

## Contents

<i>Synopsis</i>	viii
<i>Preface</i>	xi
<b>1 Introduction</b>	<b>1</b>
1.1 Preview	1
1.2 Probability	1
1.3 The scope of probability	3
1.4 Basic ideas: the classical case	5
1.5 Basic ideas: the general case	10
1.6 Modelling	14
1.7 Mathematical modelling	19
1.8 Modelling probability	21
1.9 Review	22
1.10 Appendix I. Some randomly selected definitions of probability, in random order	22
1.11 Appendix II. Review of sets and functions	24
1.12 Problems	27
<b>A Probability</b>	
<b>2 The rules of probability</b>	<b>31</b>
2.1 Preview	31
2.2 Notation and experiments	31
2.3 Events	34
2.4 Probability; elementary calculations	37
2.5 The addition rules	41
2.6 Simple consequences	44
2.7 Conditional probability; multiplication rule	47
2.8 The partition rule and Bayes' rule	54
2.9 Independence and the product rule	58
2.10 Trees and graphs	66
2.11 Worked examples	72
2.12 Odds	78

vi	<i>Contents</i>	
	2.13 Popular paradoxes	81
	2.14 Review: notation and rules	86
	2.15 Appendix. Difference equations	88
	2.16 Problems	89
<b>3</b>	<b>Counting and gambling</b>	<b>93</b>
	3.1 Preview	93
	3.2 First principles	93
	3.3 Arranging and choosing	97
	3.4 Binomial coefficients and Pascal's triangle	101
	3.5 Choice and chance	104
	3.6 Applications to lotteries	109
	3.7 The problem of the points	113
	3.8 The gambler's ruin problem	116
	3.9 Some classic problems	118
	3.10 Stirling's formula	121
	3.11 Review	123
	3.12 Appendix. Series and sums	124
	3.13 Problems	126
<b>4</b>	<b>Distributions: trials, samples, and approximation</b>	<b>129</b>
	4.1 Preview	129
	4.2 Introduction; simple examples	129
	4.3 Waiting; geometric distributions	136
	4.4 The binomial distribution and some relatives	139
	4.5 Sampling	144
	4.6 Location and dispersion	147
	4.7 Approximations: a first look	154
	4.8 Sparse sampling; the Poisson distribution	156
	4.9 Continuous approximations	158
	4.10 Binomial distributions and the normal approximation	163
	4.11 Density	169
	4.12 Distributions in the plane	172
	4.13 Review	174
	4.14 Appendix. Calculus	176
	4.15 Appendix. Sketch proof of the normal limit theorem	178
	4.16 Problems	180
<b>B Random Variables</b>		
<b>5</b>	<b>Random variables and their distributions</b>	<b>189</b>
	5.1 Preview	189
	5.2 Introduction to random variables	189
	5.3 Discrete random variables	194

<i>Contents</i>		vii
5.4	Continuous random variables; density	198
5.5	Functions of a continuous random variable	204
5.6	Expectation	207
5.7	Functions and moments	212
5.8	Conditional distributions	218
5.9	Conditional density	225
5.10	Review	229
5.11	Appendix. Double integrals	232
5.12	Problems	233
<b>6</b>	<b>Jointly distributed random variables</b>	<b>238</b>
6.1	Preview	238
6.2	Joint distributions	238
6.3	Joint density	245
6.4	Independence	250
6.5	Functions	254
6.6	Sums of random variables	260
6.7	Expectation; the method of indicators	267
6.8	Independence and covariance	273
6.9	Conditioning and dependence, discrete case	280
6.10	Conditioning and dependence, continuous case	286
6.11	Applications of conditional expectation	291
6.12	Bivariate normal density	294
6.13	Change-of-variables technique; order statistics	298
6.14	Review	301
6.15	Problems	302
<b>7</b>	<b>Generating functions</b>	<b>309</b>
7.1	Preview	309
7.2	Introduction	309
7.3	Examples of generating functions	312
7.4	Applications of generating functions	315
7.5	Random sums and branching processes	319
7.6	Central limit theorem	323
7.7	Random walks and other diversions	324
7.8	Review	329
7.9	Appendix. Tables of generating functions	329
7.10	Problems	330
	<i>Hints and solutions for selected exercises and problems</i>	336
	<i>Index</i>	365