
- column models, *see also* lattice-based models
 - ideal height, 133
 - jamming and, 104–105, 110–111, 113–114
- compaction
 - bridge collapse and, 53
 - irreversible and reversible branches, 90–92, 97–100
 - logarithmic compaction, 84, 86–87, 93, 99, 103
 - modelling, near the jamming limit, 79–93
 - tapping and the compaction curve, 83–84
- compactivity, 3–4
 - density fluctuations and, 64–65
 - shaken sand in cylinders, 210, 214–215, 220–221, 222–223
 - vibrated beds, 6
- complex bridges, 53–54, 56
- computer simulations, *see* simulations
- conductivity of graphite, 211
- configurational entropy, 3, 111–112
- configurational memory, 140, 146–147
- configurational overlap function, 98–100
- connected correlation function, 102–103
- conservation of momentum, 201
- consolidation
 - after avalanching, 125
 - by tapping, 40
 - CML model and, 133, 135–138, 140
 - isostatic behaviour and, 230
 - Monte Carlo consolidation, 26, 28–29
- contact angles, 36, 234, 240, 261
- contact forces
 - coarse-graining, 253–261
 - contact angles and, 234, 240, 261
 - deriving stresses from, 253–259
 - determination of, 209
 - effective medium theory, 259–261
 - exponential distribution of, 232, 242, 245
- contact networks, 218
- contact orientation in layers, 240–242
- continuous avalanching, 149–150, 156–162
- continuum approach
 - avalanches in rotating cylinders, 133–135
 - Boltzmann equations and, 187
 - bridge formation, 57
 - coupled continuum equations, 148–175
 - friction effects and, 203
 - ripples and dunes, 168, 171, 174
- convection processes, 5–11, 50–51
- cooperative dynamics, 27–28, 36, 38, *see also* angle of repose; bridge formation

- coordination numbers, 20–22
 - bridge formation and, 55
 - isostatic equilibrium and, 216, 222, 230
 - random graph models, 80
 - shaken granular systems, 212, 222
 - vibrational intensity and, 29–31
 - volume fraction and, 33
- correlation, molecular gas kinetics, 184–185
- correlation functions
 - autocorrelation function, 31, 33
 - cluster aggregation model, 112
 - connected, 102–103
 - coupled continuum equations, 152
 - EW model, 152–154
 - height–height correlation function, 120, 128, 150, 153
 - mass–mass correlation function, 127–128
 - random packings and, 33–36
 - transverse and longitudinal, 35
 - two-time, 102–103, 163
 - zero-temperature dynamics, 108
- Cosserat elasticity, 267
- Couette flow, 23, 51, 179, 237
- Coulomb friction, 201, 203, 236, 270
- coupled continuum equations, 148–175
- coupled map lattice (CML) model, 15, 118, 134–141, 143–146
- critical aspect ratios, 126
- critical phenomena
 - self-organised criticality, 11–13, 115–116
 - size dependence, 140
- critical slope threshold, 72, 137
- crossover behaviour
 - diffusive to asymptotic smoothing, 161–162
 - roughening to asymptotic smoothing, 165–166
- crystalline limit, 98
- crystallisation, 44–46, 90
- curve fitting, 41, 220
- cylinders, *see* rotating cylinders; shaken cylinders

- damping, 13, 15, 107
- degrees of freedom
 - fast and slow, 79
 - quadrons and, 220–221
 - translational and spin, 202, 205
- density, *see also* packing densities
 - asymptotic density, 86–87, 92, 111
 - of shaken systems, 211
- density fluctuations
 - angle of repose and, 63–66
 - bridges and, 59
 - clustering theory and, 178–180
 - compaction near the jamming limit, 84, 87
 - kinetic energy density, 186
 - variation with time, 155–156
 - zero-temperature dynamics and, 106–108
- density of states, 220
- diffusive behaviour
 - crossover to asymptotic, 161–162
 - linear bridges, 60
 - ripples, 170
- self-diffusion, 42–44
- shaking intensity and, 42
- dilatancy
 - angle of repose and, 63–66, 69
 - dynamic representations, 17
 - excess volume and, 146
 - large avalanches and, 139
 - roughness and, 74
 - sandpile collapse and, 69, 74
- dilatancy waves, 4–5
