

Cambridge University Press

052165534X - Introductory Statistics with Applications in General Insurance

I. B. Hossack, J. H. Pollard and B. Zehnwirth

Frontmatter

[More information](#)

**Introductory statistics
with applications in
general insurance**

Cambridge University Press

052165534X - Introductory Statistics with Applications in General Insurance

I. B. Hossack, J. H. Pollard and B. Zehnwirth

Frontmatter

[More information](#)

Introductory statistics with applications in general insurance

I. B. HOSSACK

Formerly Senior Lecturer in Statistics, Macquarie University

J. H. POLLARD

Professor of Actuarial Studies, Macquarie University

B. ZEHNWIRTH

Managing Director, Insurance Pty Ltd



CAMBRIDGE
UNIVERSITY PRESS

Cambridge University Press
052165534X - Introductory Statistics with Applications in General Insurance
I. B. Hossack, J. H. Pollard and B. Zehnwirth
Frontmatter
[More information](#)

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge CB2 2RU, UK
40 West 20th Street, New York, NY 10011-4211, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
Ruiz de Alarcón 13, 28014 Madrid, Spain
Dock House, The Waterfront, Cape Town 8001, South Africa
<http://www.cambridge.org>

© Cambridge University Press 1983, 1999

This book is in copyright. Subject to statutory exception
and to the provisions of relevant collective licensing agreements,
no reproduction of any part may take place without
the written permission of Cambridge University Press.

First published 1983
Second edition 1999
Reprinted 2003

Typeset in Times 10/13pt [MU]

A catalogue record for this book is available from the British Library

ISBN 0 521 65234 0 hardback
ISBN 0 521 65534 X paperback

Transferred to digital printing 2004

Cambridge University Press

052165534X - Introductory Statistics with Applications in General Insurance

I. B. Hossack, J. H. Pollard and B. Zehnwirth

Frontmatter

[More information](#)

CONTENTS

	Preface to first edition	ix
	Preface to second edition	xi
1	Introduction and mathematical preliminaries	1
1.1	Introduction	1
1.2	Summation notation	2
1.3	Factorial notation $n!$	2
1.4	Combinatorial notation $\binom{n}{r}$	3
1.5	Power notation	5
1.6	Differentiation; the slope of a curve	6
1.7	Maxima and minima	10
*1.8	Functions of more than one variable; maxima and minima	12
1.9	The exponential function e^x	14
1.10	The natural logarithm function $\ln x$	18
1.11	Exercises	23
2	Elementary probability	25
2.1	Introduction; concept of probability	25
2.2	Joint and disjoint events; intersection and union	26
2.3	Conditional probability	30
2.4	Independence of two events	31
2.5	Exercises	32
3	Random variables and their distributions	34
3.1	Discrete random variables and their distributions	34
3.2	Continuous random variables and their distributions	38
3.3	The area under a curve; integration and differentiation	42
3.4	Exercises	50
4	Location and dispersion	52
4.1	Measures of location – mean, median and mode	52
4.2	Dispersion – variance and standard deviation	57
**4.3	Expectations and moments	61
**4.4	Conditional means	63
**4.5	Conditional variances	64
**4.6	Skewness	65
4.7	Exercises	66
5	Statistical distributions useful in general insurance work	68
5.1	The normal distribution	68
5.2	The Central Limit Theorem	77

Cambridge University Press

052165534X - Introductory Statistics with Applications in General Insurance

I. B. Hossack, J. H. Pollard and B. Zehnwirth

Frontmatter

[More information](#)

<i>Contents</i>		<i>vi</i>
5.3	The log-normal distribution	81
*5.4	The Pareto distribution	84
*5.5	The gamma distribution	86
5.6	The Poisson distribution	87
5.7	Normal approximation to the Poisson distribution	91
*5.8	The binomial distribution	93
**5.9	The negative binomial distribution; heterogeneity of risk	96
5.10	The importance of theoretical distributions in general insurance	101
5.11	Exercises	102
6	Inferences from general insurance data	103
6.1	Hypothesis testing	103
6.2	Point estimation and method of moments	107
*6.3	Maximum likelihood	108
*6.4	Confidence intervals	110
*6.5	Risk factors; multivariate models; least squares	112
6.6	Exercises	120
7	The risk premium	122
7.1	Risk premium; claim frequency and claim size	122
7.2	Claim frequency rate; exposure	123
7.3	Claim size; pitfalls	128
7.4	Claim settlement pattern	137
*7.5	Excesses and excess of loss reinsurances	145
7.6	Exercises	150
8	Experience rating	151
8.1	Introduction	151
8.2	Credibility theory	156
8.3	Full credibility	157
8.4	Partial credibility	160
*8.5	Bayes' Theorem	161
**8.6	A Bayesian approach to the updating of claim frequency rates	162
8.7	No claim discount (NCD)	166
8.8	Exercises	175
9	Simulation	177
9.1	Random numbers and simulation	177
9.2	How many simulations?	182
9.3	Computer generation of random numbers	184
9.4	Linear congruential generators	184
9.5	Random observations on the normal distribution	186
9.6	Random observations on the log-normal distribution	188
9.7	Random observations on the Poisson distribution	189
*9.8	Random observations on the negative binomial distribution	192
9.9	A simulation example	193
9.10	When to simulate	195
9.11	Simulation of an NCD system	198
9.12	Limitations of the model; sensitivity analysis	203
9.13	Exercises	204

