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978-0-521-64557-7 - Level Set Methods and Fast Marching Methods: Evolving Interfaces
in Computational Geometry, Fluid Mechanics, Computer Vision, and Materials Science

J. A. Sethian

Table of Contents

[More information](#)

Contents

<i>Preface to the Second Edition</i>	<i>page xi</i>
<i>Introduction</i>	xv
Part I: Equations of Motion for Moving Interfaces	1
1 Formulation of Interface Propagation	3
1.1 A boundary value formulation	4
1.2 An initial value formulation	6
1.3 Advantages of these perspectives	8
1.4 A general framework	10
1.5 A look ahead/A look back	11
1.6 A larger perspective	12
Part II: Theory and Algorithms	15
2 Theory of Curve and Surface Evolution	17
2.1 Fundamental formulation	17
2.2 Total variation: stability and the growth of oscillations	18
2.3 The role of entropy conditions and weak solutions	20
2.4 Effects of curvature	24
3 Viscosity Solutions and Hamilton–Jacobi Equations	29
3.1 Viscosity solutions of Hamilton–Jacobi equations	30
3.2 Some additional comments and references	32
4 Traditional Techniques for Tracking Interfaces	34
4.1 Marker/string methods	34
4.2 Volume-of-fluid techniques	38
4.3 Constructing an approximation to the gradient	41
5 Hyperbolic Conservation Laws	44
5.1 The linear wave equation	44
5.2 The non-linear wave equation	48

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Table of Contents

[More information](#)

viii	<i>Contents</i>	
6	Basic Algorithms for Interface Evolution	60
6.1	Convergence of schemes for Hamilton-Jacobi equations	60
6.2	Hyperbolic schemes and Hamilton-Jacobi equations	61
6.3	The example of a propagating one-dimensional graph	63
6.4	The initial value problem: the Level Set Method	65
6.5	The boundary value problem: the stationary method	68
6.6	Schemes for non-convex speed functions	69
6.7	Approximations to geometric variables	69
6.8	Calculating additional quantities	71
6.9	Initialization	72
6.10	Computational domain boundary conditions	72
6.11	Putting it all together	73
	Part III: Efficiency, Adaptivity, and Extensions	75
7	Efficient Schemes: the Narrow Band Level Set Method	77
7.1	Parallel algorithms	77
7.2	Adaptive mesh refinement	78
7.3	Narrow banding and fast methods	80
7.4	Details of the Narrow Band implementation	84
8	Efficient Schemes: Fast Marching Methods	86
8.1	Iteration	87
8.2	Causality	87
8.3	The update procedure for the Fast Marching Method	90
8.4	Heap sorts and computational efficiency	90
8.5	Initial conditions	92
8.6	Network path algorithms	93
8.7	Optimal orderings	96
8.8	Higher accuracy Fast Marching Methods	96
8.9	Non-uniform orthogonal grids	98
8.10	General static Hamilton–Jacobi equations	99
8.11	Some clarifying comments	99
9	Triangulated Versions of Level Set Methods	101
9.1	Fundamentals and notation	102
9.2	A monotone scheme for $H(\nabla u)$	105
9.3	A positive scheme for homogeneous $H(\nabla u)$	109
9.4	A Petrov-Galerkin formulation	112
9.5	Time integration schemes	113
9.6	Algorithms	114
9.7	Schemes for curvature flow	116
9.8	Mesh adaptivity	118

Cambridge University Press

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in Computational Geometry, Fluid Mechanics, Computer Vision, and Materials Science

J. A. Sethian

Table of Contents

[More information](#)

<i>Contents</i>		ix
10	Triangulated Fast Marching Methods	120
	10.1 The update procedure	120
	10.2 A scheme for a particular triangulated domain	121
	10.3 Fast Marching Methods on triangulated domains	123
11	Constructing Extension Velocities	127
	11.1 The need for extension velocities	127
	11.2 Various approaches to extension velocities	129
	11.3 Equations for extension velocities	131
	11.4 Building extension velocities	133
	11.5 A quick demonstration	137
	11.6 Re-initialization	138
12	Tests of Basic Methods	141
	12.1 The basic Cartesian Level Set Method	141
	12.2 Triangulated Level Set Methods for H–J equations.	146
	12.3 Accuracy of Fast Marching Methods	150
	12.4 Tests of extension velocity methodology	153
13	Building Level Set and Fast Marching Applications	161
Part IV: Applications		165
14	Geometry	167
	14.1 Statement of problem	167
	14.2 Equations of motion	169
	14.3 Results	169
	14.4 Flows under more general metrics	175
	14.5 Volume-preserving flows	175
	14.6 Motion under the second derivative of curvature	177
	14.7 Triple points: variational and diffusion methods	183
15	Grid Generation	191
	15.1 Statement of problem	191
	15.2 Equations of motion	193
	15.3 Results, complications, and future work	196
16	Image Enhancement and Noise Removal	200
	16.1 Statement of problem	200
	16.2 Equations of motion	202
	16.3 Results	208
	16.4 Related work	211
17	Computer Vision: Shape Detection and Recognition	214
	17.1 Shape-from-shading	215
	17.2 Shape detection/recovery	218

Cambridge University Press

978-0-521-64557-7 - Level Set Methods and Fast Marching Methods: Evolving Interfaces
in Computational Geometry, Fluid Mechanics, Computer Vision, and Materials Science

J. A. Sethian

Table of Contents

[More information](#)

x	<i>Contents</i>	
	17.3 Surface evolution and the stereo problem	227
	17.4 Reconstruction of obstacles in inverse problems	229
	17.5 Shape recognition	231
18	Combustion, Solidification, Fluids, and Electromigration	240
	18.1 Combustion	241
	18.2 Crystal growth and dendritic solidification	249
	18.3 Fluid mechanics	255
	18.4 Additional applications	258
	18.5 Void evolution and electromigration	261
19	Computational Geometry and Computer-aided Design	267
	19.1 Shape-Offsetting	267
	19.2 Voronoi diagrams	268
	19.3 Curve flows with constraints	269
	19.4 Minimal surfaces and surfaces of prescribed curvature	270
	19.5 Extensions to surfaces of prescribed curvature	274
	19.6 Boolean operations on shapes	277
	19.7 Extracting and combining two-dimensional shapes	281
	19.8 Shape smoothing	282
20	Optimality and First Arrivals	284
	20.1 Optimal path planning	284
	20.2 Constructing shortest paths on weighted domains	289
	20.3 Constructing shortest paths on manifolds	292
	20.4 Seismic traveltimes	298
	20.5 Aircraft collision avoidance using Level Set Methods	305
	20.6 Visibility evaluations	307
21	Etching and Deposition in Microchip Fabrication	313
	21.1 Physical effects and background	313
	21.2 Equations of motion for etching/deposition	317
	21.3 Additional numerical issues	324
	21.4 Two-dimensional results	325
	21.5 Three-dimensional simulations	342
	21.6 Timings	348
	21.7 Validation with experimental results	349
22	Summary/New Areas/Future Work	357
	<i>Bibliography</i>	360
	<i>Index</i>	376