

Cambridge University Press

0521646596 - New Trends in Algebraic Geometry: EuroConference on Algebraic Geometry Warwick, July 1996

Edited by Klaus Hulek, Fabrizio Catanese, Chris Peters and Miles Reid

Frontmatter

More information

#### LONDON MATHEMATICAL SOCIETY LECTURE NOTE SERIES

Managing Editor: Professor N.J. Hitchin, Mathematics Institute, University of Oxford, 24-29 St Giles, Oxford OX1 3TG, United Kingdom

The titles below are available from booksellers, or, in case of difficulty, from Cambridge University Press.

- p-adic Analysis: a short course on recent work, N. KOBLITZ Applicable differential geometry, M. CRAMPIN & F.A.E. PIRANI
- 66 Several complex variables and complex manifolds II, M.J. FIELD
- 86
- 87
- 88
- 89
- 90
- 92
- 93

- 98
- Several complex variables and complex manifolds II, M.J. FIELD
  Topological topics, I.M. JAMES (ed)
  Surveys in set theory, A.R.D. MATHIAS (ed)
  FPF ring theory, C. FAITH & S. PAGE
  An F-space sampler, N.J. KALTON, N.T. PECK & J.W. ROBERTS
  Polytopes and symmetry, S.A. ROBERTSON
  Representations of rings over skew fields, A.H. SCHOFIELD
  Aspects of topology, I.M. JAMES & E.H. KRONHEIMER (eds)
  Diophantine equations over function fields, R.C. MASON
  Varieties of constructive mathematics, D.S. BRIDGES & F. RICHMAN
  Localization in Noetherian rings, A.V. JATEGAONKAR
  Methods of differential geometry in algebraic topology, M. KAROUBI & C. LERUSTE
  Stopping time techniques for analysts and probabilists, L. EGGHE
  Elliptic structures on 3-manifolds, C.B. THOMAS
  A local spectral theory for closed operators. I. ERDELYI & WANG SHENGWANG 99
- 100
- 104
- 105 A local spectral theory for closed operators, I. ERDELYI & WANG SHENGWANG
- Compactification of Siegel moduli schemes, C.-L. CHAI 107
- Diophantine analysis, J. LOXTON & A. VAN DER POORTEN (eds) 109
- Lectures on the asymptotic theory of ideals, D. REES 113
- Lectures on Bochner-Riesz means, K.M. DAVIS & Y.-C. CHANG Representations of algebras, P.J. WEBB (ed) 114
- 116
- 119 Triangulated categories in the representation theory of finite-dimensional algebras, D. HAPPEL
- 121 Proceedings of Groups - St Andrews 1985, E. ROBERTSON & C. CAMPBELL (eds)
- Descriptive set theory and the structure of sets of uniqueness, A.S. KECHRIS & A. LOUVEAU 128 130 Model theory and modules, M. PREST
- 131
- Algebraic, extremal & metric combinatorics, M.-M. DEZA, P. FRANKL & I.G. ROSENBERG (eds)
- 132
- 133
- Angebraic, extremal & metric combinatorics, M.-M. DEZA, F. FRANKL & Whitehead groups of finite groups, ROBERT OLIVER
  Linear algebraic monoids, MOHAN S. PUTCHA
  Number theory and dynamical systems, M. DODSON & J. VICKERS (eds)
  Analysis at Urbana, I, E. BERKSON, T. PECK, & J. UHL (eds)
  Analysis at Urbana, II, E. BERKSON, T. PECK, & J. UHL (eds) 134
- 137
- 138
- 139 Advances in homotopy theory, S. SALAMON, B. STEER & W. SUTHERLAND (eds)
- 140 Geometric aspects of Banach spaces, E.M. PEINADOR & A. RODES (eds)
- Surveys in combinatorics 1989, J. SIEMONS (ed) Introduction to uniform spaces, I.M. JAMES 141
- 144
- 146 Cohen-Macaulay modules over Cohen-Macaulay rings, Y. YOSHINO
- 148 Helices and vector bundles, A.N. RUDAKOV et al
- 149 Solitons, nonlinear evolution equations and inverse scattering, M. ABLOWITZ & P. CLARKSON
- 150
- Geometry of low-dimensional manifolds 1, S. DONALDSON & C.B. THOMAS (eds)
  Geometry of low-dimensional manifolds 2, S. DONALDSON & C.B. THOMAS (eds) 151
- 152
- 153
- 155
- Obligomorphic permutation groups, P. CAMERON
  L-functions and arithmetic, J. COATES & M.J. TAYLOR (eds)
  Classification theories of polarized varieties, TAKAO FUJITA
  Twistors in mathematics and physics, T.N. BAILEY & R.J. BASTON (eds)
  Geometry of Banach spaces, P.F.X. MÜLLER & W. SCHACHERMAYER (eds) 156
- 158 159
- Groups St Andrews 1989 volume 1, C.M. CAMPBELL & E.F. ROBERTSON (eds)
  Groups St Andrews 1989 volume 2, C.M. CAMPBELL & E.F. ROBERTSON (eds)
  Lectures on block theory, BURKHARD KÜLSHAMMER 160
- 161
- 162 Harmonic analysis and representation theory, A. FIGA-TALAMANCA & C. NEBBIA Topics in varieties of group representations, S.M. VOVSI
- 163 Topics in varieties of group representations,
- 164
- 166
- Quasi-symmetric designs, M.S. SHRIKANDE & S.S. SANE Surveys in combinatorics, 1991, A.D. KEEDWELL (ed) Representations of algebras, H. TACHIKAWA & S. BRENNER (eds) 168
- 169 Boolean function complexity, M.S. PATERSON (ed)
- 170 Manifolds with singularities and the Adams-Novikov spectral sequence, B. BOTVINNIK
- 171 Squares, A.R. RAJWADE
- 172 Algebraic varieties, GEORGE R. KEMPF
- 173 Discrete groups and geometry, W.J. HARVEY & C. MACLACHLAN (eds) Lectures on mechanics, J.E. MARSDEN
- 174
- 175 176
- Adams memorial symposium on algebraic topology 1, N. RAY & G. WALKER (eds)
  Adams memorial symposium on algebraic topology 2, N. RAY & G. WALKER (eds)
  Applications of categories in computer science, M. FOURMAN, P. JOHNST 177
- 178 Lower K- and L-theory, A. RANICKI
- 179 Complex projective geometry, G. ELLINGSRUD et al
- Lectures on ergodic theory and Pesin theory on compact manifolds, M. POLLICOTT Geometric group theory I, G.A. NIBLO & M.A. ROLLER (eds) 180
- 181
- Geometric group theory II, G.A. NIBLO & M.A. ROLLER (eds)