- dilation phase, 25–27, 54, 86
 - quench phase and, 82–84
- directed percolation model, 70, 77–78
- discretisation, continuum equations, 157–158
- disorder
 - cellular automaton models and, 14
 - compaction of disordered grains, 79–93
 - evolution of, 139
- displacement correlations, 34
- dissipation coefficient, 135
- distinct element method, 24
- distribution functions, *see* equilibrium; event-size; force; single-particle; spin distributions
- divergences, infrared, 157–158, 167
- dome formation, 58–61
- double Fourier transforms
 - Edwards–Wilkinson equation with flow, 152–154
 - temporal and spatial roughness, 150–151, 158–160
 - tilt combined with flowing grains, 164–165
- DSMC (direct simulation Monte Carlo) simulations, 188
- dunes, 167–168, 175
- dynamical arrest, 79
- dynamical exponent, 150
- dynamical heterogeneities, 34
- dynamical phase transitions, 86–87, 97
- dynamical scaling, 119–120
- dynamics, *see* fast dynamics; slow dynamics

- earthquakes, 124, 135, 142, 147
- Eden model, 19
- Edwards’ compactivity, *see* compactivity
- Edwards’ hypothesis, 108–113
- Edwards’ singularity, *see* infrared divergence
- Edwards–Wilkinson (EW) equation, 151–156
- effective medium theory, 253, 259–261, 264
- effective temperature, 2, 5, 17, 65, 67, *see also* granular temperature
- effective viscosity, 22–23
- elasticity, *see also* inelasticity
 - Cosserat elasticity, 267
 - effective medium theory, 259–261
 - isostaticity and, 230
 - stress field computation using, 264–268
- elasto-plasticity, 233, 253, 259, 261–262, 273
- elongated grains, 5
- energy
 - activation energies, 95–96
 - external sources, 24–25
- energy sink term, 203–204
- Enskog–Boltzmann equation, 184, 200

Enskog correction, 194, 206
 Enskog equations, 200, 206, *see also*
 Chapman–Enskog (CE) expansion
 entropy
 configurational entropy, 3, 111–112
 Edwards' hypothesis and, 108–113
 force probability distribution and, 245
 ground-state entropy, 106
 shaken sand in cylinders, 210, 214, 228
 equilibration times
 Boltzmann solutions and, 183
 dynamic transitions and, 86
 for jamming, 92
 number of particle collisions, 196–197
 shaken sandpiles, 100–101
 equilibrium distribution functions, 191
 equipartition in granular gases, 207–208
 equivalent temperatures, *see* compactivity
 ergodicity breakdowns, 87
 Euler angles, 217
 Euler equations, 187
 Euler relations, 221, 227, 287
 event-driven simulations, *see* hard-particle simulations
 event size distributions, 126, 137–143
 EW (Edwards–Wilkinson) equation, 151–156
 excess volume, *see* dilatancy
 excitations, 113, 220
 excitons, 68
 exit mass sizes, 126–127, 141–142, 145

 fabric tensors, 217
 fast and slow degrees of freedom, 40–42, 63–64
 fast dynamics
 non-ergodicity and, 87, 97
 simple CA model, 94–96
 SPRT and, 84–86
 FCC (face centred cubic) packing, 251
 Fibonacci numbers, 114
 flipping mechanisms, 83, 89, 95, 119
 flowing grains
 Case B model, 156–162
 Case C model, 162–167
 coupling with clusters, 148, 174
 diffusion in ripples, 170, 172
 EW model, 151–156
 molecular dynamics approaches, 22–24
 through wedges, channels and apertures, 4–5
 fluid mechanics and jamming, 209
 fluidisation, 6, 8, 45, 176–178
 fluidised regime characteristics, 99
 flux-divergence term, 170
 force and torque balance, 216–217, 223, 226–227,
 230, 234–235
 force chains, 57, 229, 236, 272
 force distribution function, 238–239, 243–244
 forces, *see also* contact forces
 contact orientation and, 240–242
 large-scale treatment of distributions, 245–273
 microscopic treatment of distributions, 233–245
 probability distribution, 237–239, 241, 245, 273
 sandpile force transmission, 216

 spatial distribution, 230–232
 volume fractions and, 215–222
 formation history, *see* preparation history
 Fourier transforms, *see* double Fourier transforms;
