

An Introduction to Nuclear Physics

This clear and concise introduction to nuclear physics provides an excellent basis for a 'core' undergraduate course in this area.

The book opens by setting nuclear physics in the context of elementary particle physics and then shows how simple models can provide an understanding of the properties of nuclei, both in their ground states and excited states, and also of the nature of nuclear reactions. The book includes chapters on nuclear fission, its application in nuclear power reactors, and the role of nuclear physics in energy production and nucleosynthesis in stars.

This new edition contains several additional topics: muon-catalysed fusion, the nuclear and neutrino physics of supernovae, neutrino mass and neutrino oscillations, and the biological effects of radiation.

A knowledge of basic quantum mechanics and special relativity is assumed. Appendices deal with other more specialised topics. Each chapter ends with a set of problems for which outline solutions are provided.

NOEL COTTINGHAM and DEREK GREENWOOD are theoreticians working in the H. H. Wills Physics Laboratory at the University of Bristol. Noel Cottingham is also a visiting professor at the Université Pierre et Marie Curie, Paris.

They have also collaborated in an undergraduate text, *Electricity and Magnetism*, and a graduate text, *An Introduction to the Standard Model of Particle Physics*. Both books are published by the Cambridge University Press.



An Introduction to

Nuclear Physics

Second edition

W. N. COTTINGHAM *University of Bristol*

D. A. GREENWOOD University of Bristol





PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS

The Edinburgh Building, Cambridge CB2 2RU, UK 40 West 20th Street, New York, NY 10011-4211, USA 10 Stamford Road, Oakleigh, VIC 3166, Australia Ruiz de Alarcón 13, 28014 Madrid, Spain Dock House, The Waterfront, Cape Town 8001, South Africa

http://www.cambridge.org

© Cambridge University Press 1986, 2001

This book is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 1986 Reprinted 1987 (with corrections and additions), 1988, 1990, 1992, 1998 Second edition 2001

Typeface Times 10/13pt 3B2 [KB]

A catalogue record for this book is available from the British Library

Library of Congress Cataloguing in Publication data

Cottingham, W. N.

An introduction to nuclear physics / W.N. Cottingham and D.A. Greenwood – 2nd ed. p. cm.
Includes bibliographical references and index.
ISBN 0 521 65149 2 (hardback) – ISBN 0 521 65733 4 (paperback)
1. Nuclear physics. I. Greenwood, D. A. II. Title.
QC776.C63 2001

539.7–dc21 00-059885

ISBN 0 521 65149 2 hardback ISBN 0 521 65733 4 paperback

Transferred to digital printing 2002



Contents

	Preface to the second edition	ix
	Preface to the first edition	X
	Constants of nature, conversion factors and notation	xii
	Glossary of some important symbols	xiii
1	Prologue	1
1.1	Fermions and bosons	2
1.2	The particle physicist's picture of nature	2
1.3	Conservation laws and symmetries: parity	3
1.4	Units	4
	Problems	5
2	Leptons and the electromagnetic and weak interactions	7
2.1	The electromagnetic interaction	7
2.2	The weak interaction	9
2.3	Mean life and half life	12
2.4	Leptons	13
2.5	The instability of the heavy leptons: muon decay	15
2.6	Parity violation in muon decay	16
	Problems	17
3	Nucleons and the strong interaction	19
3.1	Properties of the proton and the neutron	19
3.2	The quark model of nucleons	21
3.3	The nucleon–nucleon interaction: the phenomenological	
	description	22
3.4	Mesons and the nucleon-nucleon interaction	26
3.5	The weak interaction: β -decay	28
3.6	More quarks	29
3.7	The Standard Model of particle physics	31
	Problems	31