Cambridge University Press

0521646596 - New Trends in Algebraic Geometry: EuroConference on Algebraic Geometry Warwick, July 1996

Edited by Klaus Hulek, Fabrizio Catanese, Chris Peters and Miles Reid

Frontmatter

More information

```
183
             Shintani zeta functions, A. YUKIE
Arithmetical functions, W. SCHWARZ & J. SPILKER
184
185
             Representations of solvable groups, O. MANZ & T.R. WOLF
186
             Complexity: knots, colourings and counting,
                                                                                               D.J.A. WELSH
             Surveys in combinatorics, 1993, K. WALKER (ed)
Local analysis for the odd order theorem, H. BENDER & G. GLAUBERMAN
187
188
             Locally presentable and accessible categories, J. ADAMEK & J. ROSICKY Polynomial invariants of finite groups, D.J. BENSON Finite geometry and combinatorics, F. DE CLERCK et al Symplectic geometry, D. SALAMON (ed)
189
190
191
192
194
             Independent random variables and rearrangement invariant spaces, M. BRAVERMAN Arithmetic of blowup algebras, WOLMER VASCONCELOS Microlocal analysis for differential operators, A. GRIGIS & J. SJÖSTRAND
195
196
197
             Two-dimensional homotopy and combinatorial group theory, C. HOG-ANGELONI et al. The algebraic characterization of geometric 4-manifolds, J.A. HILLMAN
198
199
              Invariant potential theory in the unit ball of C^n, MANFRED STOLL
              The Grothendieck theory of dessins d'enfant, L. SCHNEPS (ed)
Singularities, JEAN-PAUL BRASSELET (ed)
200
201
202
              The technique of pseudodifferential operators, H.O. CORDES
              Hochschild cohomology of von Neumann algebras, A. SINCLAIR & R. SMITH Combinatorial and geometric group theory, A.J. DUNCAN, N.D. GILBERT & J. HOWIE (eds)
203
204
205
              Ergodic theory and its connections with harmonic analysis, K. PETERSEN & I. SALAMA (eds)
             Groups of Lie type and their geometries, W.M. KANTOR & L. DI MARTINO (eds)
Vector bundles in algebraic geometry, N.J. HITCHIN, P. NEWSTEAD & W.M. OXBURY (eds)
207
208
209
              Arithmetic of diagonal hypersurfaces over finite fields, F.Q. GOUVÊA & N. YUI
210
              Hilbert C*-modules, E.C. LANCE
Groups 93 Galway / St Andrews I, C.M. CAMPBELL et al (eds)
Groups 93 Galway / St Andrews II, C.M. CAMPBELL et al (eds)
211
212
214
              Generalised Euler-Jacobi inversion formula and asymptotics beyond all orders, V. KOWALENKO et al
              Number theory 1992–93, S. DAVID (ed)
215
216
              Stochastic partial differential equations, A. ETHERIDGE (ed)
              Stochastic partial differential equations, A. ETHERIDGE (ed)
Quadratic forms with applications to algebraic geometry and topology, A. PFISTER
Surveys in combinatorics, 1995, PETER ROWLINSON (ed)
Algebraic set theory, A. JOYAL & I. MOERDIJK
Harmonic approximation, S.J. GARDINER
Advances in linear logic, J.-Y. GIRARD, Y. LAFONT & L. REGNIER (eds)
Analytic semigroups and semilinear initial boundary value problems, KAZUAKI TAIRA
