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0521652340 - Introductory Statistics with Applications in General Insurance

I. B. Hossack, J. H. Pollard and B. Zehnwirth

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with applications in
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Introductory statistics with applications in general insurance

I. B. HOSSACK

Formerly Senior Lecturer in Statistics, Macquarie University

J. H. POLLARD

Professor of Actuarial Studies, Macquarie University

B. ZEHNWIRTH

Managing Director, Insurance Pty Ltd



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PREFACE TO FIRST EDITION

The success of the general insurance industry over several hundred years is a tribute to the judgement and skill of generations of underwriters who were able to assess diverse risks and underwrite them without extensive statistical data. The business was profitable because the premiums were more than adequate and included substantial margins for contingencies.

The twentieth century brought risks of previously unimagined magnitude, and saw the development of fierce competition between insurers in markets which were better informed, severe rate-cutting, consumerism, government control of premium rates in certain classes of business, and other factors, all of which combined to make the underwriting of insurance in modern conditions extremely difficult. Bouts of high inflation have led to claim settlements considerably higher than those allowed for in the premiums. The rapid development of new technologies has meant that new risks, for which there is insufficient experience upon which to base rates, now comprise a larger proportion of all risks. Clients have become accustomed to change, and now expect to change their insurers whenever they can see an advantage in doing so. It is no longer possible, therefore, for an insurer to charge a premium which is obviously more than adequate, and remain in business.

The twentieth century has also seen the growth of statistical theory and practice from infancy to full maturity, and the development of sophisticated computers with huge storage capacities. In the modern environment the use of these tools by insurers is essential. It needs to be emphasised, however, that in assessing premium rates, contingency margins, retention limits, provisions for outstanding claims, provisions for incurred but not reported claims, etc., *statistical methods and computer technology are tools to aid the professional; they are not substitutes for professional judgement.*

The aim of this book (which evolved from a set of lecture notes prepared for a Macquarie University Continuing Education Course with the same title) is to provide practitioners in the general insurance industry with basic statistical tools, which are of immediate application in the industry, and to demonstrate that the methods are both practical and useful.

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While our primary objective is to introduce readers with little or no statistical background to a variety of statistical methods with applications in general insurance (and we hope that they will be stimulated to pursue some of the topics in greater depth through the references), we also include a limited number of more difficult topics for the more advanced reader. *Sections and examples marked with single asterisks may be omitted at a first reading* without impairing the reader's understanding of subsequent sections and chapters. *More difficult topics for the more advanced reader are indicated by double asterisks.* Exercises are given at the end of each chapter and, in solving these problems, the reader should learn a great deal more about the methods involved. Answers to these exercises are given on pages 263–274.

We would like to record our indebtedness to colleagues in the general insurance industry and at the University for their helpful comments and discussions. In particular, we would like to thank Robert Buchanan, who read the draft manuscript critically, drew certain errors and ambiguities to our attention, and offered much useful advice. We also thank Miss Betty Thorn, who drew the diagrams.

Macquarie University,
Sydney, Australia.
December 1982

I.B.H.
J.H.P.
B.Z.

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PREFACE TO SECOND EDITION

Sixteen years have elapsed since the first edition of this book appeared, and over that time considerable advances have taken place theoretically and analytically in respect of insurance modelling. The arrival of the personal computer, the availability of huge amounts of relatively inexpensive computer memory and the vast increase in processor speed mean that more and more complex models are now readily implemented in the quiet of a user's own office. The fundamentals of insurance, however, remain unchanged, and it is with these this introductory book is primarily concerned. For this reason, whilst there are numerous changes in this new edition, none could be described as major.

It is with great sadness that we record the death of our friend and colleague Ian Hossack, who was involved in preliminary discussions about the changes required for this second edition. We hope that the changes we have made would accord with his wishes.

Several friends and colleagues helped in the necessary revision, and we would particularly like to thank Julian Leslie and Bon Clarke of Macquarie University, who made suggestions about the updating of many of the references.

December 1998

J.H.P.

B.Z.