CHAPTER 1

MARKETS AND GAMES

1.1 Strategic foundations of perfect competition

In these lectures I report on a research program that began in the early 1980s. It is part of a larger effort, underway for a much longer time, to provide strategic foundations for the theory of perfect competition.\(^1\) The theory of competition has held a central place in economic analysis since the time of Adam Smith [1776]. By providing strategic foundations for the theory of competition, economists use the principles of game theory to motivate or justify a macroscopic description of markets in which certain behavioral characteristics, such as price-taking behavior, are taken for granted. Game theory begins with individual agents and models their strategic interaction. A strategic foundation for competitive equilibrium must show how strategic interaction by rational agents leads to competitive, price-taking behavior. In practice, this research program includes the following three steps:

1. First, describe a market or a whole economy.
   In this step, the economist has to specify the commodities traded, the agents (households and firms) that make up the market or economy, their preferences, their resources, and the available technology.

2. Secondly, define an extensive-form market game describing the behavior of the agents in the market or economy.

\(^1\) When the context makes the meaning reasonably clear, I adopt the usual practice of writing \textit{competition} or \textit{competitive} when I really mean \textit{perfect competition} and \textit{perfectly competitive}.
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In this step, the economist has to specify the players, the information available to each player, the strategies available to them, the outcomes resulting from their choices, and the payoffs received.

3. Thirdly, analyze the market game to show that, under certain conditions, the equilibrium outcome corresponds to a perfectly competitive equilibrium of the original market or economy.

There are many ways in which this program can be carried out. I shall be discussing one class of models which I find particularly interesting and fruitful, the class of dynamic matching and bargaining games (DMBG). Before getting into the details, however, I want to discuss the motivation for this kind of undertaking.

1.2 Why strategic foundations?

The first question that ought to occur to anyone encountering this kind of work for the first time is “Why?” Why should anyone want to take the time to build strategic foundations for the theory of perfect competition? After all, the theory of competitive equilibrium is well defined and self-contained in its own right. The cornerstones of this theory were laid over a hundred years ago by Alfred Marshall2 and Léon Walras.3 The modern axiomatic theory developed by Kenneth Arrow, Gérard Debreu, Lionel McKenzie, and their successors is one of the most complete and definitive constructions that economics has to offer.4 Instead of pursuing steps 2 and 3, Arrow, Debreu, McKenzie, et al., simply assume that households maxi-

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mize utility subject to a budget constraint, that firms maximize profits subject to their production technology, and that prices adjust to clear markets. Why do we need more than this? There are three reasons.

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The first reason is provided by the rise of game theory. Since the foundations of general equilibrium theory were laid in the nineteenth century, game theory has assumed an increasingly central role in economics. Game theory, as defined by von Neumann and Morgenstern, offers a general and powerful framework with which to analyze interactive decision-making. To the extent that we accept the claim of game theory to be the correct framework for analyzing decision-making by individuals, we should want to use it as a tool for analyzing the behavior of agents in markets. Unfortunately, models of competitive equilibrium are not games in a strict, formal sense. In particular, games have two attractive features that models of market equilibrium do not have:

- In a strategic game, all the endogenous variables are chosen by players in the game.
- In a strategic game, any profile of strategies chosen by the players determines a unique feasible outcome of the game.

A long recognized and embarrassing lacuna in the theory of competitive equilibrium is the failure to explain where the prices come from. Sometimes we are reduced to saying that prices are chosen by an “auctioneer,” but essentially they are free parameters that are “determined,” along with the other endogenous variables, by the market-clearing condition. In other words, unlike strategic games, models of market equilibrium have endogenous variables that are not chosen by the agents.

Another weakness of the classical model of competitive equilibrium is the assumption that agents believe that they can buy and sell as much as they like at the prevailing prices. It is true that agents buy and sell as much as they
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want in equilibrium, but if an agent deviates from his equilibrium excess demand, he will find that the assumption is violated. Except in the case where agents are literally negligible, any deviation leads to an infeasible set of excess demands. As a result, the market cannot clear and some agent will be disappointed in his attempt to trade. In the language of game theory, there are some strategy profiles to which no feasible outcome corresponds (unless we abandon the assumption that agents can trade as much as they want).

So, by comparison with the framework of game theory, the model of competitive equilibrium has some loose ends when it comes to the treatment of prices and the feasibility of trades.

These loose ends do not mean that the model of competitive equilibrium is a “bad model.” On the contrary, it can be regarded as a reduced form of a more complicated model or process that describes the final outcome without giving all the details. One of the advantages of building a strategic foundation for perfect competition is that we will be forced to describe the process completely and explain how the competitive equilibrium outcome is reached. The complete model will, naturally, be an extensive-form game.

When is perfect competition appropriate?