 single Fourier transforms
 'Fredholm alternative', 191, 197
 friction
 bridge formation and, 30, 54
 convective motion and, 9
 force chains and, 230
 force probability distribution and, 237
 in granular gases, 200–206, 208
 in silos, 247–248
 in vibrated beds, 6–7
 intergrain friction, 4
 internal friction coefficient and angle, 268
 response profiles and, 252
 friction coefficients, 4, 23, 201
 granular statics, 234, 237–238, 247, 252, 268, 270
 frozen state, 98, 100–102
 frustration, 81–84, 91, 106, 228–229
 full structure factor, 150

 Gamma distributions, 244
 gases and granular media characteristics, 18, *see also*
 granular gas models
 geometric tensors, 218–219, 228
 glaciers, 168
 glasses
 displacement correlations, 34
 jamming behaviour exhibited by, 52, 92
 Lenard-Jones glasses, 261
 percolative transport, 39
 'glassy' dynamics, 98, 100–102
 golden mean, 114
 Grad expansion, 184, 187–188
 grain anticorrelations, 36, 40, 108–109
 grain inertia
 amplification and, 134, 138
 cellular automata models, 13–15, 116
 CML models, 137–140, 146–147
 grain-inertia regimes, 2–4, 22–23
 grain reorientation
 avalanches with, 115–131
 cluster reorganisation and, 70
 intracluster rearrangement, 137
 stress–strain curves, 263
 grain shapes
 aspect ratio, disordered sandpiles, 125, 127
 ground state retrievability and, 114
 non-spherical grains, 208
 orientation modes and, 102–103
 packing simulations, 19, 49
 zero-temperature dynamics and, 106–108
 grains, *see also* flowing grains
 contacts between rigid grains, 230, 234
 weight, in q -model, 243
 granular gas models, 176–208
 boundary conditions, 196–200
 friction effects, 200–206
 kinetic theory, 184–196

- granular temperature, 177
 - as a hydrodynamic field, 201
 - compactivity and, 3, 64
 - rapid shear regimes and, 3, 22, 177–178
 - vibrated beds, 6
- graphite, density and conductivity, 211
- gravity
 - bridge formation and, 54–55
 - granular gases in outer space, 208
 - packing simulations and, 19, 24
- Green function, 229
- Green–Kubo relations, 206–207
- ground states
 - entropy, 106
 - excitons, 68
 - metastability, 179
 - propagation, 107
 - retrievability, 114
 - zero-temperature dynamics, 106–107, 110
- hard-particle simulations, 22–24, 28
 - of the jammed state, 209
 - simple CA lattice model and, 98
- HCP (hexagonal close packed) packing, 237–238, 251
- HCS (homogeneous cooling state), 178–180, 187, 200, 204–205, 207
- heap formation, vibrated beds, 7
- heat flux, 183, 187, 192–194, 203–204, 206
- Heaviside functions, 35, 256, 258
- height–height correlation function, 120, 128, 150, 153
- history, *see* preparation history
- hole radius distribution, 39
- homogeneous cooling state (HCS), 178–180, 187, 200, 204–205, 207
- homogenisation, *see* coarse-graining
- hoppers, 4–5, 23, 58, 60
- hopping between potential wells, 43–44
- hopping grains in ripples, 169–171
- hourglasses, 4–5
- hydrodynamic regimes, 42–44
 - clustering in granular gases, 178–183
 - frictional granular hydrodynamics, 200, 202–208
 - kinetic theory and, 185–188, 190–191, 195, 197
- hyperbolic equations, 229, 267, 269–273
- hypergraphs, 79–80
- hysteresis
 - in granular media, 21, 200, 263
 - in sandpiles, 2, 63, 98
 - in tilted sandpiles, 73–74
- ideal height, column models, 133
- impurities (tracer particles), 9–10, 47–49
- inelasticity, 181–182, 195
 - Boltzmann applicability and, 177, 184, 199
- inertia, *see* grain inertia; particle inertia
- infrared divergence, 157–158, 167
- inhomogeneous relaxation, 96
- integrodifferential equations, 196–197
- intergrain friction, 4
- intermediate phase, 99
- intermittency, surface layer, 113–114
- intermittent avalanching, 149–150, 156, 163
- internal friction coefficient and angle, 268
- interparticle percolation, 9–10
- intrinsic size dependence, 140
- irreducible loops, 230
