

Cambridge University Press

978-0-521-65817-1 - Elliptic Curves: Function Theory, Geometry, Arithmetic

Henry McKean and Victor Moll

Table of Contents

[More information](#)

# Contents

<i>Preface</i>	<i>page xi</i>
<b>1. First Ideas: Complex Manifolds, Riemann Surfaces, and Projective Curves</b>	<b>1</b>
1.1 The Riemann Sphere	1
1.2 Complex Manifolds	3
1.3 Rational Functions	7
1.4 Luroth's Theorem	8
1.5 Automorphisms of $\mathbb{P}^1$	12
1.6 Spherical Geometry	14
1.7 Finite Subgroups and the Platonic Solids	16
1.8 Automorphisms of the Half-Plane	24
1.9 Hyperbolic Geometry	25
1.10 Projective Curves	27
1.11 Covering Surfaces	30
1.12 Scissors and Paste	33
1.13 Algebraic Functions	41
1.14 Examples	46
1.15 More on Uniformization	51
1.16 Compact Manifolds as Curves: Finale	52
<b>2. Elliptic Integrals and Functions</b>	<b>54</b>
2.1 Elliptic Integrals: Where They Come From	55
2.2 The Incomplete Integrals Reduced to Normal Form	62
2.3 The Complete Integrals: Landen, Gauss, and the Arithmetic–Geometric Mean	65
2.4 The Complete Elliptic Integrals: Legendre's Relation	68

Cambridge University Press

978-0-521-65817-1 - Elliptic Curves: Function Theory, Geometry, Arithmetic

Henry McKean and Victor Moll

Table of Contents

[More information](#)

viii	<i>Contents</i>	
2.5	The Discovery of Gauss and Abel	71
2.6	Periods in General	77
2.7	Elliptic Functions in General	81
2.8	The $\wp$ -Function	84
2.9	Elliptic Integrals, Complete and Incomplete	87
2.10	Two Mechanical Applications	89
2.11	The Projective Cubic	92
2.12	The Problem of Inversion	93
2.13	The Function Field	95
2.14	Addition on the Cubic	98
2.15	Abel's Theorem	104
2.16	Jacobian Functions: Reprise	109
2.17	Covering Tori	113
2.18	Finale: Higher Genus	118
<b>3.</b>	<b>Theta Functions</b>	<b>125</b>
3.1	Jacobi's Theta Functions	125
3.2	Some Identities	127
3.3	The Jacobi and Weierstrass Connections	131
3.4	Projective Embedding of Tori	133
3.5	Products	135
3.6	Sums of Two Squares	140
3.7	Sums of Four Squares	142
3.8	Euler's Identities: <i>Partitio Numerorum</i>	143
3.9	Jacobi's and Higher Substitutions	147
3.10	Quadratic Reciprocity	150
3.11	Ramanujan's Continued Fractions	154
<b>4.</b>	<b>Modular Groups and Modular Functions</b>	<b>159</b>
4.1	The Modular Group of First Level	159
4.2	The Modular Group of Second Level	160
4.3	Fundamental Cells	162
4.4	Generating the Groups	166
4.5	Gauss on Quadratic Forms	167
4.6	The Group of Anharmonic Ratios	169
4.7	Modular Forms	172
4.8	Eisenstein Sums	176
4.9	Absolute Invariants	177
4.10	Triangle Functions	183

Cambridge University Press

978-0-521-65817-1 - Elliptic Curves: Function Theory, Geometry, Arithmetic

Henry McKean and Victor Moll

Table of Contents

[More information](#)

<i>Contents</i>	ix
4.11 The Modular Equation of Level 2	185
4.12 Landen's Transformation	187
4.13 Modular Equations of Higher Level	189
4.14 Jacobi's Modular Equation	192
4.15 Jacobi and Legendre's Derivation: Level 5	198
4.16 Arithmetic Subgroups: Overview	200
<b>5. <i>Ikosaeder</i> and the Quintic</b>	<b>206</b>
5.1 Solvability of Equations of Degree $\leq 4$	206
5.2 Galois Groups Revisited	207
5.3 The Galois Group of Level 5	209
5.4 An Element of Degree 5	212
5.5 Hermite on the Depressed Equation	214
5.6 Hermite on the Quintic	216
5.7 A Geometric View	217
<b>6. Imaginary Quadratic Number Fields</b>	<b>224</b>
6.1 Algebraic Numbers	225
6.2 Primes and Ideal Numbers	227
6.3 Class Invariants and Kronecker's <i>Jugendtraum</i>	235
6.4 Application of the Modular Equation	237
6.5 The Class Polynomial	239
6.6 Class Invariants at a Prime Level	243
6.7 Irreducibility of the Class Polynomial	248
6.8 Class Field and Galois Group	249
6.9 Computation of the Class Invariants	250
<b>7. Arithmetic of Elliptic Curves</b>	<b>252</b>
7.1 Arithmetic of the Projective Line	252
7.2 Cubics: The Mordell–Weil Theorem	253
7.3 Examples	255
7.4 Proof of the Mordell–Weil Theorem	259
<i>References</i>	265
<i>Index</i>	278