

## Index

- Abelian group 2
- acoustic mode 393
- active representation 23
- adiabatic potential 173
- adjoint 54
  - of an operator 102
- adjoint matrix 418
- algebra of turns 228
- ambivalent class 435
- angular momentum 184, 189
- anharmonicity 160
  - constant of 160
- antibonding orbitals 106, 125
- antiferromagnetic crystal 265
- antilinear operator 252
- antipole 223
- antisymmetrical direct product 100
- antisymmetrizing operator 141
- antiunitary operator 252, 267, 405
- associated Legendre functions 194
- associative 2, 220
- axial groups 82
- axial tensor 283
- axial vector 82
  
- basic domain 331
- basis 53, 96
  - of a lattice 308
- basis functions, construction of 97
- bcc* *see* body-centred cubic
- benzene 104, 109, 174
- Bethe 80, 150, 151
- bilateral binary (BB) 232
- binary composition 1, 15, 70
- binary rotation 25
- Bloch functions 317, 357
- block-diagonal structure 404
- body-centred cubic (*bcc*) 309
- bonding orbitals 106, 125
- Born and von Kármán boundary conditions 316
- Born–Oppenheimer approximation 173
- bra 102
- Bragg reflection 358
- Bravais lattice 311, 318
- Brillouin zone 327, 329, 358, 397
- BSW notation 361, 362, 370
  
- c*-tensors 303
- Cartan gauge 204, 210, 240, 241, 242
- Cartesian tensors 360
- Cayley–Klein parameters 202, 243, 351
- celebrated theorem 79
  
- central extension 336, 337, 367
- centralizer 14, 19, 434
- centre 19
- character 74, 99
- character system 74
- character tables 76–78, 80, 447
- character vector 259
- characteristic equation 420, 441
- charge overlap 107
- charge transfer 178
- chemical bond 106
- class 5, 19
- class algebra 439
- class constants 436
- class property 440
- Clebsch–Gordan series 209, 277, 385
- closed shell 172
- closo*  $B_n H_n^{-2}$  51
- closure 1, 393
- co-factor 413
- coincidence 162
- column matrix 415
- combination bands 160
- commutation relations (CRs) 131, 187
- compatibility relations 362
- complementary IR 303
- complementary minor 413
- complementary operators 265
- complex conjugate 218
- complex conjugation operator 253
- complex number 218
- complex plane 219
- complex quaternion parameters 244
- component (of a vector) 57
- Condon and Shortley (CS) choice of phase 190
- conical transformation 195
- conjugate 18
- conjugate bases 292
- conjugate elements 5
- continuous groups 182
- conventional unit cells 309
- co-representation 257, 267, 269–273
- correlation tables 467
- corresponding elements 43
- coset 7
  - expansion 318
  - representatives 7
- coupled representation 210
- covering group 336, 337
- crystal classes 311
- crystal field
  - intermediate 134

## 482 Index

- crystal field (cont.)  
   strong 139  
   weak 152  
 crystal pattern 307  
 crystallographic orbit 321  
 crystallographic point groups 45, 46, 310  
 crystals (physical properties of) 282  
 cubic groups 244  
 cyclic group 3, 36, 86, 243  
 cyclobutadiene 130  
  
 degeneracy index 290  
 degenerate mode 161  
 delocalization energy 113  
 descending symmetry 140  
 determinants 413  
 diagonal matrix 420  
 diagonalization of the Dirac characters 440  
 diamond 378  
 dibenzene chromium 50  
 dihedral groups 36  
 dihedral planes 39, 41  
 dimension (of a representation) 70, 74  
 dimensionality (of a vector space) 53  
 dipole moment operator 159  
 Dirac character 14, 20, 434  
 Dirac notation 101, 102, 132  
 direct product  
   of groups 8, 12, 39  
   of matrices 99, 432  
   of representations 99  
   of sets 99  
 direct sum 72, 424  
 dispersion relation 393, 394  
 displacement vector 163  
 displacement vector space 162  
 double cosets 385  
 double group 82, 148, 149, 195, 248  
 dynamical matrix 392, 398  
  