Computability, enumerability, unsolvability, S.B. COOPER, T.A. SLAMAN & S.S. WAINER (eds)
A mathematical introduction to string theory, S. ALBEVERIO, J. JOST, S. PAYCHA, S. SCARLATTI
Novikov conjectures, index theorems and rigidity I. S. FERRY, A. RANICKI & J. ROSENBERG (eds)
217
218
220
221
222
223
224
225
              Novikov conjectures, index theorems and rigidity I, S. FERRY, A. RANICKI & J. ROSENBERG (eds)
Novikov conjectures, index theorems and rigidity II, S. FERRY, A. RANICKI & J. ROSENBERG (eds)
226
227
              Ergodic theory of \mathbb{Z}^d actions, M. POLLICOTT & K. SCHMIDT (eds)
228
              Ergodicity for infinite dimensional systems, G. DA PRATO & J. ZABCZYK Prolegomena to a middlebrow arithmetic of curves of genus 2, J.W.S. CASSELS & E.V. FLYNN Semigroup theory and its applications, K.H. HOFMANN & M.W. MISLOVE (eds)
229
230
231
              The descriptive set theory of Polish group actions, H. BECKER & A.S. KECHRIS Finite fields and applications, S. COHEN & H. NIEDERREITER (eds) Introduction to subfactors, V. JONES & V.S. SUNDER Number theory 1993–94, S. DAVID (ed)
The James forest, H. FETTER & B. GAMBOA DE BUEN
232
233
234
235
236
 237
              Sieve methods, exponential sums, and their applications in number theory, G.R.H. GREAVES et al
              Sieve methods, exponential sums, and their applications in number theory, G.R.H. GREAVES Representation theory and algebraic geometry, A. MARTSINKOVSKY & G. TODOROV (eds) Clifford algebras and spinors, P. LOUNESTO Stable groups, FRANK O. WAGNER Surveys in combinatorics, 1997, R.A. BAILEY (ed) Geometric Galois actions I, L. SCHNEPS & P. LOCHAK (eds) Geometric Galois actions II, L. SCHNEPS & P. LOCHAK (eds) Model theory of groups and automorphism groups. D. EVANS (ed.)
 238
 239
 240
241
242
 243
 244
               Model theory of groups and automorphism groups, D. EVANS (ed)
              Geometry, combinatorial designs and related structures, J.W.P. HIRSCHFELD et al
 245
 246
              p-Automorphisms of finite p-groups, E.I. KHUKHRO Analytic number theory, Y. MOTOHASHI (ed)
 247
              Tame topology and o-minimal structures, LOU VAN DEN DRIES
The atlas of finite groups: ten years on, ROBERT CURTIS & ROBERT WILSON (eds)
Characters and blocks of finite groups, G. NAVARRO
Gröbner bases and applications, B. BUCHBERGER & F. WINKLER (eds)
Geometry, and exhaustications, I. ROBERGER & F. WINKLER (eds)
 248
 249
 250
 251
               Geometry and cohomology in group theory, P. KROPHOLLER, G. NIBLO, R. STÖHR (eds) The q-Schur algebra, S. DONKIN
 252
 253
               Galois representations in arithmetic algebraic geometry, A.J. SCHOLL & R.L. TAYLOR (eds) Symmetries and integrability of difference equations, P.A. CLARKSON & F.W. NIJHOFF (eds)
 254
 255
 257
               An introduction to noncommutative differential geometry and its physical applications, 2ed, J. MADORE
               Groups St Andrews 1997 in Bath, I, C.M. CAMPBELL et al (eds)
Groups St Andrews 1997 in Bath, II, C.M. CAMPBELL et al (eds)
New trends in algebraic geometry, K. HULEK, F. CATANESE, C. PETERS & M. REID (eds)
 260
 261
```