- irreversible branch, compaction curve, 90–92, 97–100
- irreversible packing and plasticity, 262–264
- isostaticity
 - coordination numbers and, 216, 222, 230–231
 - force chains, 229
 - polydisperse beads, 235
- jammed systems
 - Boltzmann applicability, 209, 212–213
 - bridge formation and, 52, 57
 - configurational probabilities and, 110
 - contact network loops and voids, 218
 - grain anticorrelation and, 36
 - thermodynamics of, 209–232
 - vibrated hourglasses, 4–5
- jamming limit
 - amplitude cycling model, 90–92
 - displacement correlations and, 34
 - entropies near, 111
 - modelling compaction near, 79–93
 - shaken sand and grain shapes, 104–114
- Janssen’s model for silos, 233, 246–248, 270–271
- Kadanoff model, 144
- kernel function, 213
- kinetic theory, 183–196
- Knudsen domains, 208
- Knudsen number, 185–188, 196, 204
- Knudsen orders, 195
- KPZ (Kardar–Parisi–Zhang) equation, 16
- Landau–Ginsberg models, 206–207
- Landau’s notation, 264
- large-scale properties of granular materials, 245–273
- lattice-based models, 13, 23, 79, 85, 104, *see also* CA models; CML model; column models
- lattice gas models, 116
- lattice grain models, 13–14, 116
- least squares fit, 40, 42, *see also* curve fitting
- Lenard–Jones glasses, 261
- linear bridges
 - dome formation from, 58–61
 - formation, 53–54
 - size distribution, 55–56
- link angle, bridge formation, 58
- liquids, granular media compared to, 1, 18
- logarithmic coarsening law, 107
- logarithmic compaction, 84, 86–87, 93, 99, 103
- logarithmic growth of packing fraction, 97
- longitudinal correlation functions, 35
- loops, 54, 80, 218–221, 226–227, 230
- Love stress tensor, 217
- low-amplitude pinning, 91
- low-temperature dynamics, 113–114

- magnetic resonance imaging (MRI), 50–51
- magnitude distributions, *see* avalanche size distributions
- main axis, bridges, 55
- mass–mass correlation function, 127–128
- maximal angle of stability, 12, 63, 69, 171, 174
- ‘Maxwell demon effect’, 180
- MD (molecular dynamics) simulations, 22–25, 30, 55, 57
- mean angle, 58–61, 72, 125
- mean-field equations, 166–167, 244
- mean-field theory, 79–81, 112
- mean force, 240
- mean free path, 177, 182–185, 187–188, 196–198, 204
- mean free time, 183
- mechanical equilibrium, 210, 230, 264, 269, 273
 - coordination number and, 216
- mesoscopicity, 182
- metastability
 - dense granular systems, 179
 - equilibration of older systems, 99
 - finite lattice and mean field, 112
 - three-spin model requirement, 81–82
- minimum event size, 134
- miscibility theory, 215
- mixtures of grains, *see* segregation of mixtures
- mobile grain coupling with clusters, 148
- Mohr–Coulomb yield criteria, 233, 268–272
- molecular chaos (Stosszahlansatz), 184, 212
- molecular dynamics (MD) simulations, 22–25, 30, 55, 57
- molecular gases, 176–177, 184, 186–187, 195
- monodisperse spheres
 - Boltzmann equation, 185
 - simulations based on, 19, 21, 39
- Monte Carlo simulations, 18–22
 - compression phase, 28
 - DSMC simulations, 188
 - friction and, 54
 - size segregation, 10, 25
 - three-spin model and, 82
- mutual stabilisation, 53, 55
- Navier–Stokes equations, 116, 182, 187, 189, 195
- Newton’s third law, 217, 226, 254
- NMR (nuclear magnetic resonance), 51, 58
 - noise, 25, 88–89, 113, 151, 173
 - white, 59, 64, 157, 162
- non-Abelian models, 20–22, 116
- non-equipartition, 207
- non-ergodicity, fast dynamics, 87, 97
- non-hydrodynamic phenomena, 181–182, 207
- nonsequential packing simulations, 20–22, 28
- nucleation scenarios, 45
- one-species model, 168
- ordering length, 113–114
- orientation angle, 55, 58, 217
- orientation distribution, 36–37, 240
- oriented stress linearity (OSL) model, 271–273
- orientedness parameter, 100–102
- orthogonality principle, 42, 191–193
- oscillons, 8
- OSL (oriented stress linearity) model, 271–273