 $E_1$  (electric dipole) transition 104, 171  
 $E_2$  (electric quadrupole) transition 104, 171  
 eigenvalues 393, 420  
 eigenvector 97, 99, 420  
 Einstein summation convention 186, 282  
 elastic constants (third order) 296  
 elastic stiffness 286  
 electrical conductivity 298  
 electrochemical potential 297  
 electron spin 131  
 empty lattice approximation 366  
 energy bands 357, 360, 371  
 entropy production 288  
 equivalent matrices 420  
 equivalent points 327  
 equivalent positions 320  
 equivalent representations 72, 259  
 equivalent wave vector 331  
 Euler angles 205  
 Euler construction 223  
 Euler–Rodrigues parameters 230  
 Euler’s formula 219  
 extended zone 359  
  
 extension 320  
  
 face-centred cubic (*fcc*) 308  
 factor group 8, 12, 319, 407  
 factor system 234  
 faithful representation 58  
 ferrimagnetic crystal 265  
 ferrocene 50  
 ferromagnetic crystal 265  
 ferromagnetism 304  
*fcc* *see* face-centred cubic  
 fibre 13  
 fine structure constant 133, 173  
 flux 288  
 force constants 391  
 free-electron approximation 357, 366  
 frequency 92, 385  
 Frobenius reciprocity theorem 93  
 Frobenius–Schur test 261, 273, 405  
 function operator 63, 183, 316  
 function space 97, 104  
 fundamental theorem (Nowick) 290  
 fundamental theorem (Onsager) 288  
 fundamental transition 159  
 fundamental translations 307  
  
 galvanomagnetic effects 299  
 geometry of rotations 222  
 germanium 378, 406  
 glide plane 318  
 ground representation 88  
 group 1  
   of the Hamiltonian 68, 96  
   of the Schrödinger equation 68  
   of the wave vector 367  
 group generators 3  
 group representation 62  
 gyration tensor 294  
  
 Hall tensor 302  
 halving subgroup 265  
 Hamiltonian 133  
   invariant under  $R$  67  
 harmonic approximation 160, 391  
 Hermitian matrix 421  
 Hermitian scalar product 54  
 Herring factor group 344  
 Herring group 344, 367  
 Herring multiplication rule 345  
 Herring’s method 335, 344  
 holosymmetric space group 331  
 homomorphism 208  
 homomorphous group 14  
 Hückel 113  
 Hund’s rules 134, 144  
 hybridization 106, 116  
  
*i*-tensors 303  
 icosahedral point group 37, 244  
 identity 2, 28, 223  
 identity representation 70  
 image 13, 60  
 indicatrix 284

- induced representation 90
- improper axis 28
- improper rotation 26, 282
- indices 7, 309
- indistinguishability 3
- indium antimonide 384
- induced representation 88, 93
- infinitesimal generator 183, 189
- infinitesimal rotation 83, 284
- inner direct product 16
- integral invariance 196
- International notation 28, 36, 267
- intertwining matrix 427
- intertwining number 93
- invariant subgroup 7, 8
- inverse 2
- inverse class 22, 434
- inversion operator 58
- irreducibility criterion 92
- irreducible representation (IR) 73, 243
- irreducible volume 397
- irregular operations 233, 243
- irreversible processes 288
- isomorphous group 2, 42
  
- Jahn–Teller effect 175
- Jones symbol 58
  
- kernel 17, 336
- Kerr effect 296
- ket 102
- Kramers' theorem 151, 256
  
- LA mode 409
- LO mode 410
- Lagrange's theorem 21
- Lagrangian strain 296
- Laporte rule 171
- LCAO approximation 109
- left coset 88
- length (of a vector) 55
- Levi–Civita three-index symbol 185
- linear operator 252
- linear response 288
- linear vector space 53
- little co-group 327, 333
- little factor group 332
- little group 332, 367
- lowering operator 132
  