London Mathematical Society Lecture Note Series. 264

# New Trends in Algebraic Geometry

**EuroConference on Algebraic Geometry Warwick, July 1996** 

Edited by

Klaus Hulek (chief editor) Universität Hannover

Fabrizio Catanese Georg-August-Universität, Göttingen

Chris Peters Université de Grenoble

Miles Reid University of Warwick





## CAMBRIDGE UNIVERSITY PRESS

Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

Cambridge University Press The Edinburgh Building, Cambridge CB2 2RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org

Information on this title: www.cambridge.org/9780521646598

© Cambridge University Press 1999

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 1999

A catalogue record for this publication is available from the British Library

ISBN-13 978-0-521-64659-8 paperback ISBN-10 0-521-64659-6 paperback

Transferred to digital printing 2005



# Contents

Victor V. Batyrev: Birational Calabi–Yau $n$ -folds have equal Betti numbers	1
Arnaud Beauville: A Calabi-Yau threefold with non-Abelian fundamental group	13
K. Behrend: Algebraic Gromov-Witten invariants	19
Philippe Eyssidieux: Kähler hyperbolicity and variations of Hodge structures	71
Carel Faber: Algorithms for computing intersection numbers on moduli spaces of curves, with an application to the class of the locus of Jacobians	93
Marat Gizatullin: On some tensor representations of the Cremona group of the projective plane	111
Y. Ito and I. Nakamura: Hilbert schemes and simple singularities	151
Oliver Küchle and Andreas Steffens: Bounds for Seshadri constants	235
Marco Manetti: Degenerate double covers of the projective plane	255
David R. Morrison: The geometry underlying mirror symmetry	283
Shigeru Mukai: Duality of polarized K3 surfaces	311
Roberto Paoletti: On symplectic invariants of algebraic varieties coming from crepant contractions	327

٧



# vi Contents

Kapil H. Paranjape: The Bogomolov–Pantev resolution, an expository account	347
Tetsuji Shioda: Mordell–Weil lattices for higher genus fibration over a curve	359
Bernd Siebert: Symplectic Gromov-Witten invariants	375
Claire Voisin: A generic Torelli theorem for the quintic threefold	425
P.M.H. Wilson: Flops, Type III contractions and Gromov–Witten invariants on Calabi–Yau threefolds	465



# **Foreword**

The volume contains a selection of seventeen survey and research articles from the July 1996 Warwick European algebraic geometry conference. These papers give a lively picture of current research trends in algebraic geometry, and between them cover many of the outstanding hot topics in the modern subject. Several of the papers are expository accounts of substantial new areas of advance in mathematics, carefully written to be accessible to the general reader. The book will be of interest to a wide range of students and nonexperts in different areas of mathematics, geometry and physics, and is required reading for all specialists in algebraic geometry.

The European algebraic geometry conference was one of the climactic events of the 1995–96 EPSRC Warwick algebraic geometry symposium, and turned out to be one of the major algebraic geometry events of the 1990s. The scientific committee consisted of A. Beauville (Paris), F. Catanese (Pisa), K. Hulek (Hannover) and C. Peters (Grenoble) representing AGE (Algebraic Geometry in Europe, an EU HCM–TMR network) and N.J. Hitchin (Oxford), J.D.S. Jones and M. Reid (Warwick) representing Warwick and British mathematics. The conference attracted 178 participants from 22 countries and featured 33 lectures from a star-studded cast of speakers, including most of the authors represented in this volume.