- Oslo rice pile experiments, 127
- overlap function, 98–100
- overshoot effect
 - inhomogeneous relaxation and, 96–97
 - in silos, 247–248, 271
 - packing density and, 85, 263
- packing densities, 3, 21, 30–31, 36, 271, *see also* random close packing
- packing fractions
 - annealed cooling and, 97–100
 - bridge formation and, 61–62
 - disordered sandpiles, 125
 - monodisperse spheres, 19, 21
 - orientedness parameter and, 100
 - RCP threshold and, 44–46
 - shaken sand simulations, 25, 96–97
- packing structures, 19–22
- ‘parking-lot’ model, 89
- partial voids, 39
- particle inertia, 15
- particle size, *see* grain shape; segregation of mixtures
- Peclet numbers, 4
- percolation
 - directed percolation model of avalanching, 70, 77–78
 - interparticle percolation, 9–10
 - segregation of mixtures and, 46–48, 51
- percolation clusters, 57
- percolative transport, 39
- perfect packing, 45–46
- ‘phase diagram’
 - avalanche morphologies, 71, 77
 - dynamical phase transitions, 86–87, 97
- photoelasticity, 236, 250–251
- pinning, low-amplitude mechanical, 91–92
- plasticity
 - elasto-plasticity, 233, 253, 259, 261–262, 273
 - irreversible packing as, 264
 - onset of, 226
 - stress–strain curve, 268
- plug flow, 5–6, 179–180
- Poisson ratio, 260, 262, 265, 267, 271
- polar histograms, 241–242
- polydispersity
 - granular gases, 208
 - response function and, 251–252
 - statics of polydisperse grains, 234–235, 264, 273
- polymers and bridge models, 56, 60
- polynomials, Sonine, 191
- pore spaces, 39
- pouring
 - Maxwell demon effect, 180
 - preparation history effects, 12, 19–20
 - sandpile preparation by, 209, 246–248
 - simulation of, 40
 - size segregation and, 50

- power law behaviour
 - avalanche size distribution, 115, 129, 137, 139
 - density of states, 220
 - Fourier transforms, 159, 161, 163
 - q -mode force distribution, 245
- power spectrum fluctuations, 88–89, 115–116
- precursor piles, 121–124
- preparation history
 - angle of repose dependence on, 12, 66, 74
 - contact orientation and, 241–242
 - force probability distribution and, 237
 - packing density dependence on, 19, 271
 - pressure in jammed systems and, 209–210, 216, 248–252
 - stress–strain curves, 263
- pressure
 - force distribution function and, 238–239
 - localised overloads and, 250–252
 - preparation history and, 209–210, 248–250
- q -model, 242–245
- quadrons, 219–223, 227
- quasiperiodicity, 121, 140, 146
- quasistatic flow, 3–5, 23–24, 42–43, 137
- quench phase, 55, 82–85
- quenched disorder, 17, 92
- quenched systems
 - coarse-graining in, 226–230
 - stress tensor derivation for, 223
- quenched values, Edwards' flatness, 111
- ramp rates, 90–92, 98
- random close packing (RCP) density, 19, 27, 44
 - dynamic transitions and, 87
 - jamming limit entropies near, 111
 - ramp rates and, 90–93, 98
- random deposition in sandpiles, 141–142
- random graph models, 79–93
 - cluster aggregation model and, 112
 - Edwards' hypothesis, 113
 - three-spin model, 81–82
- random transmission coefficients, 243
- random trapping, 67
- random walks, 5, 42, 56, 60, 109
- 'rattlers', 84–86, 106–107, 110–111, 212
- RCP, *see* random close packing density
- relaxation events
 - avalanches in rotating cylinders, 133–137, 141–142, 144–145
 - inhomogeneous relaxation, 96
 - ripple formation, 169
 - vibrated powders, 40–41
- reptating, *see* hopping
- response functions, 246, 250–252, 264, 268
- response theory, 177
- 'reverse Brazil nut' effect, 50
- reversibility
 - density curve of shaken grains, 211, 215, 230
 - irreversibility for sandpiles, 223
- reversible branch, compaction curve, 90–92, 97–100
- Reynolds dilatancy, *see* dilatancy
- rheology, 263
- rice pile experiments, 127
- ripple formation, 168–174
- rotating cylinders
 - avalanches in, 132–146, 149–150
