PAPPUS OF ALEXANDRIA AND THE MATHEMATICS OF LATE ANTIQUITY

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CHAPTER I

THE OUTSIDE WORLD

The only ancient mathematicians whose lives we know anything about are those who died in spectacular and gruesome circumstances: Archimedes, slain in 212 BC by Roman soldiers on the rampage, Syracuse having been taken after more than two years of siege; Hypatia, torn to bits by a lynching Christian mob in the streets of Alexandria in AD 415. Pappus, however, as well as Euclid, Apollonius or Ptolemy, must have had a quiet life, so he remains just a name to us. And he is one of the "famous" mathematicians, as distinct from the relatively substantial number of anonymous people whose lives and professions had something to do with mathematics, who studied it or had studied it, who used it cursorily for their job or taught it at various levels.

My aim in this chapter will be to gather some evidence about the mathematical practices of late antiquity *apart* from what we find in the books of the famous mathematicians – we will take a look at the outside world.[1] This will imply looking at a variety of sources which are not usually considered relevant for the history of mathematics. Why not? It seems somewhat perverse to dismiss as irrelevant evidence taken from astrology, land-surveying, architecture, and mechanics because all those people "weren't mathematicians really." Astrologers called themselves *mathematici* and land-surveyors *geometres* and their writings are replete with calculations, praises of mathematics and statements that mathematical knowledge was an essential part of their activities – yet such claims are discounted as mere rhetoric. Since those people

[1] Of course, given my limitations of time, space and expertise, this analysis will be partial – just scratching the surface of what could be termed "public understanding" of mathematics in antiquity.

only actually used mathematics in order to carry out very simple calculations or apply elementary geometrical theorems, they should not belong in the same story as Euclid or Archimedes or, for that matter, Pappus.

I will start from the assumption that it is irrelevant for my purpose whether the mathematics they employed was simple or complex, or even whether they actually made use of mathematics at all. I am interested above all in whether they saw their work and their type of knowledge as having to do with mathematics and whether they constructed their self-image accordingly, especially when it came to contrasting their form of knowledge with other forms of knowledge. If claiming affiliation with mathematics was a rhetorical move, then we should ask ourselves how that rhetoric came into place, and why the people who were deploying it thought that their rhetoric could work.

What was the nature of the relations between "those people" and people like Pappus? When we look at the *Collection* in more detail, we will see that the presence of other people who claimed affiliation with mathematics did have an impact on it. Apart from that, I have no ready answer. I do not suggest causal links; rather, I intend to provide some background to what Pappus was doing, in the hope that the reader will share my sense that when the *Collection* was being written there was a world out there where ideas about mathematics and mathematicians loomed larger than is usually thought – a world that should not be ignored.

1.1 Mathematics and the stars

That astrological texts could be fruitfully used by ancient historians was an idea first put forward by Lynn Thorndike, revived by Ramsay MacMullen and lately applied with interesting results by Tamsyn Barton.[2] Part of the operation consists in seeing how possible star-determined destinies were

[2] Thorndike (1913); MacMullen (1971); Barton (1994a). See also Cumont (1937), 87 ff.

valued on the desirability scale, what was said about social ascent, reversals of fortune, sexual mores, and so on. I will be looking at how mathematical professions fared as a possible destiny – which ones they were, with what other professions they were grouped and whether they were generally seen as a bad or good thing.

Our principal source will be a Sicilian writer, Julius Firmicus Maternus, whose main work, *Mathesis*, was written around AD 337 in Latin for Mavortius, a government official, with the declared aim of providing the non-expert with a translation of useful bits from Egyptian and Babylonian astrological texts.[3] For his theories and methods of interpretation he draws upon previous astrologers, especially Dorotheus Sidonius (AD 25–75) and Vettius Valens (second century AD), but his frequent references to contemporary official titles and posts testify that his evaluation of such aspects is updated.[4] I have chosen to focus on Firmicus Maternus because he devotes a great deal of attention to jobs and professions, but I will use evidence from other late fourth-century astrological treatises as well: Paul of Alexandria's *Elementa Apotelesmatica* (*Elements of astrology*) and Hephaestio's *Apotelesmatica*.

