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Steps in Commutative Algebra
Second edition

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CAMBRIDGE
UNIVERSITY PRESS

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge CB2 2RU, UK
40 West 20th Street, New York, NY 10011–4211, USA
10 Stamford Road, Oakleigh, VIC 3166, Australia
Ruiz de Alarcón 13, 28014 Madrid, Spain
Dock House, The Waterfront, Cape Town 8001, South Africa

http://www.cambridge.org

First published 1990
Reprinted with corrections 1994
Second edition 2000

A catalogue record for this book is available from the British Library

ISBN 0 521 64623 5 paperback

Transferred to digital printing 2004

To the memory of my parents

William Yorke Sharp (27th July 1912 – 2nd June 1998)

and

Dora Sharp (*née* Willis) (25th March 1912 – 23rd May 2000)

Cambridge University Press
0521646235 - Steps in Commutative Algebra, Second Edition
R. Y. Sharp
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Preface to the 1st Edition

Why write another introductory book on commutative algebra? As there are so many good books already available on the subject, that seems to be a very pertinent question.

This book has been written to try to persuade more young people to study commutative algebra by providing 'stepping stones' to help them into the subject. Many of the existing books on commutative algebra, such as M. F. Atiyah's and I. G. Macdonald's [1] and H. Matsumura's [13], require a level of experience and sophistication on the part of the reader which is rather beyond what is achieved nowadays in a mathematics undergraduate degree course at some British universities. This is sad, for students often find some undergraduate topics in ring theory, such as unique factorization in Euclidean domains, attractive, but this undergraduate study does leave something of a gap which needs to be bridged before the student can approach the established books on commutative algebra with confidence. This is an attempt to help to bridge that gap.

For definiteness, I have assumed that the reader's knowledge of commutative ring theory is limited to the contents of the book 'Rings and factorization' [20] by my colleague David Sharpe. Thus the typical reader I have had in mind while writing this book would be either a final year undergraduate or first year postgraduate student at a British university whose appetite for commutative ring theory has been whetted by a course like that provided by [20], but whose experience (apart from some basic linear algebra and vector space theory) does not reach much beyond that. It should be emphasized that, for a reader who has these prerequisites at his or her fingertips, this book is largely self-contained.

Experienced workers in commutative algebra will probably find that the book makes slow progress; but then, the book has not been written for them! For example, as [20] does not work with ideals, this topic is introduced from scratch, and not until Chapter 2; modules are not studied until Chapter 6; there is a digression in Chapter 10 to discuss finitely generated modules over a principal ideal domain, in the hope that this will help to strengthen readers' experience in the techniques introduced earlier in the book; the ideas of Chapter 10 are applied in Chapter 11 to the study of canonical forms for square matrices over fields; and the theory of transcendence degrees of field extensions is developed in Chapter 12, for use in connection with the dimension theory of finitely generated commutative algebras over fields. Otherwise, the topics included are central ones for commutative algebra.

The hope is that a reader who completes this book will feel inspired and encouraged to turn to a more advanced book on commutative algebra, such as H. Matsumura's [13]. It must be emphasized that the present book will not in itself provide complete preparation, because it does not include any introduction to the homological algebra of the functors Ext and Tor, and a good understanding of these is highly desirable for the serious student of commutative algebra. The student will have to turn elsewhere for these, and even for the theory of tensor products. The latter have been avoided in this book because the risk of putting youthful readers off with unnecessary technicalities at an early stage (after all, one can do a lot of commutative algebra without tensor products) seemed to outweigh the advantages that would be gained by having them available.

The reader's attention is drawn to possible further avenues of study, such as tensor products, homological algebra, applications of the Nullstellensatz to algebraic geometry, or even, in Chapter 12, to the Galois theory and ruler and compass constructions which often feature in books on field theory, in items called 'Further Steps' towards the ends of some of the later chapters. No attempt has been made in this book to provide historical background comments, as I do not feel able to add anything to the comments of this type which are already available in the books listed in the Bibliography.