Cambridge University Press

052165534X - Introductory Statistics with Applications in General Insurance

I. B. Hossack, J. H. Pollard and B. Zehnwirth

Frontmatter

[More information](#)

<i>Contents</i>		<i>vii</i>
10	Estimation of outstanding claim provisions	206
10.1	Delays in claim reporting and claim settlement; run-off	206
10.2	The run-off triangle	207
10.3	Chain-ladder method without inflation adjustment	208
10.4	Does the chain-ladder model fit the data?	211
10.5	Chain-ladder method with inflation adjustment	213
10.6	The separation method (direct future payments approach)	223
*10.7	The separation method (two other approaches)	231
10.8	IBNR, and the chain-ladder and separation methods	233
10.9	Alternative methods of assessing outstanding claim provisions	234
10.10	The tail	234
10.11	Estimation of IBNR claim provisions	237
10.12	Exercises	241
11	Elementary risk theory	243
11.1	Introduction	243
11.2	Portfolio with constant (fixed) claim size	245
11.3	Variable claim size	248
**11.4	The expectation and variance of C	255
11.5	The assumption of normality	255
11.6	Summary and further reading	257
11.7	Exercises	259
	References	260
	Solutions to exercises	262
	Author index	275
	Subject index	277

Cambridge University Press

052165534X - Introductory Statistics with Applications in General Insurance

I. B. Hossack, J. H. Pollard and B. Zehnwirth

Frontmatter

[More information](#)

PREFACE TO FIRST EDITION

The success of the general insurance industry over several hundred years is a tribute to the judgement and skill of generations of underwriters who were able to assess diverse risks and underwrite them without extensive statistical data. The business was profitable because the premiums were more than adequate and included substantial margins for contingencies.

The twentieth century brought risks of previously unimagined magnitude, and saw the development of fierce competition between insurers in markets which were better informed, severe rate-cutting, consumerism, government control of premium rates in certain classes of business, and other factors, all of which combined to make the underwriting of insurance in modern conditions extremely difficult. Bouts of high inflation have led to claim settlements considerably higher than those allowed for in the premiums. The rapid development of new technologies has meant that new risks, for which there is insufficient experience upon which to base rates, now comprise a larger proportion of all risks. Clients have become accustomed to change, and now expect to change their insurers whenever they can see an advantage in doing so. It is no longer possible, therefore, for an insurer to charge a premium which is obviously more than adequate, and remain in business.

The twentieth century has also seen the growth of statistical theory and practice from infancy to full maturity, and the development of sophisticated computers with huge storage capacities. In the modern environment the use of these tools by insurers is essential. It needs to be emphasised, however, that in assessing premium rates, contingency margins, retention limits, provisions for outstanding claims, provisions for incurred but not reported claims, etc., *statistical methods and computer technology are tools to aid the professional; they are not substitutes for professional judgement.*

The aim of this book (which evolved from a set of lecture notes prepared for a Macquarie University Continuing Education Course with the same title) is to provide practitioners in the general insurance industry with basic statistical tools, which are of immediate application in the industry, and to demonstrate that the methods are both practical and useful.

Cambridge University Press

052165534X - Introductory Statistics with Applications in General Insurance

I. B. Hossack, J. H. Pollard and B. Zehnwirth

Frontmatter

[More information](#)*Preface*

x

While our primary objective is to introduce readers with little or no statistical background to a variety of statistical methods with applications in general insurance (and we hope that they will be stimulated to pursue some of the topics in greater depth through the references), we also include a limited number of more difficult topics for the more advanced reader. *Sections and examples marked with single asterisks may be omitted at a first reading* without impairing the reader's understanding of subsequent sections and chapters. *More difficult topics for the more advanced reader are indicated by double asterisks.* Exercises are given at the end of each chapter and, in solving these problems, the reader should learn a great deal more about the methods involved. Answers to these exercises are given on pages 263–274.

We would like to record our indebtedness to colleagues in the general insurance industry and at the University for their helpful comments and discussions. In particular, we would like to thank Robert Buchanan, who read the draft manuscript critically, drew certain errors and ambiguities to our attention, and offered much useful advice. We also thank Miss Betty Thorn, who drew the diagrams.

Macquarie University,
Sydney, Australia.
December 1982

I.B.H.
J.H.P.
B.Z.

Cambridge University Press

052165534X - Introductory Statistics with Applications in General Insurance

I. B. Hossack, J. H. Pollard and B. Zehnwirth

Frontmatter

[More information](#)

PREFACE TO SECOND EDITION

Sixteen years have elapsed since the first edition of this book appeared, and over that time considerable advances have taken place theoretically and analytically in respect of insurance modelling. The arrival of the personal computer, the availability of huge amounts of relatively inexpensive computer memory and the vast increase in processor speed mean that more and more complex models are now readily implemented in the quiet of a user's own office. The fundamentals of insurance, however, remain unchanged, and it is with these this introductory book is primarily concerned. For this reason, whilst there are numerous changes in this new edition, none could be described as major.

It is with great sadness that we record the death of our friend and colleague Ian Hossack, who was involved in preliminary discussions about the changes required for this second edition. We hope that the changes we have made would accord with his wishes.

Several friends and colleagues helped in the necessary revision, and we would particularly like to thank Julian Leslie and Bon Clarke of Macquarie University, who made suggestions about the updating of many of the references.

December 1998

J.H.P.

B.Z.