Firmicus Maternus is a good example of the care ancient astrologers devoted to their self-image: his work contains a section about the opponents of astrology and an outline of the profession (*Qualis vita et quale institutum esse debet mathematicis*) which includes a whole code of behaviour for practitioners.[5] As well as possessing generic moral and social virtues, they must be close to the divinity, be available to the public, turn down pecuniary rewards and avoid trouble by refusing to make predictions about the Emperor.[6] A bound-

[3] The significance of Firmicus being Sicilian is that he was fluent in Greek and felt particularly close to the Hellenic heritage: Archimedes is by him called "my fellow citizen from Syracuse," *Mathesis* II 148.23 (II refers to the second volume of the Teubner edition; I have modified the translations in parts). Mavortius is mentioned by Ammianus Marcellinus, *Res Gestae* 16.8.5; cf. Rhys Bram (1975) for epigraphical evidence about him.

[4] MacMullen (1971), 221.

[5] *Mathesis* 4.9 ff. and 85.5 ff. respectively.

[6] *Math.* 85.3–89.5. Cf. Barton (1994a).

ary is drawn between bad astrologers and good ones: those latter are distinguished by their ethical qualities, as well as by the extent of their knowledge. Incompetent astrologers bring the discipline into disrepute: Maternus contrasts the *prudentissimus mathematicus* with the inept one both in terms of expertise (as against "fallacious and heedless ... ignorance") and of honesty in pursuing one's art.[7] Astrology is equated to "philosophy and divine knowledge"[8] and, because of its certainty, he claims, it compares favourably with the disputes and inconsistencies of the very people who attack it – for instance, the enemies of astrology have never been able to agree among themselves about the nature of the gods, and even Plato and Aristotle are at variance about the essence of the soul.[9]

The astrologer's expertise includes mathematical knowledge – the question of *what* mathematical knowledge is problematic. Most of the actual horoscopes on papyrus or ostrakon that have come down to us (ranging from the first century BC to the fourth AD, with the majority dating to the second or third AD) are not very detailed, and suggest that they were cast and interpreted by people who did not necessarily know much about the geometrical complexities of planetary motions and may have had a minimum of calculating skills. Many practitioners of astrology must have carried out their job with the help of planetary tables and instruments, for which there is quite extensive evidence, both material and textual.[10] On the

[7] *Math.* 10.15–21 and 11.2–11.

[8] *Math.* 4.14.

[9] *Math.* 4.29–5.31. Cf. Ptolemy, *Apotelesmatica* 2.7.20–8.1 about bad astrologers: "[Most] deceive the vulgar, because they are reputed to foretell many things, even those that cannot naturally be known beforehand, while to the more thoughtful they have thereby given occasion to pass equally unfavourable judgement upon the natural subjects of prophecy. Nor is this deservedly done; it is the same with philosophy – we need not abolish it because there are evident rascals among those that pretend to it." Hephaestion refers to the "truth" which can be reached in astrological observations: e.g. *Apotelesmatica* 81.17; 145.17. See also Hübner (1990).

[10] On horoscopes see Neugebauer & Van Hoesen (1959); Baccani (1992); Jones (1994a). On tables see Neugebauer (1975), section V A 2 and the more recent bibliography in Jones (1994a). On astronomical instruments the most recent is Turner (1994).

other hand, we have several more detailed horoscopes, where the number of data considered was greater and a substantial amount of knowledge, and possibly observations, were involved. It would seem that there was a range of competences which may have corresponded to different customers or different demands: some astrologers would have commanded a higher price for highly personalized, complex birth-charts, some others would have put together more ordinary horoscopes. We can also imagine that the same astrologer could have catered differently for different clients.[11]

Firmicus Maternus confirms this picture: he contrasts calculations with purely visual observations,[12] and distinguishes between the task of working out the motions of the heavenly bodies, which is more difficult and therefore only practicable for the expert practitioner who wants to make a very accurate prediction, and the mere *interpretation* of the effects of the motions, which is easier.[13] Also, he and Paul and Hephaestio all repeatedly insist on accuracy; they frequently refer to, or report, calculations and often draw up tables or mention instruments that can help achieve better results (the astrolabe, the gnomon, the so-called *sfaera barbarica*, which was probably a type of armillary sphere).[14] The type of mathematics involved may not have been very complex, but that need not interest us: what matters is that mathematics is presented as a

[11] For a similar scenario among medical practitioners, see Pearcy (1984); Lloyd (1993); Barton (1994b).

[12] *Math.* 69.11.

[13] *Math.* 13.29–15.6.

