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Thomas Peterfalvi

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Preface

This book comprises two parts, each devoted to the revision of the proof of a theorem about finite groups. The two theorems are among the results taken as foundation material in the revision of the classification of finite simple groups undertaken by D. Gorenstein, R. Lyons and R. Solomon.

The famous theorem of W. Feit and J. G. Thompson states that every finite group of odd order is solvable. It is the most important among the initial theorems in the classification of finite simple groups. The proof of this theorem divides into two parts. The first consists of the study of the maximal subgroups of a minimal counterexample to the theorem. This part, which is of considerable difficulty, has been revised by H. Bender and G. Glauberman; their work has appeared as a book in this series. The second part of the proof of the Feit-Thompson Theorem uses character theory to show that the existence of a simple group of odd order is impossible. In Part I of this book, we give a revision of this portion of the proof. Thus, with the book of Bender and Glauberman, a complete proof of the theorem is provided.

In Part II of this book, a revised proof of a theorem of M. Suzuki is given. This theorem characterizes certain groups which have a split BN-pair of rank 1. Let G be a 2-transitive group of permutations of a set X of odd order. Assume that the stabilizer in G of a point x of X has a normal subgroup acting regularly on $X - \{x\}$, and that subgroups of G fixing two points of X have odd order. The theorem shows that, if G is simple, then G is a group of Lie type of rank 1 in characteristic 2. This part is based on an earlier version which appeared, in French, in 1986.

I wish to record my warmest gratitude to Professor G. Glauberman who has both encouraged me to undertake, and assisted me in effecting, the preparation of each of these texts. I am also very grateful to R. Sandling who has carried out the translation into English with great care and who has suggested various improvements.