V



vi Contents

4	Nuclear sizes and nuclear masses	33
4.1	Electron scattering by the nuclear charge distribution	33
4.2	Muon interactions	36
4.3	The distribution of nuclear matter in nuclei	37
4.4	The masses and binding energies of nuclei in their ground	
	states	39
4.5	The semi-empirical mass formula	41
4.6	The β -stability valley	44
4.7	The masses of the β -stable nuclei	48
4.8	The energetics of α -decay and fission	50
4.9	Nuclear binding and the nucleon-nucleon potential	52
	Problems	52
5	Ground-state properties of nuclei: the shell model	56
5.1	Nuclear potential wells	56
5.2	Estimates of nucleon energies	58
5.3	Energy shells and angular momentum	60
5.4	Magic numbers	65
5.5	The magnetic dipole moment of the nucleus	66
5.6	Calculation of the magnetic dipole moment	67
5.7	The electric quadrupole moment of the nucleus	68
	Problems	72
6	Alpha decay and spontaneous fission	74
6.1	Energy release in α -decay	74
6.2	The theory of α -decay	75
6.3	Spontaneous fission	83
	Problems	87
7	Excited states of nuclei	89
7.1	The experimental determination of excited states	89
7.2	Some general features of excited states	93
7.3	The decay of excited states: γ -decay and internal conversion	97
7.4	Partial decay rates and partial widths	99
7.5	Excited states arising from β -decay	100
	Problems	101
8	Nuclear reactions	103
8.1	The Breit-Wigner formula	103
8.2	Neutron reactions at low energies	107
8.3	Coulomb effects in nuclear reactions	109
8.4	Doppler broadening of resonance peaks	111
	Problems	113
9	Power from nuclear fission	115
9.1	Induced fission	115



		Contents	vii
9.2	Neutron cross-sections for ²³⁵ U and ²³⁸ U		116
9.3	The fission process		118
9.4	The chain reaction		119
9.5	Nuclear fission reactors		121
9.6	Reactor control and delayed neutrons		122
9.7	Production and use of plutonium		124
9.8	Radioactive waste		125
9.9	The future of nuclear power		126
	Problems		127
10	Nuclear fusion		130
10.1	The Sun		130
10.2	Cross-sections for hydrogen burning		132
10.3	Nuclear reaction rates in a plasma		135
10.4	Other solar reactions		139
10.5	Solar neutrinos		140
10.6	Fusion reactors		143
10.7	Muon-catalysed fusion		146
	Problems		148
11	Nucleosynthesis in stars		151
11.1	Stellar evolution		151
11.2	From helium to silicon		155
11.3	Silicon burning		156
11.4	Supernovae		157
11.5	Nucleosynthesis of heavy elements		160
	Problems		161
12	Beta decay and gamma decay		163
12.1	What must a theory of β -decay explain?		163
12.2	The Fermi theory of β -decay		166
12.3	Electron and positron energy spectra		168
12.4	Electron capture		171
12.5	The Fermi and Gamow-Teller interactions		173
12.6	The constants $V_{\rm ud}$ and $g_{\rm A}$		177
12.7	Electron polarisation		178
12.8	Theory of γ -decay		179
12.9	Internal conversion		184
	Problems		185
13	Neutrinos		186
13.1	Neutrino cross-sections		186
13.2	The mass of the electron neutrino		188
13.3	Neutrino mixing and neutrino oscillations		189



VIII	Contents	
13.4	Solar neutrinos	193
13.5	Atmospheric neutrinos	195
	Problems	196
14	The passage of energetic particles through matter	199
14.1	Charged particles	199
14.2	Multiple scattering of charged particles	206
14.3	Energetic photons	207
14.4	The relative penetrating power of energetic particles	211
	Problems	212
15	Radiation and life	214
15.1	Ionising radiation and biological damage	214
15.2	Becquerels (and curies)	215
15.3	Grays and sieverts (and rads and rems)	216
15.4	Natural levels of radiation	217
15.5	Man-made sources of radiation	218
15.6	Risk assessment	219
	Problems	220
	Appendix A: Cross-sections	222
A .1	Neutron and photon cross-sections	222
A.2	Differential cross-sections	224
A.3	Reaction rates	225
A.4	Charged particle cross-sections: Rutherford scattering	226
	Appendix B: Density of states	227
	Problems	230
	Appendix C: Angular momentum	230
C.1	Orbital angular momentum	230
C.2	Intrinsic angular momentum	232
C.3	Addition of angular momenta	233
C.4	The deuteron	234
	Problems	235
	Appendix D: Unstable states and resonances	235
D.1	Time development of a quantum system	236
D.2	The formation of excited states in scattering: resonances	
	and the Breit-Wigner formula	241
	Problems	244
	Further reading	245
	Answers to problems	246
	Index	267