- $M_1$  (magnetic dipole) transition 104, 171
- magnetic point groups 265, 303
  - crystal-field theory for 280
- magnetoelectric polarizability 304
- many-electron atom 133
- mapping 13, 60
- matrices 415
  - special 418, 420
- matrix element 102, 103
- matrix multiplication 415
- matrix representation 53, 70, 424
- matrix representative 56, 57, 415
- metric 55, 311
  
- Miller indices 309, 328
- mixing coefficient 115
- modulus 219
- molecular orbital 107, 115
- molecular point groups 48
- Morse potential 160
- Mulliken 81
- Mulliken–Herzberg notation 151
- multiplet 133
- multiplication table 1, 34
- multiplier representation 400
  
- negative hemisphere 223
- negative rotation 24
- Nernst tensor 301
- Neumann's principle 282, 288
- non-symmorphic space group 318, 344, 367, 378
- norm 221
- normal matrix 420
- normal mode coordinates 156, 162, 163, 164
- normal modes 156
  - symmetry of 156
- normalization 112
- normalized basis 285
- normalized vector 55
- normalizer 19
  
- $O(3)$  203, 208, 240
- occupation number representation 159
- octahedral complex 117, 174
- octahedral point group 37
- Onsager 288
- Onsager reciprocal relations (ORR) 288, 298
- Opechowski's rules 149
- optic mode 393
- optical activity 294
- optical energies 178
- orbital approximation 133
- order
  - of a class 5, 21
  - of a group 2
  - of an axis 23
- orthogonal group  $O(3)$  *see*  $O(3)$
- orthogonal matrix 61, 421
- orthogonality theorem 73, 425, 428, 430
  - for the characters 76, 77, 195
- orthonormal basis 55
- orthonormal eigenvectors 393
- outer direct product 15
- overlap integral 112
- overtone 161
  
- $\pi$  bond 106, 126
- $\pi$  electron systems 109, 113
- parity 136, 164, 167, 209
- parity selection rule 171, 174
- passive representation 23
- Pauli exclusion principle 133, 140
- Pauli gauge 204, 211, 240, 242, 243
- Pauli matrices 200
- Pauli repulsion 145
- Peltier effect 298
- pentagonal dodecahedron 37

## 484 Index

- periodic boundary conditions (PBCs) 316, 326, 357, 366, 397  
 periodicity (of the reciprocal lattice) 394  
 permutation 3  
 permutation matrix 88, 419  
 permutation representation 372, 375  
 phase factor 67  
 phenomenological coefficients 288  
   relations 288  
 piezomagnetic effect 305  
 plane waves 392  
 Pockels effect 296  
 point group 30, 48  
   of a space group 318  
   of the wave vector 327, 332, 360, 407  
 point group generators 286  
 point subgroup 317  
 point symmetry operations 28  
 polar vector 26  
 polarizability 161  
 pole (of a rotation) 222  
 pole conventions 245  
 poles (choice of) 223  
 positive hemisphere 222  
 positive rotation 24  
 primitive lattice 308  
 principal axis transformation 284  
 principal minor 423  
 projection (of a vector) 56  
 projection diagram 27  
 projection operator 98, 366, 368, 403, 408  
 projective factor 233, 274  
 projective representation 67, 195, 218, 233, 234, 333, 335, 400  
 prongs (of a star) 385, 386  
 proper point group 36  
 proper rotations 282  
 properties of the characters 74  
 pseudoscalar 26, 211, 282  
 pseudovector 26, 82, 220  
  
 quaternion 220  
 quaternion conjugate 221  
 quaternion group 226, 443  
 quaternion parameters 230, 351  
 quaternion units 220  
  
 raising operator 132  
 Raman scattering 161  
 range (of  $\phi$ ) 23  
 rearrangement theorem 1  
 reciprocal lattice 324  
 reduced zone 359  
 reduction (of a representation) 78  
 reflection 59  
 regular classes 233, 384  
 regular operation 233  
 regular representation 79  
 repeat index 290  
 representation domain 332  
 required representations 345, 346  
 right-handed axes 23  
 Rodrigues 225  
  
 rotation 23  
 rotation operator 23  
 rotation parameter 223  
 rotational motion 156  
 rotational symmetry 310  
 rotations  
   in  $\mathbb{R}^2$  182  
   in  $\mathbb{R}^3$  184  
 rotoreflection axis 28  
 rotoreflection operator 27  
 row matrix 415  
 Russell–Saunders coupling 132, 133  
 Russell–Saunders multiplets 148, 152  
  
