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Prologue

More than 100 elements are now known to exist, distinguished from each other by the electric charge Ze on the atomic nucleus. This charge is balanced by the charge carried by the Z electrons which together with the nucleus make up the neutral atom. The elements are also distinguished by their mass, more than 99% of which resides in the nucleus. Are there other distinguishing properties of nuclei? Have the nuclei been in existence since the beginning of time? Are there elements in the Universe which do not exist on Earth? What physical principles underlie the properties of nuclei? Why are their masses so closely correlated with their electric charges? Why are some nuclei radioactive? Radioactivity is used to man's benefit in medicine. Nuclear fission is exploited in power generation. But man's use of nuclear physics has also posed the terrible threat of nuclear weapons.

This book aims to set out the basic concepts which have been developed by nuclear physicists in their attempts to understand the nucleus. Besides satisfying our appetite for knowledge, these concepts must be understood if we are to make an informed judgment on the benefits and problems of nuclear technology.

After the discovery of the neutron by Chadwick in 1932, it was accepted that a nucleus of atomic number Z was made up of Z protons and some number N of neutrons. The proton and neutron were then thought to be elementary particles, although it is now clear that they are not but rather are themselves structured entities. We shall also see that in addition to neutrons and protons several other particles play an important, if indirect, role in the physics of nuclei. In this and the following two chapters, to provide a background to our subsequent study of the nucleus, we shall describe the elementary particles of nature, and their interactions, as they are at present understood.

1.1 Fermions and bosons

Elementary particles are classified as either *fermions* or *bosons*. Fermions are particles which satisfy the Pauli exclusion principle: if an assembly of identical fermions is described in terms of single-particle wave-functions, then no two fermions can have the same wave-function. For example, electrons are fermions. This rule explains the shell structure of atoms and hence underlies the whole of chemistry. Fermions are so called because they obey the Fermi–Dirac statistics of statistical mechanics.

Bosons are particles which obey Bose–Einstein statistics, and are characterised by the property that *any* number of particles may be assigned the same single-particle wave-function. Thus, in the case of bosons, coherent waves of macroscopic amplitude can be constructed, and such waves may to a good approximation be described classically. For example, photons are bosons and the corresponding classical field is the familiar electromagnetic field **E** and **B**, which satisfies Maxwell's equations.

At a more fundamental level, these properties are a consequence of the possible symmetries of the wave-function of a system of identical particles when the coordinates of any two particles are interchanged. In the case of fermions, the wave-function changes sign; it is completely anti-symmetric. In the case of bosons the wave-function is unchanged; it is completely symmetric.

There is also an observed relation between the intrinisc angular momentum, or spin, of a particle and its statistics. The intrinsic spin $\mathbf{s}$ is quantised, with spin quantum number s (see Appendix C). For a fermion, s takes one of the values $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$; for a boson, s takes one of the values $0, 1, 2, \ldots$. A theoretical explanation of this relationship can be given within the framework of relativistic quantum field theory.

1.2 The particle physicist's picture of nature

Elementary particle physics describes the world in terms of elementary fermions, interacting through fields of which they are sources. The particles associated with the interaction fields are bosons. To take the most familiar example, an electron is an elementary fermion; it carries electric charge $-e$ and this charge produces an electromagnetic field **E**, **B**, which

Table 1.1. *Types of interaction field*

Interaction field	Boson	Spin
Gravitational field	'Gravitons' postulated	2
Weak field	W^+, W^-, Z particles	1
Electromagnetic field	Photons	1
Strong field	'Gluons' postulated	1

exerts forces on other charged particles. The electromagnetic field, quantised according to the rules of quantum mechanics, corresponds to an assembly of *photons*, which are bosons. Indeed, Bose–Einstein statistics were first applied to photons.

Four types of interaction field may be distinguished in nature (see Table 1.1). All of these interactions are relevant to nuclear physics, though the gravitational field becomes important only in densely aggregated matter, such as stars. Gravitational forces act on all particles and are important for the physics on the large scale of macroscopic bodies. On the small scale of most terrestrial atomic and nuclear physics, gravitational forces are insignificant and except in Chapter 10 and Chapter 11 we shall ignore them.

Nature provides an even greater diversity of elementary fermions than of bosons. It is convenient to divide the elementary fermions into two classes: *leptons*, which are not sources of the strong fields and hence do not participate in the strong interaction; and *quarks*, which take part in all interactions.

The electron is an example of a lepton. Leptons and their interactions are described in Chapter 2. Quarks are always confined in compound systems which extend over distances of about 1 fm. The term *hadron* is used generically for a quark system. The proton and neutron are hadrons, as are mesons. The proton and neutron are the subject matter of Chapter 3.

