

Contents

<i>Preface</i>	<i>page</i>	xiii
1 Introduction		1
2 Pure Substances		7
2.1 Planar interfaces		7
2.1.1 Mathematical model		7
2.1.2 One-dimensional freezing from a cold boundary		9
2.1.3 One-dimensional freezing from a cold boundary: Small undercooling		13
2.1.4 One-dimensional freezing into an undercooled melt		15
2.1.5 One-dimensional freezing into an undercooled melt: Effect of kinetic undercooling		18
2.2 Curved interfaces		21
2.2.1 Boundary conditions		21
2.2.2 Growth of a nucleus in an undercooled melt		26
2.2.3 Linearized instability of growing nucleus		32
2.2.4 Linearized instability of a plane front growing into an undercooled melt		35
2.2.5 Remarks		39
3 Binary Substances		42
3.1 Mathematical model		42
3.2 Directional solidification		45

x	<i>Contents</i>	
	3.3 Basic state and approximate models	46
	3.4 Linearized instability of a moving front in directional solidification	48
	3.5 Mechanism of morphological instability	56
	3.6 More general models	57
	3.7 Remarks	59
4	Nonlinear theory for directional solidification	62
	4.1 Bifurcation theory	62
	4.1.1 Two-dimensional theory	62
	4.1.2 Two-dimensional theory for wave number se- lection	66
	4.1.3 Three-dimensional theory	72
	4.2 Long-scale theories	76
	4.2.1 Small segregation coefficient	77
	4.2.2 Small segregation coefficient and large surface energy	78
	4.2.3 Near absolute stability	80
	4.3 Remarks	82
5	Anisotropy	86
	5.1 Surface energy and kinetics	86
	5.2 Directional solidification with “small” anisotropy	91
	5.3 Directional solidification with “small” anisotropy: Stepwise growth	97
	5.4 Unconstrained growth with “small” anisotropy	105
	5.4.1 Two-dimensional crystal and one-dimensional front	110
	5.4.2 Three-dimensional crystal and two-dimensional front	111
	5.5 Unconstrained growth with “large” anisotropy – One-dimensional interfaces	121
	5.6 Unconstrained growth with “large” anisotropy – Two-dimensional interfaces	135
	5.7 Faceting with constant driving force	139
	5.8 Coarsening	152
	5.9 Remarks	156
6	Disequilibrium	162
	6.1 Model of rapid solidification	164
	6.2 Basic state and linear stability theory	167

<i>Contents</i>		xi
6.3	Thermal effects	171
6.4	Linear-stability theory with thermal effects	172
6.4.1	Steady mode	173
6.4.2	Oscillatory mode	173
6.4.3	The two modes	177
6.5	Cellular modes in the FTA: Two-dimensional bifurcation theory	181
6.6	Oscillatory modes in the FTA: Two-dimensional bifurcation theory	183
6.7	Strongly nonlinear pulsations	189
6.7.1	Small β	190
6.7.2	Large β	198
6.7.3	Numerical simulation	203
6.8	Mode coupling	204
6.8.1	Pulsatile–cellular interactions	204
6.8.2	Oscillatory–cellular interactions	205
6.8.3	Oscillatory–pulsatile interactions	206
6.9	Phenomenological models	208
6.10	Remarks	211
7	Dendrites	215
7.1	Isolated needle crystals	217
7.2	Approximate selection arguments	221
7.3	Selection theories	229
7.4	Arrays of needles	237
7.5	Remarks	251
8	Eutectics	255
8.1	Formulation	256
8.2	Approximate theories for steady growth and selection	261
8.3	Instabilities	267
8.4	Remarks	270
9	Microscale Fluid Flow	274
9.1	Formulation	276
9.2	Prototype flows	279
9.2.1	Free convection	279
9.2.2	Bénard convection	280
9.3	Directional solidification and volume-change convection	283

xii	<i>Contents</i>	
	9.4 Directional solidification and buoyancy-driven convection	287
	9.5 Directional solidification and forced flows	292
	9.6 Directional solidification with imposed cellular convection	304
	9.7 Flows over Ivantsov needles	311
	9.8 Remarks	319
10	Mesoscale Fluid Flow	324
	10.1 Formulation	325
	10.2 Planar solidification between horizontal planes	326
	10.3 Mushy-zone models	331
	10.4 Mushy zones with volume-change convection	336
	10.5 Mushy zones with buoyancy-driven convective instability	341
	10.6 An oscillatory mode of convective instability	349
	10.7 Weakly nonlinear convection	356
	10.8 Chimneys	357
	10.9 Remarks	363
11	Phase-Field Models	366
	11.1 Pure materials – A model system	367
	11.2 Pure materials – A deduced system	372
	11.3 Pure materials – Computations	374
	11.4 Remarks	376
	<i>Index</i>	379