 - Case A model, 151–156
 - Case B model, 156–162
 - Case C model, 162–167
- roughness
 - anomalous roughening, 74–75, 165–168
 - asymptotic roughness, 119–129
 - entropic landscape, 108–113
 - ripple formation and, 171, 174
 - sand in rotating cylinders, 149–150
 - sandpile collapse and, 68–69, 74
 - scaling relations for interfacial roughness, 150–151
 - silos walls, 247
 - Type II avalanches and, 118–119, 126–131
- roughness exponents, 150, 163, 165, 167
- spatial roughening, 128–130, 158
- surface roughening, 17
- temporal roughening, 119
- saltation, 168–169, 171–172, 174
- sand dunes, 167–168
- sand ripples, 168–174
- sandpiles, *see also* shaken sand
 - bridge formation in, 52–54
 - cellular automaton models, 13–15, 70–78, 116–117
 - collapse, 67–69
 - coupled continuum equations, 148–175
 - dip problem, 248, 271
 - disorder in, 79–92, 122–124
 - force transmission in, 216
 - in rotating cylinders, 132–135
 - random deposition, 141–147
 - realistic models, 17
 - shape of critical, 136, 147
 - spatial roughening exponent, 128–130
 - stresses in, 226, 248–250
 - surface dynamics, 148–175
 - theoretical studies of, 15–17
 - tilting effects on, 71–76
- saturation
 - sandpile surfaces, 129, 154–156
 - saturated interfaces, 151
 - saturation mass in silos, 247–248, 271
 - sheared granular systems, 180
- scale invariance
 - absence of disorder and, 119, 121, 127
 - Kadanoff model, 144–145
 - SOC model and, 12–13, 16–17
 - spatial and temporal, 17
- scale separation, 182–183, 185, 188, 195
- scaling behaviour, *see also* coarse-graining
 - dynamical scaling, 119–120
 - q -model, 245
- scaling relations, interfacial roughness, 150–151
- screening effect, silos, 247–248, 271

- segregation of mixtures, 4
 - cellular automata and, 8–11, 23
 - processes other than shaking, 50–51
 - shaking-induced, 25, 46–51
- self-diffusion, 42–44
- self-organised criticality (SOC), 11–13, 115–116
- sequential packing simulations, 19–20
- shaken cylinders, 210–211
- shaken sand, *see also* vibrated beds
 - contact network topologies, 32
 - entropy of, 210
 - jamming limit effects, 104–114
 - lattice model with long-range interactions, 104–106
 - segregation of mixtures, 25, 46–51
 - simple lattice model, 94–103
 - simulations, 24–26, 29–40
 - transient response, 40–44
- shape-dependent ageing, 99–100, 102–103
- shear modulus, 267
- shear transformation zones, 261
- shear waves, 178–179
- sheared flows, 176–208
 - boundary conditions, 196–200
 - correlation in, 185
 - q -model and, 245
 - response functions, 252
 - spin distribution function, 205
- shock waves, 182
- silo geometry, 233, 246–248, 270–272
- simulations, *see also* Monte Carlo simulations
 - amplitude cycling, 90–92
 - contact orientation in layers, 240–242
 - molecular dynamics simulations, 22–24
 - random packing, 19–22
 - shaken sand, 24–26
 - size segregation and, 9
 - vibrated powders, 7, 27–29, 40–42
- single Fourier transforms
 - coupled continuum equations, 152
 - mean-field equations, 166
 - temporal and spatial roughness, 149–150, 158–159, 161–162
 - tilt combined with flowing grains, 163–164
- single-particle distribution function, 185–186
- single-particle relaxation threshold (SPRT), 84–86, 97
- size segregation, *see* segregation of mixtures
- slow degrees of freedom, 40–42, 63–64
- slow dynamics
 - as cascade process, 97
 - of granular clusters, 86–87
 - simple CA model, 94–96
 - three-spin model requirement, 81–82
- slow relaxation modes, 63–64
- smoothing
 - asymptotic smoothing, 149, 162–163, 165, 167
 - fixed points, 143–154, 156
- SOC (self-organised criticality), 11–13, 115–116
- soft-particle simulations, 22, 24
- soil mechanics, 210, 262
- solidification fronts, 16
- solids, granular media compared to, 1, 18
- Sonine polynomials, 191, 204
- spaces and partial voids, 39