[14] In his commentary on Ptolemy's *Syntaxis* 5.1 and 5.12, Pappus describes in detail an astrolabe (3.11 ff.) and a parallactic instrument (70.10 ff.), and often mentions ἀκριβεῖα (mostly of data). Paulus Alexandrinus, *Elementa Apotelesmatica* 33.11–12; 79.10–11, uses Ptolemy's *Handy Tables* as a guarantee of the accuracy of some data. Hephaestio also mentions accuracy often: e.g. *Ap.* 84.31; 85.13–16; 94.26–95.7 (he recommends repeated experience and practice in order to reach greater accuracy and come closer to the truth); 133.1–2; 197.24–25; 327.23, and *Epitomae* 41.3; 216.22–23. The importance of accurate calculations is also stressed by Manilius, *Astronomica* e.g. III.218 ff. (early first century AD). As for instruments, there are relevant passages at Firmicus Maternus, *Math.* II 315.8 (gnomon); 278.15, II 174.25, II 284.1, II 288.14, II 294.12–359.11 (*sfaera barbarica*); Paulus Alexandrinus, *El. Ap.* 80.13, 20 (astrolabe); Hephaestio, *Ap.* 51.10–12; 88.24 (both about an astrolabe); 234.24 (in connection with a ὑδροσκοπίον, perhaps a water-clock) and *Ep.* 2.12; 269.10.

kind of knowledge that grants better results and a higher degree of expertise.

Let us see now how mathematical professions are represented in the treatise. Astrologers figure both as *astrologi* and as *mathematici*. Some distinction between the two terms probably existed (perhaps *astrologi* indicated a lower level of competence), but they generally share the same associations, apart from a couple of cases in which *mathematici* (without *astrologi*) are grouped with long-haired philosophers, or with those "who discover and learn by themselves what has not been handed down to them on the authority of others."[15] *Astrologi* are involved in a range of other activities: most often with *haruspices* and dream interpreters and, in the same crowd, with doctors or *archiatri* (court doctors),[16] but at times also with teachers and geometers, orators, "ingenious inventors" and grammarians and poets.[17]

On the whole, being an astrologer is a rather positive prospect, comparable to that of being a doctor or an orator. Indeed, such associations reinforce our knowledge of the links between astrology and medicine, and underline not only the predictive nature of both disciplines, but also the public persona of astrologers, the fact that casting and, above all, interpreting birth-charts implied some capacity for rhetorical performance.[18] Out of a total of fourteen occurrences in the

[15] *Math.* 157.17: "sacerdotes magos archiatros mathematicos et per se invenientes atque discentes, quicquid illis non est alieno ⟨traditum⟩ magisterio."

[16] *Math.* 162.3; 165.25 (both with doctors); 185.26; 252.12; II 25.8; II 335.6; II 340.21 (this latter on their own). Cf. the same association in Paulus, *El. Ap.* 64.6 (who puts together diviners, dream interpreters, astrologers, augurs and those who partake in mysteries) and Hephaestio, *Ap.* 147.4–7 (where a certain configuration of Venus and Mercury produces souls apt to investigate things that are hidden, i.e. magicians, meteorologers, mechanicians, astrologers, wonder-workers, augurs, dream-interpreters, philosophers) and 171.9 (magicians, astrologers, people who speak in precepts and make predictions).

[17] *Math.* 159.1; 263.18 and II 24.12; 263.25; II 24.12, respectively.

[18] For these themes in astrology and medicine, cf. Barton (1994b). Hephaestio, *Ap.* 146.13–21 has astrological souls in a group of souls inclined to politics, love of fame and divination, and at *Ap.* 3.7 says that the ancient Egyptians combined astronomy and medicine in making predictions (hence, the ἰατρομαθηματικοί already mentioned by Ptolemy *Ap.* 16.7 ff.).

Mathesis, doctors are associated with predictive professions (*haruspices*, astrologers) four times,[19] as well as once with *herbarii*,[20] once with dyers, perfume-makers, musicians and athletes[21] and once with orators and lawyers. Those latter, however, are the doctors who are "ennobled by their talents in their professions."[22] The two occurrences of *archiatrus*, a title which apparently denoted the top of the medical profession, are both in the company of astrologers and *haruspices.*[23]

Architects are mentioned three times, of which twice in connection with sculptors and builders;[24] mechanicians three times, of which once with sculptors, poets and musicians.[25] Several categories of technicians are present (*artifex, ex-inventor, fabricator, inventor, organarius*), but the differences between them are unclear. The destinies associated with them are as a rule quite favourable: some craftsmen will make a living out of their art and learn by themselves what is not handed down to them; others, more favoured by the stars, will be friends of kings; some public builders will get the greatest dignity from their job.[26]