Preface to the second edition

The main structure of the first edition has been retained, but we have taken the opportunity in this second edition to update the text and clarify an occasional obscurity. The text has in places been expanded, and also additional topics have been added. The growing interest of physics students in astrophysics has encouraged us to extend our discussions of the nuclear and neutrino physics of supernovae, and of solar neutrinos. There is a new chapter devoted to neutrino masses and neutrino oscillations. In other directions, a description of muon-catalysed fusion has been included, and a chapter on radiation physics introduces an important applied field.

We should like to thank Dr John Andrews and Professor Denis Henshaw for their useful comments on parts of the text, Mrs Victoria Parry for her secretarial assistance, and Cambridge University Press for their continuing support.

W. N. Cottingham D. A. Greenwood *Bristol. March 2000*



Preface to the first edition

In writing this text we were concerned to assert the continuing importance of nuclear physics in an undergraduate physics course. We set the subject in the context of current notions of particle physics. Our treatment of these ideas, in Chapters 1 to 3, is descriptive, but it provides a unifying foundation for the rest of the book. Chapter 12, on β -decay, returns to the basic theory. It also seems to us important that a core course should include some account of the applications of nuclear physics in controlled fission and fusion, and should exemplify the role of nuclear physics in astrophysics. Three chapters are devoted to these subjects.

Experimental techniques are not described in detail. It is impossible in a short text to do justice to the ingenuity of the experimental scientist, from the early discoveries in radioactivity to the sophisticated experiments of today. However, experimental data are stressed throughout: we hope that the interdependence of advances in experiment and theory is apparent to the reader.

We have by and large restricted the discussion of processes involving nuclear excitation and nuclear reactions to energies less than about 10 MeV. Even with this restriction there is such a richness and diversity of phenomena that it can be difficult for a beginner to grasp the underlying principles. We have therefore placed great emphasis on a few simple theoretical models that provide a successful description and understanding of the properties of nuclei at low energies. The way in which simple models can elucidate the properties of a complex system is one of the surprises of the subject, and part of its general educational value.

We have tried to keep the mathematics as simple as possible. We assume a knowledge of the basic formulae of special relativity, and

Χ



Preface to the first edition

Х

some basic quantum mechanics: wave-equations, energy levels and the quantisation of angular momentum. A few topics which may not be covered in elementary courses in quantum mechanics are treated in appendices. We consider the technicalities of angular momentum algebra, phase shift analysis and isotopic spin to be inappropriate to a first course in nuclear physics. Equations are written to be valid in SI units; results are usually expressed in MeV and fm. Each chapter ends with a set of problems intended to amplify and extend the text; some refer to further applications of nuclear physics. We have covered the bulk of the material in this book in 35 lectures of the core undergraduate curriculum at Bristol; these are given in the second and third years of the honours physics course.

We thank colleagues and students who read drafts of the text and drew our attention to errors and obscurities, which we have tried to eliminate. We are grateful to Margaret James and Mrs Lilian Murphy for their work on the typescript.

There is a less obvious debt: to the sometime Department of Mathematical Physics of the University of Birmingham where, under Professor Peierls, we first learned about physics.