- spatial roughening exponent, 128–130, 158
- spatial structure factor, 158–159
- spin distribution function, 205
- spin models on random graphs, 80
- spin variable, 201–202
- spontaneous crystallisation, 44–46
- SPRT (single-particle relaxation threshold), 84–86, 97
- stabilising angle, 37–38
- stability criteria, 25
- static equilibrium conditions, 234, *see also* force and torque balance
- static properties of granular materials, 233–273
- statistical mechanics, 2–4, 211–215
- steady-state shear, 22
- stick–slip motion, 132–147
- strain tensors, 259, 261, 265
- stratification, 51
- stress fields, 222–230
 - coarse-graining and, 226–230
 - elasticity formalism for, 264–268
 - Mohr–Coulomb computation, 268–270
- stress indeterminacy, 235
- stress response problem, 252
- stress–strain relations, 263, 266–268
- stress tensors, 23, 253–256, 259
 - frictional effects and, 203–204
 - isostaticity and, 217, 223
 - kinetic theory formulation, 193–195
 - Love stress tensor, 217
 - Mohr–Coulomb assumption, 268–269
- stresses
 - deriving from contact forces, 253–259
 - exponential form, 232
 - layered granular systems, 251
 - ratio of stress components, 271
 - scale dependency of, 183
 - sheared granular systems, 180
 - static pilings, 246–252
- ‘stuck grains’, 151, 162, 167, 174
- superconductors, 2
- ‘supercooled’ behaviour, 45–46
- supercritical slopes, 117
- supersonic systems, 182
- surface roughness, *see* roughness
- surface tension, 16
- surface width, sandpile automata, 119–120, 126, 130
- sustainability
 - angle of repose and, 68
 - bridges, friction and, 30, 54
- symmetry breaking, 6
- system-spanning avalanches, 128
- tapping, 40, 82–84, 211
- temperatures, *see* effective; granular; low; tensorial and zero-temperature
- temporal roughening exponent, 119, 158
- temporal structure factor, 159
- tensorial temperature, 215
- texture tensors, 241–242, 259–260

- theoretical descriptions, 261–273
- thermal averaging, 2
- thermodynamics of the jammed state, 209–232
- three-spin model, 81–82, 89
- threshold driving forces, 134, 137–138
- threshold instability, 115
- tilting
 - avalanche morphology and, 72–77
 - combined with flowing grains, 162–167
 - effects on sandpile stability, 70–78
 - ripples, 170
 - rotated cylinder model, 135–136, 162–167
- torque, *see* force and torque balance
- tracer particles, 9–10, 47–49
- transient response, vibrated beds, 40–44
- transitions, glassy and fluidised regimes, 99
- transmission coefficients, q -model, 243
- triangular avalanches, 72–77
- triaxial tests, 262, 264
- TRUBAL software, 23
- two-peak behaviour, 121, 139–141
- two-species model, ripples and dunes, 174
- two-time correlation function, 102–103, 163
- Type I avalanches, 115–117
- Type II avalanches, 118–131
- unilaterality, contact forces, 234–236
- uphill avalanches, 72–77, 121, 124
- upward stabilisations, 38
- velocity field and spin, 202
- vibrated beds, *see also* shaken sand; tapping
 - as non-hydrodynamic, 207
 - attainable packing fractions, 44–6
 - convective instabilities in, 5–8
 - transient response, 40–44
- viscosity, *see also* Bagnold number
 - effective viscosity, 22–23
- void loops, 226
- voids
 - contact networks, 218
 - excess void space, 106, 146
 - lattice model solutions and, 95
 - nonsequential dynamics and, 31
 - partial, 39
 - propagation, 5
- volume fractions, 20, *see also* packing fractions
 - coordination number and, 33
 - forces in granular systems and, 215–222
 - vibration intensity and, 29–30, 40–41
- volume functions, 213, 215, 217–221, 228
- vortices, 51
- wave speed, 173–174
- wavelength, ripple merger, 173
- wedges, flow through, 4–5, 23
- weighted particle heights, 9, 47–48
- wet sand, 69
- white noise, 59, 64, 157, 162
- yield criteria, 233, 268–272
- Young modulus, 261–263, 265, 267
- zero-temperature dynamics, 106–108