Geometers are side by side with philosophers, teachers of grammar, star-gazers, experts in *sacrae litterae*, orators and lawyers, or with astrologers and other diviners or, again, with scholarly grammarians, orators and teachers.[27] Some people's horoscopes indicate them as *tabularii* (accountants), tax experts[28] or discoverers: of calculations and instruments; of calculation, music, signs (*notae*) and difficult letters; together with orators, grammarians, doctors and musicians or with

[19] *Math.* 162.3; 165.25; 157.17; 263.18.
[20] *Math.* 168.8. Hephaestio, *Ap.* 170.23–29 has doctors, smiths, architects and measurers together.
[21] *Math.* 220.27.
[22] *Math.* II 338.11: "quos professionis suae nobilitet ingenium."
[23] *Math.* 157.17; 263.18. On *archiatri* see Nutton (1977).
[24] *Math.* II 349.1; II 331.21; II 322.18 (this latter has naval architects).
[25] *Math.* II 23.24 (with sculptors, etc.); 186.23; II 341.6–7 ("mechanicus qui instrumenta bellis faciat necessaria").
[26] *Math.* 162.15; II 344.12 and II 72.8, respectively.
[27] *Math.* 155.24; 159.1; 164.3, respectively. Hephaestio, *Ap.* 149.28–30 has philologists, geometers, mathematicians and unspecified "wise people" together.
[28] *Math.* II 307.4; II 75.15, respectively.

orators and teachers.[29] Of some people it is forecast that they will make a living out of reckoning and calculation,[30] while those who, more vaguely, "excel in numbers" are grouped together with public teachers, orators, grammarians and jurists.[31]

On the whole, a remarkable number of people had some involvement with mathematics written in the stars for them, and such a career would have constituted quite a desirable destiny. Also, jobs to do with numbers and calculations, often carried out for the general public, were usually included among the educated professions. In sum, activities associated with mathematics were highly visible, had a rather positive public profile and were generally linked to intellectual practices.[32]

Firmicus Maternus himself shows that within his discipline expertise and good, accurate practice were explicitly linked to the use of mathematics. Moreover (and we will see that astrology is not the only case) boundaries were drawn between good and bad practice on the basis both of ethical qualities and of the extent of a person's knowledge.

1.2 Mathematics in the workshop

Land-surveyors, architects, mechanicians, "technicians" of various sorts were all pretty visible in the scenario of late antiquity, as we noticed in looking at Firmicus' treatise and will continue to see in the section on laws. Sharp distinctions be-

[29] *Math.* 172.6; 161.3; 233.17; 170.21, respectively. There also people expert in calculation who interpret the course of the stars, 211.4 and 233.18. Hephaestio, *Ap.* 170.2–3 groups together lawyers, people in charge of offices – possibly of account offices (λογιστηρίων προϊσταμένους) – teachers and leaders of crowds.

[30] *Math.* 158.12 and 183.25: "computus aut calculus." Cf. also souls described as ἐπιλογιστικάς, together with, among others, souls which are high-minded and apt for science (μεγαλόφρονας and ἐπιστημονικάς), in Hephaestio, *Ap.* 146.24–26.

[31] *Math.* 182.2.

[32] Gregory of Nyssa (second half of fourth century AD; *Epistula* 17.24), wanting to make the point that the virtues of the master become the virtues of the pupils, declares that a smith will not become a weaver, nor will a weaver become a rhetorician or a geometer. Note that, although the two artisanal categories of smith and weaver are kept well separated, rhetor and geometer are taken to be somewhat alike.

tween one category and the other are hard to draw:[33] the architects for whom we have evidence include people who were credited with ship-building and supervision of war-machines – an architect could probably double as an engineer whenever need required. Also, some terms are ambiguous: the Latin *geometres* or *geometra*, whose primary meaning is "geometer," is often translated as "land-surveyor" unproblematically. A more balanced view would be to accept that the term retained some ambiguity, thus reflecting the combination of skills, "practical" and "theoretical," that the practitioners themselves often possessed. It is sometimes clear from the context that a land-surveyor is meant, sometimes a geometer; at other times pinpointing one meaning in preference to the other is simply not possible.[34]

Generally speaking, the social status of technicians in late antiquity seems to have been upwardly mobile.[35] Architects, for instance, received a great deal of recognition. Symmachus, who was prefect of Rome between AD 384 and 391, had to investigate a case involving two people who had senatorial rank and had been trusted with public money in order to build a basilica and a bridge: Auxentius and Cyriades, the latter a "comes et mechanicae professor."[36] They were suspected of embezzling funds, and pretty soon started to accuse each other. The equal status of the two made a separate inquiry necessary, and Symmachus decided to charge the master craftsmen ("fabrilis artis magistros") with an assessment of the situation as far as the works were concerned. Before the results were produced, Auxentius fled town, and was replaced by another senator, the notary Aphrodisius.