A feature of the book is the large number of exercises included, not only at the ends of chapters, but also throughout the text. These range from the routine checking of easy properties to some quite tricky problems; in some cases, a linked series of problems form almost a 'mini-project', in which a line of development related, but peripheral, to the work of the chapter, is explored. Some of the exercises are needed for the main development later in the book, and these exercises are marked with a symbol '$\sharp$': those so marked which, in my opinion, seemed among the harder ones have been provided with hints. (Exercises which are used later in the book but only in other exercises have not been marked with a '$\sharp$'; also, there are some unmarked exercises which are not particularly easy but have not been provided with hints.) It is hoped that this policy will help the reader to make steady progress through the book without becoming seriously held up on details which are important for the subsequent work. Indeed, I have tried to provide very full and complete arguments for all the proofs presented in the book, in the hope that this will enable the reader to develop his or her own expertise: there are plenty of substantial exercises available for consolidation of that expertise.

The material included (none of which is new or due to me) has been selected to try to give prominence to topics which I found exciting when I was a postgraduate student. In this connection, I would like to record here my gratitude to three British mathematicians who considerably influenced my own development, namely Ian Macdonald, who first excited my interest in commutative algebra (in an Oxford lecture course which was a forerunner to the book [1]), Douglas Northcott, from whose writings I have benefited greatly over the years, and David Rees, whose infectious enthusiasm for local ring theory has often been a source of inspiration to me.

The presentation of the material in the book reflects my experiences in teaching postgraduate students at the University of Sheffield, both through MSc lecture courses and through reading programmes for beginning PhD students, over the past 15 years. Most of the presentation has grown out of MSc lectures gradually refined over many years. Preliminary versions of most of the chapters have been tried out on classes of postgraduate students at Sheffield University during the sessions 1988-89 and 1989-90, and I am grateful to those students for acting as 'guinea pigs', so to speak. I would particularly like to thank Ian Staniforth and Paul Tierney, whose eagle eyes spotted numerous misprints in preliminary versions.

I would also like to thank David Tranah, the Senior Mathematics Editor of Cambridge University Press, for his continual encouragement over many years, without which this book might never have been completed; Chris Martin of Sheffield University Computing Services and my colleague Mike Piff for their patient advice over many months which has helped to make the world of computers less daunting for me than it was at the outset of this project; and my wife, Alice Sharp, not only for many things which have nothing to do with mathematics, but also for casting her professional mathematical copy-editor's eye over preliminary versions of this book and providing very helpful advice on the layout of the material.

Rodney Y. Sharp
Sheffield
April 1990

Cambridge University Press
0521646235 - Steps in Commutative Algebra, Second Edition
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Preface to the 2nd Edition

The decade since the appearance of the first edition of this book has seen the publication of some important books in commutative algebra, such as D. Eisenbud's 'Commutative algebra with a view toward algebraic geometry' [5], which stresses the geometric heritage of the subject, and W. Bruns' and J. Herzog's 'Cohen–Macaulay rings' [2]. There is therefore even more motivation to encourage young people to study commutative algebra, and so, in my opinion, the *raison d'être* for this book – to provide 'stepping stones' to help young people into the subject so that they can go on to study more advanced books with confidence – is as strong as ever.

This second edition contains two new chapters, namely Chapter 16 on 'Regular sequences and grade' and Chapter 17 on 'Cohen–Macaulay rings'. These chapters are just ideal-theoretic introductions to the topics of their titles: a complete treatment of them would involve significant use of homological algebra, and that is beyond the scope of the book. Nevertheless, there are some ideal-theoretic aspects which can be developed very satisfactorily within the framework of the book, and, indeed, which provide good applications of ideas developed in earlier chapters; it is those aspects which receive attention in these new chapters. It is hoped that they will whet the reader's appetite to explore Bruns' and Herzog's [2], a book which provides ample evidence of the importance of the Cohen–Macaulay condition.

I have taken the opportunity to make a few improvements to, and correct a small number of misprints in, the fifteen chapters which formed the first edition. It is again a pleasure for me to record my gratitude to David Tranah and Roger Astley of Cambridge University Press for their continued encouragement and support.

Rodney Y. Sharp
Sheffield
July 2000