1.3 Conservation laws and symmetries: parity

The total energy of an isolated system is constant in time. So also are its linear momentum and angular momentum. These conservation laws are derivable from Newton's laws of motion and Maxwell's equations, or from the laws of quantum mechanics, but they can also, at a deeper level, be regarded as consequences of 'symmetries' of space and time. Thus the law of conservation of linear momentum follows from the homogeneity of space, the law of conservation of angular momentum from the isotropy of space; it does not matter where we place the origin of our coordinate axes, or in which direction they are oriented.

These conservation laws are as significant in nuclear physics as elsewhere, but there is another symmetry and conservation law which is of particular importance in quantum systems such as the nucleus: reflection symmetry and parity. By reflection symmetry we mean reflection about the origin, $\mathbf{r} \to \mathbf{r}' = -\mathbf{r}$. A single-particle wave-function $\psi(\mathbf{r})$ is said to have parity +1 if it is even under reflection, i.e.

$$\psi(-\mathbf{r}) = \psi(\mathbf{r}),$$

and parity −1 if it is odd under reflection, i.e.

$$\psi(-\mathbf{r}) = -\psi(\mathbf{r}).$$

More generally, a many-particle wave-function has parity +1 if it is even under reflection of all the particle coordinates, and parity −1 if it is odd under reflection.

Parity is an important concept because the laws of the electromagnetic and of the strong interaction are of exactly the same form if written with respect to a reflected left-handed coordinate system $(0x', 0y', 0z')$ as they are in the standard right-handed system $(0x, 0y, 0z)$ (Fig. 1.1). We shall see in Chapter 2 that this is not true of the weak interaction. Nevertheless, for many properties of atomic and nuclear systems the weak interaction is unimportant and wave-functions for such systems can be chosen to have a definite parity which does not change as the wave-function evolves in time according to Schrödinger's equation.

1.4 Units

Every branch of physics tends to find certain units particularly congenial. In nuclear physics, the size of the nucleus makes 10^{-15} m = 1 fm (femtometre) convenient as a unit of length, usually called a *fermi*. However, nuclear cross-sections, which have the dimensions of area, are measured in *barns*; 1 b = 10^{-28} m^2 = 100 fm^2. Energies of interest are usually of the order of MeV. Since mc^2 has the dimensions of energy, it is convenient to quote masses in units of MeV/c^2.

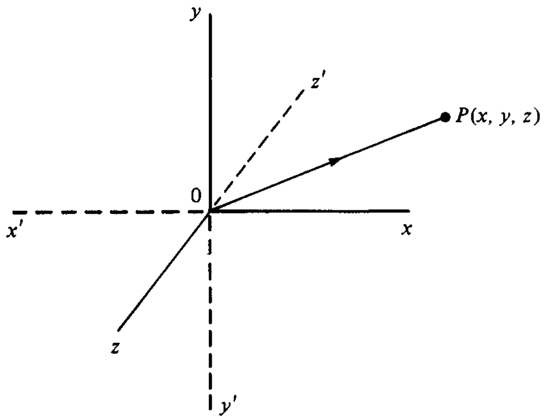

Fig. 1.1 The point P at **r** with coordinates (x, y, z) has coordinates $(-x, -y, -z)$ in the primed, reflected coordinate axes. $(0x', 0y', 0z')$ make up a *left-handed* set of axes. In the figure, the $0z$ axis is out of the plane of the page.

For order-of-magnitude calculations, the masses m_e and m_p of the electron and proton may be taken as

$$m_e \approx 0.5 \text{ MeV}/c^2$$
$$m_p \approx 938 \text{ MeV}/c^2$$

and it is useful to remember that

$$\hbar c \approx 197 \text{ MeV fm}, \qquad e^2/4\pi\varepsilon_0 \approx 1.44 \text{ MeV fm},$$
$$e^2/4\pi\varepsilon_0 \hbar c \approx 1/137, \qquad c \approx 3 \times 10^{23} \text{ fm s}^{-1}.$$

The student will perhaps be surprised to find how easily many expressions in nuclear physics can be evaluated using these quantities.

Problems

1.1 Show that the ratio of the gravitational potential energy to the Coulomb potential energy between two electrons is $\approx 2.4 \times 10^{-43}$.

1.2(*a*) Show that in polar coordinates (r, θ, ϕ) the reflection $\mathbf{r} \to \mathbf{r}' = -\mathbf{r}$ is equivalent to $r \to r' = r$
$\theta \to \theta' = \pi - \theta, \phi \to \phi' = \phi + \pi$.