[33] Downey (1946–48), 109, argues that μηχανικός denotes someone superior to ἀρχιτέκτων. Cf. also Coulton (1977); Bulmer-Thomas (1981); Mansuelli (1985); Donderer (1996); Anderson (1997).

[34] Cf. Dilke (1971), 44; Schindel (1992), 377 and 377n18 for analogous views.

[35] Cf. Clarke (1971), 113; Cracco Ruggini (1971); Williams (1985), ch. 2; Bowman (1992).

[36] Symmachus, *Relationes* 25–26; the case went on from AD 382 to 387. Auxentius has been identified with the person celebrated in *IGR* 3.887, an inscription which commemorates a bridge built on the river Sarus in Cilicia, see Grégoire (1927–8); Vera (1981). Donderer (1996) has further examples of third- and fourth-century architects with quite a high social status based in e.g. Antioch and Rome.

The people who had laid the foundations for the bridge were then interrogated, and it turned out that a part of the bridge, left unfinished (or defective, *incohata*), had been destroyed by the violence of the river, with the cost for repairs assessed by the *artifices* (a further step down the hierarchy at the building site) at twenty solids. Cyriades assured the authorities that repairs could be easily carried out, but further investigation revealed that in part of the bridge the stones were not fitting together. Cyriades then claimed that the job had initially been done properly according to his instructions ("consilio suo et ratione artis"), but that Auxentius had later deliberately spoiled the work, filling the gaps with straw and weeds, in order to make him (Cyriades) look incompetent. A diver ("urinandi artifex"), however, testified that the use of straw and weeds was not aimed at discrediting Cyriades, but at ensuring stability – it was a matter of using a different technique. Finally, the witness whose testimony was at variance with the others (it is not clear who the poor fellow was – maybe the diver) was subjected to torture and confessed that Cyriades had threatened him, but he was not believed because it was thought that the confession was made simply to put an end to the torture. The *comes et professor mechanicae*, for his part, kept maintaining that the bridge could be repaired, and that it had been Auxentius who embezzled the money.

We do not know whether or not the case was solved: our evidence from Symmachus stops here, and he turns to other matters. What emerges from this episode is first of all the obvious fact that architects could have senatorial rank and that there was such a thing as a *professor* of mechanics, a title which seems to denote some official recognition of expertise, and possibly of teaching capacities. Moreover, we are given glimpses into the several ranks at the building site: the architects first of all, trusted with administration of money (and responsibility for it); then, probably next in status, the master craftsmen, who are trusted with the important task of assessing the works when the two chief people start accusing each other; finally, the *artifices* themselves, who are again able to function in a semi-official capacity by giving a financial esti-

mate, and the diver, someone with a degree of specialization or skill, who is interrogated simply as witness (not as expert witness) – given their low social status, such people could be tortured if their testimony sounded suspicious.

Another edifying story is told by a contemporary of Symmachus, Augustine. A pupil of his in Carthage, Alypius, had been unjustly apprehended by an angry mob with the accusation of burglary. An architect, who was the top man in charge of public buildings, happened to pass by as the student was being taken away and recognized him from having met at the house of a common acquaintance – a senator. Having listened to Alypius' version of the story, the architect "ordered all the people there, who were in an uproar and making threatening shouts, to come along with him," and led them to the house of the real offender, who was brought to justice. What I find notable in this episode is not so much that the architect was a frequent visitor to a senator's house, as that his authority with the man in the street was such that he could use it to persuade an angry crowd to do as he said.[37]

Another late witness is Cassiodorus, who, in letters dating from the sixth century, charged an architect with the task of repairing some communal baths, which involved the administration of public money, and on another occasion requested another architect for public work in Rome. In this second case, Cassiodorus reminded his addressee of the seven wonders of the ancient world and expressed the wish that the architect chosen for the job "apply himself to books" in order to stand comparison with such a glorious past.[38] We can only guess what the books in question may have been: in another letter Cassiodorus exhorts a steward of the imperial retinue to take inspiration from Euclid and Archimedes for "beautiful shapes" with which to adorn the imperial palace.[39]

Cassiodorus provides evidence for *mechanici* as well: he mentions a mechanician who had been paired with a water-

