Representations of Reductive Groups

The representation theory of reductive algebraic groups and related finite reductive groups is a subject of great topical interest and has many applications. The articles in this volume provide introductions to various aspects of the subject, including algebraic groups and Lie algebras, reflection groups, abelian and derived categories, the Deligne–Lusztig representation theory of finite reductive groups, Harish-Chandra theory and its generalisations, quantum groups, subgroup structure of algebraic groups, intersection cohomology, and Lusztig's conjectured character formula for irreducible representations in prime characteristic.

The articles are carefully designed to reinforce one another, and are written by a team of distinguished authors: M. Broué, R. W. Carter, S. Donkin, M. Geck, J. C. Jantzen, B. Keller, M. W. Liebeck, G. Malle, J. C. Rickard, and R. Rouquier. This volume as a whole should provide a very accessible introduction to an important, though technical, subject.

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The Isaac Newton Institute of Mathematical Sciences of the University of Cambridge exists to stimulate research in all branches of the mathematical sciences, including pure mathematics, statistics, applied mathematics, theoretical physics, theoretical computer science, mathematical biology and economics. The four six-month long research programmes it runs each year bring together leading mathematical scientists from all over the world to exchange ideas through seminars, teaching and informal interaction.

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Contents

Preface

This volume gives an account of the representation theory of reductive algebraic groups over algebraically closed fields and over finite fields. It contains carefully coordinated chapters written by 9 leading workers in the area of algebraic groups.

The volume begins with an article by R.W. Carter introducing the basic concepts in the theory of linear algebraic groups. This includes the properties of well known subgroups such as maximal tori, Borel subgroups and parabolic subgroups, and a description of the classification of the simple algebraic groups by means of root systems and Dynkin diagrams.

There is a class of abstract groups, the Coxeter groups, which play a key role in the theory of algebraic groups. An article by R. Rouquier discusses the properties of Coxeter groups in general, and also the particular Coxeter groups such as Weyl groups and affine Weyl groups which appear in the theory of algebraic groups.

Various concepts from homological algebra are frequently used in the representation theory of algebraic groups. A chapter by B. Keller introduces these concepts, including abelian categories, derived categories and triangulated categories.

Finite reductive groups are defined as fixed point sets of reductive algebraic group under a Frobenius map. The representation theory in characteristic 0 of these groups was developed by Deligne and Lusztig. An article by M. Geck explains the basic properties of Frobenius maps and expounds the Deligne-Lusztig theory, including a parametrization of all irreducible representations of finite reductive groups.

An elegant theory has been developed by Broué, Malle and Michel in recent years in the context of the modular representation theory of finite reductive groups in the cross characteristic case. There is an exposition of this theory, known as the d–Harish Chandra or generalized Harish–Chandra theory, by M. Broué and G. Malle.

There is also an article by J.C. Jantzen giving an introduction to quantum groups. Quantum groups are now proving useful in many varied branches of mathematics, in particular the modular representation theory of reductive groups in the equal characteristic case. Jantzen explains the main ideas, beginning with quantum sl_2 and leading up to the properties of the canonical basis.

There is an intimate connection between the representation theory of algebraic groups and the subgroup structure of such groups. An article by M.W. Liebeck discusses the subgroup structure of simple algebraic groups both over algebraically closed fields and over finite fields.

Many of the deeper results on representations of algebraic groups, particularly those of G. Lusztig, and various positivity results on Hecke algebras

and canonical bases of quantum groups, use methods of intersection cohomology. A chapter by J.C. Rickard provides an introduction to the subject of intersection cohomology.

Properties of intersection cohomology are used in the final article by S. Donkin. There is at present no known character formula for the irreducible modules for reductive groups over fields of prime characteristic. However, there is a conjectural formula due to G. Lusztig, which is known to hold in certain situations. Donkin describes Lusztig's conjectured character formula and what is at present known about it.