 $\sigma$  bond 106  
*sc* see simple cubic lattice  
 scalar 209, 282  
 scalar product 101, 102  
 Schmidt orthogonalization 112  
 Schönflies notation 28, 80, 267  
 Schur's lemma 259, 270, 291, 425, 426  
 screw rotation 318  
 Seebeck effect 298  
 Seitz operator 314  
 semidirect product 13  
 setting 322  
 shell model 411  
 shift operators 188  
 Shubnikov 265  
 silicon 378, 384, 406  
 similarity of orientation 289  
 similarity transformation 72, 416, 420  
 simple cubic (*sc*) lattice 368  
 singlet state 141  
 singular matrix 416  
 site symmetry 321  
 space group 314  
 space group representations 331, 336, 339  
 space group symmetry 394  
 space lattice 307  
 special orthogonal groups  
   SO(2) 182, 184  
   SO(3) 61, 182, 184, 192, 203, 208, 231  
 special orthogonal (SO) matrices 61  
 special unitary groups  
   SU(2) 200, 202, 208  
   SU'(2) 203  
 spectral term 133  
 spherical harmonics 193  
 spherical vector 194  
 spin eigenvector 132, 133  
 spin–orbit coupling 104, 133, 148, 173, 281  
 spin orbital 103  
 spin pairing 145  
 spin postulate 131  
 spin quantum number 131  
 spin selection rule 103, 171  
 spinor 209  
 spinor representation 82, 149, 232, 236, 237  
 standard parameters 235  
 standard representation 246  
 standardization 240  
 star 332, 333

- stereographic projection 213  
Stern–Gerlach experiment 131  
subduced representation 93  
subduction 93, 138, 223  
subgroup 6  
sum rule 391  
symmetric group 4, 5  
symmetric tensor 284  
symmetrical direct product 100  
symmetrizing operator 141  
symmetry coordinates 289, 401, 403  
symmetry element 27, 28  
symmetry groups  
  lower 294  
  upper 294  
symmetry operations 23, 26  
symmorphic space group 318, 333, 367
- TA mode 411  
TO mode 411  
tensor 209, 282  
  of rank 2 209  
  of rank  $n$  283  
tensor properties of crystals 282  
tetrahedral point group 37  
thermal conductivity 298  
thermodynamic force 288  
thermoelectric effects 297  
thermoelectric power 298  
thermomagnetic effects 299  
time-evolution operator 253  
time reversal 252, 255, 358, 404  
time-reversal symmetry 262  
total angular momentum 131, 148  
totally symmetric representation 70  
trace of a matrix 416  
*trans*-dichloroethylene 50
- transform 5, 21  
transformation  
  of functions 63, 64  
  of operators 102  
transition metal complexes 117  
transition probability 104, 171, 388  
translation subgroup 316  
translational motion 156  
translational symmetry 307, 357, 391  
translations 27  
transposed matrix 61  
triplet state 141  
turn 225
- uniaxial groups 294  
unimodular 201  
unit cell 307–310, 325, 327  
unitary basis 55  
unitary matrix 61, 422  
unitary operator 252  
unitary representation 424  
upper cubic groups 301
- vector representations 81  
vibrational motion 156  
  degeneracy of 158  
vibrational quantum number 159  
vibronic coupling 104, 173  
vibronic interaction 173, 175  
Voigt notation 284, 286  
von Laue conditions 358
- wave vector 392  
Wigner–Seitz cell 309, 327  
Wyckoff position 321
- zero overlap approximation (ZOA) 112  
zone boundary 368