Cambridge University Press
978-0-521-65375-6 - Introduction to the Analysis of Normed Linear Spaces
J. R. Giles
Frontmatter
More information

Introduction to the Analysis of Normed Linear Spaces

AUSTRALIAN MATHEMATICAL SOCIETY LECTURE SERIES

1 Introduction to Linear and Convex Programming, N. CAMERON
2 Manifolds and Mechanics, A. JONES, A GRAY & R. HUTTON
3 Introduction to the Analysis of Metric Spaces, J. R. GILES
4 An Introduction to Mathematical Physiology and Biology, J. MAZUMDAR
5 2-Knots and their Groups, J. HILLMAN
6 The Mathematics of Projectiles in Sport, N. DE MESTRE
7 The Petersen Graph, D. A. HOLTON & J. SHEEHAN
8 Low Rank Representations and Graphs for Sporadic Groups, C. PRAEGER & L. SOICHER
9 Algebraic Groups and Lie Groups, G. LEHRER (ed)
10 Modelling with Differential and Difference Equations, G. FULFORD, P. FORRESTER & A. JONES
11 Geometric Analysis and Lie Theory in Mathematics and Physics, A. L. CAREY & M. K. MURRAY (eds)
12 Foundations of Convex Geometry, W. A. COPPEL
13 Introduction to the Analysis of Normed Linear Spaces, J. R. GILES
14 The Integral: An Easy Approach after Kurzweil and Henstock, L. P. YEE & R. VYBORNY

Australian Mathematical Society Lecture Series. 13

Introduction to the Analysis of Normed Linear Spaces

J. R. Giles
Department of Mathemtics
University of Newcastle, Australia

CAMBRIDGE
UNIVERSITY PRESS

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge, CB2 2RU, UK www.cup.cam.ac.uk
40 West 20th Street, New York, NY 10011-4211, USA www.cup.org
10 Stamford Road, Oakleigh, Melbourne 3166, Australia
Ruiz de Alarcón 13, 28014 Madrid, Spain

First published 2000

Printed in the United Kingdom at the University Press, Cambridge

A catalogue record for this book is available from the British Library

ISBN 0 521 65375 4 paperback

for Zeny

wife, mother and
γιαγια

CONTENTS

Cambridge University Press
978-0-521-65375-6 - Introduction to the Analysis of Normed Linear Spaces
J. R. Giles
Frontmatter
More information

PREFACE

This text is designed as a basic course in functional analysis for senior undergraduate or beginning postgraduate students. For students completing their final undergraduate year, it is aimed at providing some insight into basic abstract analysis which more than ever, is the contextual language of much modern mathematics. For postgraduate students it is aimed at providing a foundation and stimulus for their further research development.

It is assumed that the student will be familiar with real analysis and have some background in linear algebra and complex analysis. It is also assumed that the student will have studied a course in the analysis of metric spaces such as that given in the author's text

Introduction to the Analysis of Metric Spaces, Cambridge University Press, 1987.

Reference to this text will be made under the abbreviation AMS § __ .

In AMS, most of the example spaces introduced are normed linear spaces and many of the implications of linear structure were explored. For example when closure in metric spaces was discussed it was natural to consider the closure of linear subspaces in normed linear spaces and when continuity was considered it was logical to study the continuity of linear mappings on normed linear spaces. In order to make this text as self-contained as possible, the example spaces are again introduced and the elementary properties of normed linear spaces are treated but in a more sophisticated way. For example, in AMS the properties of finite dimensional normed linear spaces were deduced in a contrived manner from compactness properties of the real numbers, but here the properties are established in a more expeditious manner using topological isomorphism with Euclidean space. Nevertheless, this text may be regarded as a sequel to AMS especially in that we assume all those properties generally associated with the metric topology of the space.

In AMS we were concerned with the particular topological structure of normed linear spaces as metric spaces. The cohesive theme in this text is with the structural properties of normed linear spaces in general, associated especially with dual spaces and the algebra of continuous linear operators on normed linear spaces. The course given here depends fundamentally on the Hahn–Banach Theorem which assumes the Axiom of Choice.

The author follows the approach that a great deal of the analysis of normed linear spaces can be handled with a knowledge of metric space topology and is usefully done so before the student has studied general topology. In fact the wide applicability of the analysis of normed linear spaces would argue for this approach. He would suggest that from an analysis point of view the most useful application of the analysis of general topological spaces is in the analysis of linear topological spaces which, for a proper understanding, requires a knowledge of normed linear space theory.