[37] *Confessiones* VI.9. Architects are mentioned as public officers in the *Codex Justiniani* 12.19.12.1 (a law of the emperor Anastasius, AD 491–518).
[38] *Variae* II.39 and VII.15 respectively.
[39] *Variae* VII.5.

diviner "so that the waves discovered by one can be lifted by the other, and what nature does not allow to go upwards can rise artificially."[40] In another letter written on behalf of King Theodoric (*ca.* 507–511), he invites two *spectabiles viri* between whom a boundary dispute has arisen to entrust their case to the capable hands of a land-surveyor, who will solve it "by means of geometrical forms and land-surveying knowledge" rather than with weapons, and will enclose the contested land as diligently as the speech is enclosed by each letter. Cassiodorus continues with a micro-history of land-surveying: how it originated with the Chaldeans, how it was taken up by the Egyptians and eventually by Augustus, who carried out an extensive programme of land-division. The "metrical author" Hero is then mentioned as the person who "made [land-surveying] into a written doctrine, so that the person involved in this study could learn by reading what he would have fully to demonstrate to the eyes." The discipline enjoys a great reputation indeed when compared to other branches of knowledge: arithmetic, Cassiodorus says, is taught to empty schoolrooms; geometry "insofar as it discusses heavenly things" is only known to scholars; astronomy and music are learnt just for their own sake, but the *agrimensor* "shows what he says, and proves what he has learnt."[41]

More light on the public persona of land-surveyors is cast by the texts collected in the so-called *Corpus Agrimensorum Romanorum*, which includes treatises ranging from the first century BC to the early fifth century AD at least, but chiefly from the second century AD.[42] Unlike most ancient scientific practices, our evidence for land-surveying is not limited to the

[40] *Variae* III.53, esp. 6; quoted in Oleson (1984), 34.

[41] *Variae* III.52.

[42] There are two main editions of the *Corpus* so far: one by K. Thulin includes Frontinus, Agennius Urbicus, Hyginus, Siculus Flaccus and Hyginus Gromaticus. The other, by F. Blume, K. Lachmann and A. Rudorff is complete but arguably less accurate. There is also an edition with French translation of Siculus Flaccus' *De condicionibus agrorum*, and Brian Campbell (see (1996)) is preparing an English translation of the *Corpus*. See also Gabba (1984); Hinrichs (1992); Toneatto (1992). References are to Thulin's edition unless otherwise specified.

texts themselves. We have both further testimonies in legal, historical and literary works and abundant material evidence: inscriptions recording official decisions about, say, the territorial boundaries of two neighbouring cities; boundary stones; maps; surveying instruments; tombstones of land-surveyors. Traces of land-division detectable by means of, for instance, aerial photography are numerous, especially in Italy, France, the former Yugoslavia and North Africa.[43]

Even though late antiquity hardly saw the centuriation of new territories, everyday general administration involved settling disputes about boundaries, as we have seen in the case of Cassiodorus, or division of heredity.[44] We have entries in the collections of laws known as *Codex Theodosii* and *Digesta* on the "Administration of boundaries": land-surveyors were among the main arbitrators in such cases.[45] If a dispute arose between two parties and one of the two brought in a land-surveyor, the fee had to be shared by both.[46] A *mensor* was also instrumental in cases where a flood had obliterated the borders between properties: in this case he worked for the governor of the province, i.e. in an official "public" capacity. If necessary, the governor would inspect the situation personally.[47] Another entry in another law collection, the *Codex Justiniani*, runs thus: "The chief (*primicerius*) of the land surveyors after completing two years [of service] is assigned the lowest office of *agens in rebus*."[48] And again, *mensores* and *metatores* are mentioned as part of the staff of several governmental departments – their duties included dividing prop-

[43] See e.g. Dilke (1971); Hinrichs (1974); Chouquer & Favory (1992).

[44] Centuriation, used loosely, means the division of a territory in squares or, less frequently, rectangles of a given size. A late example of land (re)division was with the emperor Julian, who, according to Eunapius, *Vitae sophistarum* 493, measured the land with the aim of relieving the Greeks from part of their tribute.

[45] *De finium regundorum*, in *Codex Theodosii* 2.26, with laws ranging from AD 330 to 392, and *Digesta* 10.1.

[46] *Dig.* 10.1.4.1.

[47] *Dig.* 10.1.8: "si ita res exigit, oculisque suis subiectis locis."