A second reason for wanting a strategic foundation of competitive equilibrium is to provide a theoretical rationale for perfect competition. A formal statement of the theory may be aesthetically pleasing, but it does not tell us why or under what circumstances the theory is appropriate. For example, we can define a perfectly competitive equilibrium for an Edgeworth Box economy with two goods and two agents, even though perfect competition is unlikely to be

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5 Ostrov (1980) and Makowski (1983) have characterized competitive equilibrium in terms of the no-surplus condition. They have argued that finite economies such as the Edgeworth Box economy sometimes satisfy the no-surplus condition and hence are perfectly competitive. However, the finite economies that satisfy the no-surplus condition are very special, and might be considered pathological.
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achieved in a market consisting of only two agents. There is nothing in the formal definition to tell us whether the model applies to economies consisting of two, or a hundred, or a million agents.

One advantage of providing strategic foundations for perfect competition is that we are forced to construct a game in which perfect competition is only one of many possible outcomes. Then we show that under certain conditions competition arises as the unique equilibrium outcome. Deriving perfect competition from a more general framework provides us with a rationale for the competitive equilibrium: it shows under what conditions the model of competitive equilibrium is a good description of rational interaction among economic agents.

This aspect of the research program is illustrated by the distinction between limit theorems and theorems in the limit. Perfect competition is an idealized state, one which is only more or less imperfectly approximated by reality and then only under certain special conditions. It is a non-trivial task to decide when the theory is a reasonable approximation. Some markets may be approximately perfectly competitive, others are not. How do we know which is which? Consider a sequence of economies, in which the number of agents is growing without bound. In the limit there is a continuum of individually insignificant agents. At what point does the economy become “competitive?” Theorems in the limit characterize the exact conditions under which a perfectly competitive outcome may be expected to occur, whereas limit theorems tell us that, as we approach those conditions, the observed outcome will approach the competitive outcome.

By embedding different accounts of equilibrium behavior in a single theoretical framework, we are able to distinguish and classify the conditions under which different forms of competition arise. So another use for strategic foundations of perfect competition is to understand better the conditions under which perfect competition is an appropriate description of market behavior.
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Normative economics

Still another use for strategic foundations of competition comes from the role of the competitive equilibrium as a normative ideal. The classical theorems of welfare economics tell us that, under certain conditions, every perfectly competitive equilibrium allocation is Pareto-efficient and every Pareto-efficient allocation can be decentralized as a perfectly competitive equilibrium with lump-sum transfers. But how is perfect competition brought about? What practical institutions will achieve the desired outcome? In order to use the theory of perfect competition as a practical tool for achieving the efficient allocation of resources, we need more than a definition of a perfectly competitive outcome. We also need a theory of the institutions and conditions under which perfect competition may arise as the result of interactive decision-making by rational agents. In other words, we need a theory of the strategic foundations of perfect competition. Because game-theoretic models describe more completely the institutions that underly the market, they give us insight into the reasons why perfect competition may or may not be achieved. This knowledge may suggest policies to increase the degree of competition and, in the limit, achieve perfect competition. Alternatively, it may convince us that perfect competition is not the optimum in certain circumstances.

1.3 Cooperative market games

Attempts to provide a game-theoretic foundation for competition are almost as old as the theory of competition itself. In Mathematical Psychics, Francis Ysidro Edgeworth (1881) addresses a broad theme, the applicability of mathematics in the social sciences. In particular, he addresses the question of whether the behavior of individuals is “determinate,” by which he means “predictable using mathematical models.” He argues that social processes, which might be indeterminate when small numbers of economic agents are involved, became determinate, and
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hence subject to mathematical analysis, when the number of agents is large. As an example, Edgeworth studies trade between two agents in a setting we have since come to call an Edgeworth Box economy. He argues that the outcome of such trade is indeterminate, because only efficiency and individual rationality can be counted on. But as the number of traders increases, the outcome of trade becomes increasingly determinate. The possibilities for recontracting among a large number of agents restrict the possible outcomes and, in the limit, allow for only competitive outcomes.

In *Mathematical Psychics*, Edgeworth follows the three steps of the program listed above. He describes an economy, represents the behavior of the agents by a coalition-forming game and shows that under certain conditions the solution of the game corresponds to a perfectly competitive equilibrium outcome. In modern terminology, he describes a model of an exchange economy consisting of a finite number of economic agents. Each agent has an initial endowment of the commodities available in the economy, a consumption set that specifies the possible commodity bundles the agent can consume, and preferences over his consumption set. The allocation of resources is undertaken by coalitions of agents. Formally, a coalition is any non-empty set of agents. An allocation is attainable for a coalition if the commodity bundle assigned to each agent belongs to his consumption set and the bundles add up to the coalition’s total endowment. A coalition can improve on an attainable allocation if there exists an allocation that is attainable for the coalition and makes every member of the coalition better off. Edgeworth introduces the concept of the *contract curve* to describe the outcomes of the recontracting process. The contract curve is what we now call the *core* of the market game. It consists of the set of all attainable allocations that cannot be improved upon by any coalition. Edgeworth shows that as the number of agents grows unboundedly large, the core of the market game shrinks until it contains only the perfectly competitive equilibrium allocations.
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This profound result, which stands virtually alone in nineteenth-century contributions to economic theory for its depth and beauty, was re-discovered by Shubik [1959] who showed the equivalence of the core and the contract curve. For the next fifteen years the theory was extended and refined until it reached a more or less definitive state with the publication of Hildenbrand [1974].