W. N. Cottingham D. A. Greenwood *Bristol, August 1985*



Constants of nature, conversion factors and notation

Velocity of light	c	$2.99792 \times 10^8 \text{ m s}^{-1}$
Planck's constant	$\hbar = h/2\pi$	$1.05457 \times 10^{-34} \text{ J s}$
Proton charge	e	$1.60218 \times 10^{-19} \text{ C}$
Boltzmann's constant	k_{B}	$1.3807 \times 10^{-23} \text{ J K}^{-1}$
		$= 8.617 \times 10^{-5} \text{ eV K}^{-1}$
Gravitational constant	G	$6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$
Fermi coupling	$G_{ m F}$	$1.166 \times 10^{-11} (hc)^3 \text{ MeV}^{-2}$
constant		
Electron mass	$m_{\rm e}$	$9.1094 \times 10^{-31} \text{ kg}$
		$= 0.51100 \text{ MeV}/c^2$
Proton mass	$m_{\rm p}$	1.007 276 amu
	•	$= 938.27 \text{ MeV}/c^2$
Neutron mass	$m_{\rm n}$	1.008 66 amu
		$= 939.57 \text{ MeV}/c^2$
Atomic mass unit	(mass ¹² C atom)/12	$1.660 54 \times 10^{-27} \text{ kg}$
		$= 931.49 \text{ MeV}/c^2$
Bohr magneton	$\mu_{\rm B} = e \hbar/2m_{\rm e}$	$5.78838 \times 10^{-11} \text{ MeV T}^{-1}$
Nuclear magneton	$\mu_{\rm N} = e \hbar/2m_{\rm p}$	$3.15245 \times 10^{-14} \text{ MeV T}^{-1}$
Bohr radius	$a_0 = 4\pi\varepsilon_0 \hbar^2 / m_{\rm e} e^2$	$0.529177\times10^{-10}\mathrm{m}$
Fine-structure	$e^2/4\pi\varepsilon_0\hbar c$	1/137.036
constant		
$\hbar c = 197.327 \text{ MeV fm},$	$a^2/4\pi a = 1.420.04$	S MaV fm
mc = 197.327 IVIC V IIII,	$\epsilon / + n\epsilon_0 = 1.43990$	DIMICA LIII

 $hc = 197.327 \text{ MeV fm}, \quad e^2/4\pi\varepsilon_0 = 1.43996 \text{ MeV fm}$ $1 \text{ MeV} = 1.60218 \times 10^{-13} \text{ J}$ $1 \text{ fm} = 10^{-15} \text{ m}, \quad 1 \text{ barn} = 10^{-28} \text{ m}^2 = 10^2 \text{ fm}^2$ (Source: Review of Particle Physics (1998), Eur. Phys. J. C3, 1–794.)

Notation

 \mathbf{r} , \mathbf{k} , etc., denote vectors (x, y, z), (k_x, k_y, k_z) , and $r = |\mathbf{r}|$, $k = |\mathbf{k}|$, $d^3\mathbf{r} = dx dy dz$, $d^3\mathbf{k} = dk_x dk_y dk_z$.

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} = \frac{1}{r} \frac{\partial^2}{\partial r^2} r + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \sin \theta \frac{\partial}{\partial \theta} + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \phi^2},$$

 $d\Omega = \sin\theta \, d\theta \, d\phi$ denotes an infinitesimal element of solid angle. xii



Glossary of some important symbols

```
nuclear mass number (= N + Z)
       \boldsymbol{A}
               electromagnetic vector potential
  \mathbf{A}(\mathbf{r}, t)
               §4.1 nuclear surface width; §4.5 bulk binding coefficient
B(Z, N)
               binding energy of nucleus
               magnetic field
  \mathbf{B}(\mathbf{r},t)
               §4.5 surface tension coefficient; §14.1 impact parameter
               electric field
  \mathbf{E}(\mathbf{r}, t)
               energy; E_n, E_p neutron energy, proton energy; E_n^F, E_p^F
       E
               neutron, proton Fermi energy, measured from the bot-
               tom of the shell-model neutron potential well; E_G §8.3
F(Z, E_e)
               §12.3 Coulomb correction factor in \beta-decay
               §12.3 kinematic factor in total \beta-decay rate
f(Z, E_0)
               §6.2 exponent in the tunnelling formula
       G
               §12.2 weak interaction coupling constant (= G_{\rm F}V_{\rm ud})
      G_{\mathrm{w}}
               §8.1 statistical factor in Breit-Wigner formula
        g
               §5.6 orbital and intrinsic magnetic moment coefficients
   g_L, g_s
               §12.5 axial coupling constant
      g_{\rm A}
\mathcal{G}(r_{\rm s}/r_{\rm c})
               §6.2 tunnelling integral
               §C.3 total angular momentum operator
               quantum number associated with J^2
        j
               quantum number of J_z
       j_z
       k
               wave vector
               value of k = |\mathbf{k}| at the Fermi energy
      k_{\rm F}
               §C.1 orbital angular momentum operator
       L
               quantum number associated with L^2; Chapter 9,
               Chapter 14 mean free path
```