(*b*) What are the parities of the following electron states of the hydrogen atom:

(*i*) $\psi_{100} = \frac{1}{\sqrt{\pi}}\left(\frac{1}{a_0}\right)^{\frac{3}{2}} \mathrm{e}^{-r/a_0},$

(*ii*) $\psi_{210} = \frac{1}{4\sqrt{(2\pi)}}\left(\frac{1}{a_0}\right)^{\frac{3}{2}} \frac{r}{a_0} \mathrm{e}^{-r/2a_0} \cos\theta,$

(*iii*) $\psi_{21-1} = \frac{1}{8\sqrt{\pi}}\left(\frac{1}{a_0}\right)^{\frac{3}{2}} \frac{r}{a_0} \mathrm{e}^{-r/2a_0} \sin\theta\, \mathrm{e}^{-\mathrm{i}\phi}?$

($a_0 = (4\pi\varepsilon_0)\hbar^2/m_e e^2$ is the Bohr radius.)

1.3(*a*) Show that the wavelength of a photon of energy 1 MeV is $\approx$ 1240 fm.

(*b*) The electrostatic self-energy of a uniformly charged sphere of total charge e, radius R, is $U = (3/5)e^2/(4\pi\varepsilon_0 R)$. Show that if $R = 1$ fm, $U \approx 0.86$ MeV.

2

Leptons and the electromagnetic and weak interactions

2.1 The electromagnetic interaction

The electromagnetic field is most conveniently described by a vector potential **A** and a scalar potential ϕ. For simplicity, we consider only the potential $\phi(\mathbf{r}, t)$. Using Maxwell's equations, this may be chosen to satisfy the wave-equation

$$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = -\frac{\rho(\mathbf{r}, t)}{\varepsilon_0}. \tag{2.1}$$

Here $\rho(\mathbf{r}, t)$ is the electric charge density due to the charged particles, which in atomic and nuclear physics will usually be electrons and protons, and c is the velocity of light.

In regions where $\rho = 0$, equation (2.1) has solutions in the form of propagating waves; for example, the plane wave

$$\phi(\mathbf{r}, t) = (\text{constant})\,\mathrm{e}^{\mathrm{i}(\mathbf{k}\cdot\mathbf{r}-\omega t)}. \tag{2.2}$$

This satisfies

$$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = 0 \tag{2.3}$$

provided

$$\omega^2 = c^2k^2. \tag{2.4}$$

The wave velocity is therefore c, as we should expect. In quantum theory, unlike classical theory, the total energy and momentum of the wave are quantised, and can only be integer multiples of the basic quantum of energy and momentum given by the de Broglie relations:

$$E = \hbar\omega, \quad \mathbf{p} = \hbar\mathbf{k}. \tag{2.5}$$

Such a quantum of radiation is called a *photon*. A macroscopic wave can be considered to be an assembly of photons, and we can regard photons as particles, each carrying energy E and momentum $\mathbf{p}$.

Using (2.4) and (2.5), E and $\mathbf{p}$ are related by

$$E^2 = p^2c^2. \tag{2.6}$$

For a particle of mass m, the Einstein equation gives

$$E^2 = p^2c^2 + m^2c^4.$$

We therefore infer that the photon is a particle having zero mass.

A second important type of solution of (2.1) exists when charged particles are present. If these are moving slowly compared with the velocity of light, so that the term $\partial^2\phi/(c^2\,\partial t^2)$ can be neglected, the solution is approximately the Coulomb potential of the charge distribution. For a particle with charge density ρ_1, we can take

$$\phi(\mathbf{r}, t) \approx \frac{1}{4\pi\varepsilon_0}\int \frac{\rho_1(\mathbf{r}', t)}{|\mathbf{r} - \mathbf{r}'|}\,\mathrm{d}^3\mathbf{r}'. \tag{2.7}$$

Another charged particle with charge density ρ_2 will have a potential energy given by

$$\begin{aligned} U_{12} &= \int \rho_2(\mathbf{r}, t)\phi(\mathbf{r}, t)\mathrm{d}^3r \\ &= \frac{1}{4\pi\varepsilon_0}\int \frac{\rho_1(\mathbf{r}', t)\rho_2(\mathbf{r}, t)}{|\mathbf{r} - \mathbf{r}'|}\,\mathrm{d}^3\mathbf{r}\,\mathrm{d}^3\mathbf{r}'. \end{aligned} \tag{2.8}$$

