
INDEX OF NOTATION

- $B(X)$, closed unit ball, 22
 $B(x_0, r)$, closed ball of radius r and centre x_0 , 22
 $B_r(x_0)$, closed ball of radius r and centre x_0 , 22
 $\mathfrak{B}(X)$, space of bounded linear operators on X , 28
 $\mathfrak{B}(X, Y)$, space of bounded linear operators, 28
 $\mathfrak{B}_0(X, Y)$, the space of compact operators, 186
 $\mathfrak{B}_{00}(X, Y)$, the space of finite rank operators, 186
 $\mathfrak{B}_2(X)$, the space of Hilbert–Schmidt operators, 187
 c_σ , the barycentre of the simplex σ , 215
 $\overline{\text{co}} S$, the closed convex hull of S , 55
 $\text{co} S$, the convex hull of S , 55
 C^* -algebra, 167
 $C(K)$, space of continuous functions on a compact Hausdorff space K , 23
 $C(L)$, space of bounded continuous functions on L , 23
 $C_c(L)$, space of continuous functions with compact support, 91
 $C_c^{\mathbb{R}}(L)$, space of continuous real-valued functions with compact support, 93
 $C_0(L)$, space of continuous functions vanishing at infinity, 91
 $C^{\mathbb{R}}(L)$, space of bounded continuous real-valued functions, 93
 $C_0^{\mathbb{R}}(L)$, space of continuous real-valued functions vanishing at infinity, 93
 $d(X, Y)$, Banach–Mazur distance between X and Y , 66
 $D(x, r)$, open ball, 20
 $D(x_0, r)$, open ball of radius r and centre x_0 , 22
 $D_r(x_0)$, open ball of radius r and centre x_0 , 22
 $\delta_A(a)$, the resolvent set of a in the algebra A , 167
 Δ , closed unit disc in the complex plane, 96
 $f \vee g$, the join of f and g , 93
 $f \wedge g$, the meet of f and g , 93
 $f|S$, restriction of a f to S , 25
 $\mathcal{F}_b(S)$, space of bounded functions on S , 23
 $\text{Im } T$, image of T , 28
 k -simplex, 213
 K^a , annihilator of K , 164
 K^0 , polar of K , 158

- l_1 -norm, 23
 l_p -norm, 23
 $\text{lin } S$, linear span, 38
 $\overline{\text{lin } S}$, closed linear span, 38
 $\text{lin } Z$, linear span, 21
 L_∞ , one-point compactification of L , 96
 aL , preannihilator of L , 164
 0L , prepolar of L , 158
 $\mathcal{L}(X, Y)$, space of linear operators, 28

 mesh K , the mesh of the simplicial complex K , 216

 P_F , orthogonal projection onto F , 136
 φ -mean, 5

 $r_n(t)$, Rademacher function, 143
 $r(T)$, the spectral radius of T , 174
 $\text{Rad } B$, the radical of the Banach algebra B , 178
 $\rho(T)$, the resolvent set of the operator T , 168

 $\text{sd } K$, the barycentric subdivision of K , 215
 S^\perp , set of vectors orthogonal to S , 135
 $S(X)$, unit sphere, 22
 $S(x_0, r)$, sphere of radius r and centre x_0 , 22
 $S_r(x_0)$, sphere of radius r and centre x_0 , 22
 $\sigma^{(<\omega)}$, set of finite subsets of σ , 114
 $\sigma(T)$, the spectrum of the operator T , 167

 $\sigma(X, \mathcal{F})$, weak topology generated by \mathcal{F} , 115
 $\sigma(X, X^*)$, weak topology on a normed space, 115
 $\sigma(X^*, X)$, weak-star topology, 116
 $\sigma_A(a)$, the spectrum of a in the algebra A , 167
 $\sigma_{\text{ap}}(T)$, the approximate point spectrum of the operator T , 169
 $\sigma_c(T)$, the continuous spectrum of the operator T , 169
 $\sigma_{\text{com}}(T)$, the compression spectrum of the operator T , 168
 $\sigma_p(T)$, the point spectrum of the operator T , 168
 $\sigma_r(T)$, the residual spectrum of the operator T , 169

 T -invariant, 226
 T^* , adjoint of T , 31
 $\|T\|_2$, Hilbert–Schmidt norm, 187
 $\|T\|_{\text{HS}}$, Hilbert–Schmidt norm, 187

 $v(T)$, the numerical radius of T , 202
 $V(T)$, the numerical range of T , 201

 x^\perp , set of vectors orthogonal to x , 135
 $\|x\|_p$, l^p -norm of x , 23
 \tilde{X} , completion of X , 35
 X' , space of linear functionals on X , 28, 45
 X^* , dual of X , 31
 X^* , space of bounded linear functionals on X , 28, 45

INDEX OF TERMS

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