xiii



xiv Glossary of some important symbols

m	quantum number of L_z ; reduced mass
$m_{ m s}$	quantum number of s_z
m_{lpha}	mass of α -particle; m_a , m_{nuc} mass of atom, nucleus
N	number of neutrons in nucleus
n(E)	density of states
$\mathcal{N}(E)$	integrated density of states
p	momentum
Q	§5.7 nuclear electric quadrupole moment; §6.1 kinetic
	energy release in nuclear reaction
q	§9.4 fission probability
R	§4.3 nuclear radius; §12.3 reaction rate
$r_{\rm s}, r_{\rm c}$	§6.2 potential barrier parameters
$S_{\rm n}(N,Z)$	§5.2 neutron separation energy
S(E)	§8.3 parameter of nuclear reaction cross-section for
	energies below the Coulomb barrier
$S_0(E), S_c(E)$	§12.3 electron (positron) energy spectrum without and
	with Coulomb correction
S	§C.2 intrinsic angular momentum operator
S	quantum number associated with s; §4.5 symmetry
	energy coefficient
T	kinetic energy
$T_{1/2}$	decay half life
$t_{ m nuc}$	§5.2 nuclear time scale
$t_{\rm p}$	§9.4 prompt neutron life
$\overset{_{\mathbf{r}}}{U}$	potential energy; \bar{U} mean proton–neutron potential
	energy difference in nucleus
$u_l(r)$	radial wave-function
V	normalisation volume; $\S 3.3 \ V(r)$ nucleon–nucleon
	potential
$V_{ m ud}$	§12.5 element of Kobayashi–Maskawa matrix
v	velocity
Z	atomic number (number of protons in nucleus)
Γ, Γ_i	width, partial width, of an excited state
γ	§14.1 relativistic factor $(1-v^2/c^2)^{-\frac{1}{2}}$
δ	§4.4 coefficient of pairing energy
$arepsilon_0$	permittivity of free space
$arepsilon_{ m F}$	§11.1 Fermi energy of electron gas
θ	§13.3 neutrino mixing angle
$ heta_{ m w}$	§13.1 Weinberg angle
μ	§5.5 magnetic dipole operator
r-	0



Glossary of some important symbols

ΧV

$\mu_{ m n}, \mu_{ m p}$	neutron, proton magnetic moment
$\overset{\cdot}{\mu}$	§5.5 magnetic dipole moment; §11.1 stellar mass per
	electron; §14.3 photon linear attenuation coefficient
μ_0	permeability of free space
$\nu, \nu_{ m d}$	§9.3 mean number of prompt neutrons, delayed
	neutrons, per fission
ho	§2.1 electric charge density; §14.1 mass density
$ ho_{ m ch}$	$\S4.1$ electric charge density in units of e
$ ho_0$	§4.3 nucleon <i>number</i> density in nuclear matter
$\rho_{\mathrm{nuc}}, \rho_{\mathrm{n}}, \rho_{\mathrm{p}}$	number density of nuclei, neutrons, protons
σ	§C.2 Pauli spin matrices
σ	cross-section; σ_{tot} , σ_{e} , σ_{f} total, elastic, fission cross-
	section
τ	mean life; τ_{E1} , τ_{M1} electric, magnetic, dipole transition
	mean life; §7.4 $(\tau_i)^{-1}$ partial decay rate
Φ	§3.4 meson field
ϕ	electromagnetic scalar potential
$\psi(\mathbf{r})$	single particle wave-function
ψ_m	§D.1 general wave-function
$\Omega_{\mathrm{S0}},\Omega_{\mathrm{T}}$	§3.3 angular terms in the nucleon–nucleon potential
ω	angular frequency