[48] *Cod. Just.* 12.27.1 (AD 405, addressed to the *magister officiorum*): "Primicerius mensorum biennio expleto agentis in rebus ultimi militiam sortiatur." For the use of military terms (i.e. *militia*) for civilian purposes, see Kelly (1998), 168, who also gives further bibliography on bureaucracy in the late Roman Empire.

erty for billeting purposes.[49] This picture seems to be confirmed by the late fourth-century *Notitia dignitatum*, where land-surveyors figure on the staff of both military and civilian departments.[50]

The *Digesta* also contain a heading "Si mensor falsum modum dixerit" ("If a land-surveyor declares the wrong measure") which establishes various sanctions against land-surveyors who do not do their job properly, and considers the case where instruments are used. The jurists (Ulpian and Paul) mention, as cases parallel to those of surveyors, the professional circumstances of other practitioners, namely architects and accountants (*tabularii*), against whom it was also possible to take legal action on the basis of fraud (*dolo malo*).[51] The reason why the type of legal action appropriate to the case would have been for fraud, not for a job badly done, is because the relation between a *mensor* and a customer was not a commercial one: it did not fall under the heading of *locatio conductio*, which included transactions for manual jobs or hired labour, but was defined more in terms of a favour, *opera beneficii*. Thus, the emolument of a *mensor* or of anyone who performed a favour rather than a paid job was called *honorarium* rather than *merces*, i.e. it was intended to be more a thank-you gift than remuneration. Reality was, as it happens, rather different: the entry in the *Digesta* comments that, although officially land-surveyors should not be paid, in fact

[49] The laws in question are *Cod. Just.* 12.40.1 (mentions "mensores nostri"); 12.40.3; 12.40.5; 12.40.9 (the latter two mention *metatores*) and 12.59.10, where they are listed as part of the staff of the *scrinium sacrorum libellorum* along with pedagogists, *cellarii* and *lampadarii*. The laws date from the fourth to the fifth century. *Metatores* seems to have been the specific term for military *mensores*: in Vegetius (*ca* AD 400), *Epitoma rei militaris* 2.7, the *metatores* go in front of the rest of the army and choose the best site for encampment, while the *mensores* and *agrimensores* divide up the camp once the site has been chosen, *ibid.* 3.8.

[50] *Notitia dignitatum Orientis* 7.66 (*mensores* in the office of the *magister militum*) and *Or.* 11.12 (with the *magister officiorum*, they share an entry with *lampadarii*).

[51] *Dig.* 11.6.7 (Ulpianus): "This action is also given against a surveyor who uses mechanical instruments, if he deceives ... By analogy, the action should also be given against an architect who deceives ... I think actions should also be given against accountants who deceive in their calculations." ("Et si mensor machinarius fefellerit, haec actio dabitur. ... Hoc exemplo etiam adversus architectum actio dari debet qui fefellit ... Ego etiam adversus tabularium puto actiones dandas, qui in computatione fefellit.") On these issues see Visky (1959).

they are, but this still does not make them liable to the conditions that hold for hired labour. The legal nature of the land-surveyor's job thus seems to indicate that his task was seen, at least in principle, as quite prestigious.

Land-surveyors as represented by the authors of the *Corpus* come across as remarkably self-aware – there are several references to "professio nostra"[52] – and as knowledgeable about philosophy, geography and mathematics. Hyginus Gromaticus (second century AD) has a rare mention of Archimedes' *Arenarius*;[53] Balbus, an army-trained land-surveyor who fought with Trajan or perhaps with Domitian in the Dacian wars (or perhaps against the Germanic tribes),[54] tells his addressee Celsus that it would seem disgraceful to him if, when asked how many types of angle there are, he could only answer: "many", since it is part of their job to have more than just trivial geometrical knowledge.[55]

Marcus Junius Nipsus' treatise (second century AD, maybe later)[56] contains a number of problems which involve finding a certain element of a geometrical construction or object when some of its other elements are given. The problems are set in a specific, rather than a general form, so that the elements which are given are expressed as numbers, the procedure is by calculation of numbers, and the element which is sought will also be expressed as a specific number. For example,

> [i]f a right-angled triangle is given, and the cathetus and the basis are given and are together 23 feet, and the area of this triangle is 60 feet and the hypotenuse 17 feet, [suppose it is required] to state the cathetus and the basis separately. We would find out in this way. I multiply the number of the hypotenuse by itself. It makes 289. From this I take away four areas, which makes 240. There is a rest of 49. Of this I always take the side [i.e. the

[52] E.g. Siculus Flaccus, who, according to Dilke (1971), 44, dates from the third century AD, *De condicionibus agrorum* 98.9; Balbus, *Ad Celsum expositio et ratio omnium formarum* 93.14 (Blume).