The expository papers Five of the articles are expository in intention: among these a beautiful short exposition by Paranjape of the new and very simple approach to the resolution of singularities; a detailed essay by Ito and Nakamura on the ubiquitous ADE classification, centred around simple surface singularities; a discussion by Morrison of the new special Lagrangian approach giving geometric foundations to mirror symmetry; and two deep and informative survey articles by Behrend and Siebert on Gromov–Witten invariants, treated from the contrasting viewpoints of algebraic and symplectic geometry.

Some main overall topics Many of the papers in this volume group around a small number of main research topics. Gromov-Witten invariants



## viii Foreword

was one of the main new breakthroughs in geometry in the 1990s; they can be developed from several different starting points in symplectic or algebraic geometry. The survey of Siebert covers the analytic background to the symplectic point of view, and outlines the proof that the two approaches define the same invariants. Behrend's paper explains the approach in algebraic geometry to the invariants via moduli stacks and the virtual fundamental class, which essentially amounts to a very sophisticated way of doing intersection theory calculations. The papers by Paoletti and Wilson give parallel applications of Gromov–Witten invariants to higher dimensional varieties: Wilson's paper determines the Gromov–Witten invariants that arise from extremal rays of the Mori cone of Calabi–Yau 3-folds, whereas Paoletti proves that Mori extremal rays have nonzero associated Gromov–Witten invariants in many higher dimensional cases. The upshot is that extremal rays arising in algebraic geometry are in fact in many cases invariant in the wider symplectic and topological setting.

Another area of recent spectacular progress in geometry and theoretical physics is Calabi-Yau 3-folds and mirror symmetry. This was another major theme of the EuroConference that is well represented in this volume. The paper by Voisin, which is an extraordinary computational tour-de-force, proves the generic Torelli theorem for the most classical of all Calabi-Yau 3-folds, the quintic hypersurface in  $\mathbb{P}^4$ . The survey by Morrison explains, among other things, the Strominger-Yau-Zaslow special Lagrangian interpretation of mirror symmetry. Beauville's paper gives the first known construction of a Calabi-Yau 3-fold having the quaternion group of order 8 as its fundamental group. The paper by Batyrev proves that the Betti numbers of a Calabi-Yau 3-fold are birationally invariant, using the methods of p-adic integration and the Weil conjectures; the idea of the paper is quite startling at first sight (and not much less so at second sight), but it is an early precursor of Kontsevich's idea of motivic integration, as worked out in papers of Denef and Loeser. Several other papers in this volume (those of Ito and Nakamura, Mukai, Shioda and Wilson) are implicitly or explicitly related to Calabi-Yau 3-folds in one way or another.

Other topics The remaining papers, while not necessarily strictly related in subject matter, include some remarkable achievements that illustrate the breadth and depth of current research in algebraic geometry. Shioda extends his well-known results on the Mordell-Weil lattices of elliptic surfaces to higher genus fibrations, in a paper that will undoubtedly have substantial repercussions in areas as diverse as number theory, classification of surfaces, lattice theory and singularity theory. Faber continues his study of tautological classes on the moduli space of curves and Abelian varieties, and gives an algorithmic treatment of their intersection numbers, that parallels in many



Foreword ix

respects the Schubert calculus; he obtains the best currently known partial results determining the class of the Schottky locus. Gizatullin initiates a fascinating study of representations of the Cremona group of the plane by birational transformations of spaces of plane curves. Eyssidieux gives a study, in terms of Gromov's Kähler hyperbolicity, of universal inequalities holding between the Chern classes of vector bundles over Hermitian symmetric spaces of noncompact type admitting a variation of Hodge structures. Küchle and Steffens' paper contains new twists on the idea of Seshadri constants, a notion of local ampleness arising in recent attempts on the Fujita conjecture; they use in particular an ingenious scaling trick to provide improved criteria for the very ampleness of adjoint line bundles.