Now we have theorems that show that the Shapley value (Aumann and Shapley, 1974), the bargaining set (Mas-Colell, 1989), the set of fair trades (Schmeidler and Vind, 1972), and other solution concepts also shrink to the set of Walras allocations as the number of players becomes very large. The fact that a variety of different solution concepts lead to the same result is a strong argument for the robustness of the competitive equilibrium.

All of these attempts to provide strategic foundations for perfect competition make use of cooperative game theory.\(^6\) The cooperative approach to game theory has its limitations. Some of these are peculiar to particular cooperative solution concepts such as the core. Others are common to cooperative games in general.

At the general level, one of the attractions of cooperative game theory is that it provides a criterion for strategic stability that leads directly to a solution of the game, without the tedious business of specifying an extensive-form game. There is no need to specify the strategy set of each player, the order of moves, the information sets, or the players’ assumptions about their opponents’ behavior. All one needs is a convenient definition of what counts as a plausible outcome of the game.

In particular, there is no need to specify a well defined maximization problem for each individual player. However, this could also be considered a weakness. As

\(^6\) For present purposes we can think of a cooperative game as being defined by a set of players, a specification of the coalitions or non-empty subsets of players that can form, and a specification of the actions or payoffs that can be achieved by each coalition of players. A solution of the game is a set of profiles of actions or payoffs that satisfies some plausible criterion of stability.
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John Hicks remarked in his “A Suggestion for Simplifying Monetary Theory” [reprinted in Hicks, 1967], anything seems to go when no one is required to maximize anything. A well defined maximization problem for each agent is one of the characteristic building blocks of modern economics. Cooperative game theory lacks the discipline that comes from having to specify a maximization problem for each agent. As a result, it begs a number of questions.

This can be seen when we look at particular applications of cooperative game theory to market games. Take the core, for example. The formal definition of the core provides a criterion for stability, not a description of the process of coalition formation. An allocation belongs to the core if no coalition can improve on it. The definition suggests that if an allocation does not belong to the core it could never be an “equilibrium” because the improving coalition, whose existence is guaranteed by the definition, would do something to prevent it. Why the improving coalition should do that is not clear. Indeed, it is hard to think about this question without knowing how an allocation comes to be chosen in the first place, but suppose, for the sake of argument, that a non-core allocation has somehow occurred. It is easy to find allocations that (1) do not belong to the core, (2) make some agents better off than they would be in any core allocation, and (3) can be improved on only by coalitions which include the agents who will be worse off at any core allocation. In this case, the improving coalition would be very myopic to veto this allocation if it really believed in the core as a solution concept, because it will certainly be worse off when a core allocation is finally reached. It appears that the core concept requires agents to behave myopically, rushing to join improving coalitions so that they can cut their own throats.

More sophisticated cooperative solution concepts try to eliminate such myopia, but the cooperative framework itself is the obstacle to a consistent theory, because it does not provide each agent with a well defined maximization problem. Without an extensive-form game, many of these problems can never be resolved satisfactorily.
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For this reason, Nash [1951] proposed that cooperative games should be reduced to non-cooperative games. He interpreted a cooperative game as one in which there is unlimited pre-play communication and binding agreements can be entered into before the game is played. A non-cooperative game is one without pre-play communication or agreements. [This classification is clearly not exhaustive and there are other features that make it less than satisfactory, but the terminology has stuck so I will continue to use it.] The communication and commitment that precede the play of a cooperative game are not explicitly modeled, but they should be regarded as part of the game and analyzed using the same principles as the formal game itself. To do this, we model explicitly the pre-play communication and commitments, and then analyze the behavior of players in this game in the same way as if they were playing a non-cooperative game. This procedure, of reducing the cooperative game to a non-cooperative game by making explicit the informal parts of the cooperative theory, is known as the Nash Program.

The Nash Program is something that should appeal to economists, because it adopts Hicks' principle that every agent should be given a well defined maximization problem to solve. The use of non-cooperative game theory to provide a strategic foundation for competition is a natural extension of the earlier use of cooperative game theory. Ultimately, a satisfactory foundation for competitive markets requires a non-cooperative [extensive-form] game. This is what I shall be doing here, using non-cooperative game theory to provide a more complete description of what goes on in markets and then deriving the familiar competitive outcomes as an implication of particular conditions and assumptions.

1.4 Non-cooperative market games

The first non-cooperative approach to competition antedates the core by about fifty years. It began with the analysis of duopoly by Antoine Augustin Cournot [1838, 1960]. Cournot analyzes the problem of the owners of