Electric potential energy is basically responsible for the binding of electrons in atoms and molecules. We shall see that, in nuclear physics, it is responsible for the instability of heavy nuclei. If magnetic effects due to the motion of the charges are included, equation (2.8) is modified to

$$U_{12} = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho_1' \rho_2 + (1/c^2)\mathbf{j}_1' \cdot \mathbf{j}_2}{|\mathbf{r} - \mathbf{r}'|} \, d^3\mathbf{r} \, d^3\mathbf{r}', \tag{2.9}$$

where $\mathbf{j} = \rho\mathbf{v}$ is the current associated with the charge distribution which has velocity $\mathbf{v}(\mathbf{r})$. Thus this magnetic contribution to the energy is of relative order v^2/c^2.

The electromagnetic interaction also gives rise to the scattering of charged particles. For example, for two electrons approaching each other the interaction gives a mutual repulsion which leads to a transfer of momentum between the particles. The process can be represented by a diagram such as Fig. 2.1. In quantum electrodynamics, these diagrams, invented by Feynman, have a precise technical interpretation in the theory. We shall use them only to help visualise the physics involved. The scattering of the two electrons may be thought of as caused by the emission of a 'virtual' photon by one electron and its absorption by the other electron. In a virtual process the photon does not actually appear to an observer, though it appears in the mathematical formalism that describes the process.

2.2 The weak interaction

There are three *weak interaction* fields associated with the W^+, W^- and Z particles. Each one, like the electromagnetic field, is described by a vector and a scalar potential. However, the bosons associated with the weak

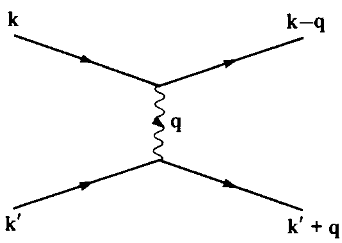

Fig. 2.1 The scattering of two electrons of momenta $\hbar\mathbf{k}$, $\hbar\mathbf{k}'$ by the exchange of a virtual photon carrying momentum $\hbar\mathbf{q}$. Time runs from left to right in these diagrams. (In principle, the exchange of a Z particle (§2.2) also contributes to electron–electron scattering, but the very short range and weakness of the weak interaction makes this contribution almost completely negligible: the electrons are in any case kept apart by the Coulomb repulsion induced by the photon exchange.)

fields all have mass, and the W^- and W^+ bosons are electrically charged. The Z boson is neutral, and most similar to the photon, but it has a mass

$$M_Z = (91.187 \pm 0.007)\ \mathrm{GeV}/c^2 \sim 100 \text{ proton masses},$$

which is very large by nuclear physics standards.

The interactions between leptons and the electromagnetic and weak fields were combined into a unified 'electro-weak' theory by Weinberg and by Salam. The existence of the Z and $W^{\pm}$ bosons was predicted by the theory, and the theory together with experimental data from neutrino–nuclear scattering also suggested values for their masses. These predictions were confirmed by experiments at CERN in 1983.

The wave equation satisfied by the scalar potential ϕ_Z associated with the Z boson is a generalisation of (2.1) and includes a term involving M_Z:

$$\left[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \left(\frac{M_Z c}{\hbar}\right)^2\right]\phi_Z(\mathbf{r}, t) = -\frac{\rho_Z(\mathbf{r}, t)}{\varepsilon_0}, \tag{2.10}$$

where ρ_Z is the neutral weak-charge density. There is a close, but not exact, analogy between weak-charge density and electric-charge density, and particles carry weak charge somewhat as they carry electric charge. In the case of a nucleus, ρ_Z will extend over the nuclear dimensions.

In free space where $\rho_Z = 0$ there exist plane wave solutions of (2.10),

$$\phi_Z(\mathbf{r}, t) = (\text{constant})\, \mathrm{e}^{\mathrm{i}(\mathbf{k}\cdot\mathbf{r} - \omega t)},$$

but now to satisfy the wave equation we require

$$\omega^2 = c^2 k^2 + c^2 (M_Z c/\hbar)^2,$$

and with the de Broglie relations (2.5) for the field quanta we obtain the Einstein energy–momentum relation for the Z boson:

$$E^2 = p^2 c^2 + M_Z^2 c^4.$$

The static solution of (2.10) which corresponds to a point unit weak charge at the origin is

$$\phi_Z(r) = \frac{1}{4\pi\varepsilon_0}\frac{\mathrm{e}^{-\kappa r}}{r}, \quad \text{writing } \kappa = \frac{M_Z c}{\hbar}. \tag{2.11}$$