Linear Analysis

LINEAR ANALYSIS

An Introductory Course

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To Márk

Qui cupit, capit omnia

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PREFACE

This book has grown out of the Linear Analysis course given in Cambridge on numerous occasions for the third-year undergraduates reading mathematics. It is intended to be a fairly concise, yet readable and down-to-earth, introduction to functional analysis, with plenty of challenging exercises. In common with many authors, I have tried to write the kind of book that I would have liked to have learned from as an undergraduate. I am convinced that functional analysis is a particularly beautiful and elegant area of mathematics, and I have tried to convey my enthusiasm to the reader.

In most universities, the courses covering the contents of this book are given under the heading of Functional Analysis; the name Linear Analysis has been chosen to emphasize that most of the material in on *linear* functional analysis. Functional Analysis, in its wide sense, includes partial differential equations, stochastic theory and non-commutative harmonic analysis, but its core is the study of normed spaces, together with linear functionals and operators on them. That core is the principal topic of this volume.

Functional analysis was born around the turn of the century, and within a few years, after an amazing burst of development, it was a well-established major branch of mathematics. The early growth of functional analysis was based on 19th century Italian function theory, and was given a great impetus by the birth of Lebesgue's theory of integration. The subject provided (and provides) a unifying framework for many areas: Fourier Analysis, Differential Equations, Integral Equations, Approximation Theory, Complex Function Theory, Analytic Number Theory, Measure Theory, Stochastic Theory, and so on.

From the very beginning, functional analysis was an international subject, with the major contributions coming from Germany, Hungary, Poland, England and Russia: Fisher, Hahn, Hilbert, Minkowski and Radon from Germany, Fejér, Haar, von Neumann, Frigyes Riesz and Marcel Riesz from Hungary, Banach, Mazur, Orlicz, Schauder, Sierpiński and Steinhaus from Poland, Hardy and Littlewood from England, Gelfand, Krein and Milman from Russia. The abstract theory of normed spaces was developed in the 1920s by Banach and others, and was presented as a fully fledged theory in Banach's epoch-making monograph, published in 1932.

The subject of Banach's classic is at the heart of our course; this material is supplemented with a body of other fundamental results and some pointers to more recent developments.

The theory presented in this book is best considered as the natural continuation of a sound basic course in general topology. The reader would benefit from familiarity with measure theory, but he will not be at a great disadvantage if his knowledge of measure theory is somewhat shaky or even non-existent. However, in order to fully appreciate the power of the results, and, even more, the power of the point of view, it is advisable to look at the connections with integration theory, differential equations, harmonic analysis, approximation theory, and so on.

Our aim is to give a fast introduction to the core of linear analysis, with emphasis on the many beautiful general results concerning *abstract* spaces. An important feature of the book is the large collection of exercises, many of which are testing, and some of which are quite difficult. An exercise which is marked with a plus is thought to be particularly difficult. (Needless to say, the reader may not always agree with this value judgement.) Anyone willing to attempt a fair number of the exercises should obtain a thorough grounding in linear analysis.

To help the reader, definitions are occasionally repeated, various basic facts are recalled, and there are reminders of the notation in several places.

The third-year course in Cambridge contains well over half of the contents of this book, but a lecturer wishing to go at a leisurely pace will find enough material for two terms (or semesters). The exercises should certainly provide enough work for two busy terms.

There are many people who deserve my thanks in connection with this book. Undergraduates over the years helped to shape the course; numerous misprints were found by many undergraduates, including John Longley, Gábor Megyesi, Anthony Quas, Alex Scott and Alan Stacey.

I am grateful to Dr Pete Casazza for his comments on the completed manuscript. Finally, I am greatly indebted to Dr Imre Leader for having suggested many improvements to the presentation.

Cambridge, May 1990 Béla Bollobás

For this second edition, I have taken the opportunity to correct a number of errors and oversights. I am especially grateful to R. B. Burckel for providing me with a list of errata.

B. B.