However, to follow this approach means that some topics have to be omitted, in particular any discussion of weak topologies, which includes the Banach–Alaoglu Theorem and compactness characterisations of reflexivity. But such material is naturally included in a subsequent course on the analysis of linear topological spaces.

Spaces involving the Lebesgue integral are mentioned in the text but the student needs no background knowledge of integration theory for this course and references to the Lebesgue integral can be glossed over without loss of understanding.

A glance at the table of contents shows that the text consists mostly of well established material. However, at this stage of his mathematical education the student needs occasionally to be made aware of the great problem areas in the subject and to glimpse the frontiers where recent advances have been made. It is important that the serious student feel that he is studying a subject which is still on the move and one where he may be able to follow where the current ground is being broken.

So at several points the student is introduced to material which is more recent and is given a guide to literature where major problems have been tackled. For example, in Section 8 the Bishop–Phelps Theorem is proved; this is a result which is constantly used by researchers in the analysis of Banach spaces. When discussing Schauder bases in Section 7 it is of interest to mention the Basis Problem and its solution by Enflo. When treating the representation of compact operators by finite rank operators in Section 15 it is useful to refer to Enflo's contribution. The Invariant Subspace Problem arises naturally in Spectral Theory and it is convenient to give a proof of Lomonosov's Theorem for compact operators in Section 18.

A lecture course using this text can be given in between 25 and 30 lecture hours given an adequate extra tutorial program and provided the students have sufficient background. The material can be tailored to suit such a course length by omitting Section 8 completely and Chapter VI on Spectral Theory.

At the end of each section there is a set of exercises which follows the order of presentation of material in the section.

At the end of the text there is an appendix which includes the set theory results used at various points of the course.

Historical notes are included to show how the subject developed from late nineteenth century beginnings into the analysis of an abstract structure using the axiomatic method. Two recent advances are included to demonstrate the continued vigour of research in the area.

As with AMS, this text has a full index. Rarely are students told that when a mathematician uses a textbook he regularly consults its index. So the index in this text gives references to all the significant places where a particular concept is used.

There are many texts in functional analysis which cover the material presented in this text book in a variety of different ways.

The great standard text for many years was

Angus E. Taylor, *Introduction to functional analysis*, John Wiley, 1958

which was updated by Angus E. Taylor and David C. Lay in 1980. This is a great reference but is generally pitched at more advanced students than our text.

A text more accessible to students of our course is

Erwin Kreysig, *Introductory functional analysis with applications*, John Wiley, 1978.

This text is very popular with engineers and does not rely to any great extent on previous knowledge of general topology.

The text which had a formative influence on the author is

A.L. Brown and A. Page, *Elements of functional analysis*, van Nostrand Reinhold, 1970.

This text is pitched at the same level as ours and does not have a general topology or general integration prerequisite. Our text often gives more detail and introduces recent results.

An expansive text is

George Bachman and Lawrence Narici, *Functional Analysis*, Academic Press, 1966.

But this text covers spectral theory with greater detail and depth.

The text

George F. Simmons, *Introduction to topology and modern analysis*, McGraw–Hill, 1963

is very readable, but it is written with more of a general topology background.

An excellent text, but one also written from a topological point of view is

G.J.O. Jameson, *Topology and normed linear spaces*, Chapman–Hall, 1974.

There are more modern texts such as

Walter Rudin, *Functional Analysis*, McGraw–Hill, 2nd edn, 1991.

This presents a far more sophisticated course than ours, but will be found to be a useful reference.

There are two modern British texts which should be mentioned.

Nicholas Young, *An introduction to Hilbert space*, Cambridge University Press, 1988

concentrates on Hilbert space with applications in mind.

Béla Bollobás, *Linear Analysis*, Cambridge University Press, 1990.

This contains a very fine lecture course and does refer to new results. It moves somewhat faster than our text and with less detail than many students would find comfortable.

I have been influenced by the text

J.R. Ringrose, *Compact non-self-adjoint operators*, van Nostrand Reinhold, 1971

in my approach to spectral theory for compact operators.

The author has given lectures on a course such as this to third year honours students at the University of Newcastle for a period of more than ten years. The lecture notes were first produced in duplicated form but have been modified and expanded considerably in the light of experience, to the form presented here.

Thanks are due to my colleagues and students in the department for their conversations over many years which have had their effect on the final result. My thanks to Philip Charlton for producing the diagrams which appear in the text. I am particularly indebted to Jan Garnsey who has so patiently and competently typed and retyped the final copy from my handwritten manuscript. I am grateful to Brailey Sims for supplying source material for the historical notes.

J.R. Giles
The University of Newcastle
NSW Australia.