Manetti's paper continues his long-term study of surfaces of general type constructed as iterated double covers of  $\mathbb{P}^2$ . He obtains many constructions of families of surfaces, and proves that these give complete connected components of their moduli spaces, provided that certain naturally occuring degenerations of the double covers are included. This idea is used here to establish a bigger-than-polynomial lower bound on the growth of the number of connected components of moduli spaces. In more recent work, he has extended these ideas in a spectacular way to exhibit the first examples of algebraic surfaces that are proved to be diffeomorphic but not deformation equivalent.

The Fourier–Mukai transform is now firmly established as one of the most important new devices in algebraic geometry. The idea, roughly speaking, is that a sufficiently good moduli family of vector bundles (say) on a variety Xinduces a correspondence between X and the moduli space  $\widehat{X}$ . In favourable cases, this correspondence gives an equivalence of categories between coherent sheaves on X and on  $\widehat{X}$  (more precisely, between their derived categories). The model for this theory is provided by the case originally treated by Mukai, when X is an Abelian variety and  $\hat{X}$  its dual; Mukai named the transform by analogy with the classical Fourier transform, which takes functions on a real vector space to functions on its dual. It is believed that, in addition to its many fruitful applications in algebraic geometry proper, this correspondence and its generalisations to other categories of geometry will eventually provide the language for mathematical interpretations of the various "dualities" invented by the physicists, for example, between special Lagrangian geometry on a Calabi-Yau 3-fold and coherent algebraic geometry on its mirror partner (which, as described in Morrison's article, is conjecturally a fine moduli space for special Lagrangian tori). Mukai's magic paper in this volume presents a Fourier-Mukai transform for K3 surfaces, in terms of moduli of semi-rigid sheaves; under some minor numerical assumptions, he establishes the existence of a dual K3 surface, the fact that the Fourier-Mukai transform is an equivalence of derived categories, and the biduality result in appropriate cases.



## x Foreword

The paper of Ito and Nakamura is the longest in the volume; it combines a detailed and wide-ranging expository essay on the ADE classification with an algebraic treatment of the McKay correspondence for the Kleinian quotient singularities  $\mathbb{C}^2/G$  in terms of the G-orbit Hilbert scheme. The contents of their expository section will probably come as a surprise to algebraic geometers, since alongside traditional aspects of simple singularities and their ADE homologues in algebraic groups and representation of quivers, they lay particular emphasis on partition functions in conformal field theory with modular invariance under  $SL(2, \mathbb{Z})$  and on  $II_1$  factors in von Neumann algebras. Their study of the G-Hilbert scheme makes explicit for the first time many aspects of the McKay correspondence relating the exceptional locus of the Kleinian quotient singularities  $\mathbb{C}^2/G$  with the irreducible representations of G; for example, the way in which the points of the minimal resolution can be viewed as defined by polynomial equations in the character spaces of the corresponding irreducible representations, or the significance in algebraic terms of tensoring with the given representation of G. Ito and Nakamura and their coworkers are currently involved in generalising many aspects of the G-orbit Hilbert scheme approach to the resolution of Gorenstein quotient singularities and the McKay correspondence to finite subgroups of  $SL(3,\mathbb{C})$ , and this paper serves as a model for what one hopes to achieve.

Thanks to all our sponsors The principal financial support for the Euro-Conference was a grant of ECU40,000 from EU TMR (Transfer and Mobility of Researchers), contract number ERBFMMACT 950029; we are very grateful for this support, without which the conference could not have taken place. The main funding for the 1995–96 Warwick algebraic geometry symposium was provided by British EPSRC (Engineering and physical sciences research council). Naturally enough, the symposium was one of the principal activities of the Warwick group of AGE (European Union HCM project Algebraic Geometry in Europe, Contract number ERBCHRXCT 940557), and financial support from Warwick AGE and the other groups of AGE was a crucial element in the success of the symposium and the EuroConference. We also benefitted from two visiting fellowships for Nakamura and Klyachko from the Royal Society (the UK Academy of Science). Many other participants were covered by their own research grants.

The University of Warwick, and the Warwick Mathematics Institute also provided substantial financial backing. All aspects of the conference were enhanced by the expert logistic and organisational help provided by the Warwick Math Research Centre's incomparable staff, Elaine Greaves Coelho, Peta McAllister and Hazel Graley.

Klaus Hulek and Miles Reid, November 1998