[53] Hyginus Gromaticus, *De limitibus constituendis* 148.4–7.

[54] For the uncertainties of interpretation, see Dilke (1971), 42.

[55] Balbus, *Ad Celsum* 93.11–15 (Blume): "foedum enim mihi videbatur, si genera angulorum quot sint interrogatus responderem 'multa': ideoque rerum ad professionem nostram pertinentium, in quantum potui occupatus, species qualitates condiciones modos et numeros excussi."

[56] For the dating, see Dilke (1971), 60.

square root]. It is 7. This I always add to the two together, that is, to 23. It makes 30 feet. Of this I always take the half. It is 15. This is the basis of the triangle in question. From the two together, that is, from the 23, I take away 15 feet. The rest is 8 feet. It will be the cathetus.[57]

Indeed, the mathematics of the surveyors often applies numbers to geometrical problems and makes extensive use of instruments and of "material" ways of defining concepts. For instance, Balbus describes some geometrical objects thus: "There are three kinds of lines, right, circular, curving ... A curving line is multi-shaped, like the line of ploughed fields or ridges or rivers."[58]

The land-surveyors often compare and contrast their kind of knowledge with that of other people: for instance, Agennius Urbicus (originally thought to be from the second century AD, now dated to the fourth or fifth)[59] says that the Stoics assert that the world is one, yet, if one wants to know what the world is like, and how big ("*qualis quantus*"), one needs geometrical knowledge.[60] About geometry in general, again Agennius Urbicus says:

Thus, of all the honourable arts, which are carried out naturally or proceed in imitation of nature, geometry takes the skill of reasoning as its field. It is

[57] Marcus Iunius Nipsus, *Podismus* 298–299 (Blume): "Si datum fuerit trigonum hortogonium, et dati fuerint cathetus et basis in se ped. XXIII, embadum huius trigoni ped. LX et hypotenusa ped. XVII dicere cathetum et basim separatim. s.q. facio hypotenusae numerum in se. fit CCLXXXVIIII. hinc tollo quattuor embada, quod fit CCXL. reliquum XLVIIII. huius semper sumo latus. fit VII. hoc semper adicio ad duas iunctas, id est ad XXIII. fiunt pedes XXX. huius semper sumo dimidiam. fit XV. erit basis eiusdem trigoni. de duabus iunctis, id est de XXIII, tollo ped. XV. reliqui ped. VIII. erit cathetus." The procedure is reminiscent of Hero's *Metrica* and, to a lesser extent, of Pappus himself (as we shall see in chapter five). Links with Hero have been suggested, and seem indeed quite likely at least for some texts in the *Corpus*, cf. Clavel-Lévêque (1992); Folkerts (1992); Guillaumin (1992); Høyrup (1996c).

[58] E.g. Balbus, 99.3–7 (Blume): "Linearum genera sunt trea, rectum, circum ferens, flexuosum ... flexuosa linea est multiformis, velut arvorum aut iugorum aut fluminum." Cf. Guillaumin (1988); Folkerts (1992).

[59] Campbell (1996), quoting an unpublished PhD dissertation by Mauro de Nardis, *The Writings of the Roman Land Surveyors: Technical and Legal Aspects*, University College London, 1994 (*non vidi* this latter).

[60] Agennius Urbicus, *De controversiis agrorum* 22.7–8. See also Santini (1990).

hard at the beginning and difficult of access, delightful in its regularity, full of beauty, unsurpassable in its effect. For with its clear processes of reasoning it illumines the field of rational thinking, so that it may be understood that geometry belongs to the arts or that the arts are from geometry.[61]

And again:

In making a judgment the land surveyor must behave like a good and just man, must not be moved by any ambition or meanness, must preserve his reputation both by his art and by his conduct. . . . [F]or some err because of inexperience, some because of impudence: indeed this whole business of judging requires an extraordinary man and an extraordinary practitioner.[62]

Indeed, land surveyors concur in presenting themselves not just as experts in measuring, but also as people able to resolve controversies and to bring unorderly, unmeasured space to order. Frontinus (first century AD and the object of two later commentaries) even talks about the operation of centuriating a piece of land as an expression or a restoration of its truth (*veritas*) – a truth evidently equivalent to the land becoming geometrized, as well as being brought under Roman control.[63]

The presence of land surveyors in military contexts is paralleled by testimonies about other technical professions: Vegetius includes *librarii* for the keeping of accounts and *artifices* and *exercitati homines* for the responsibility for war engines in his catalogue of members of an army.[64] He also describes two methods to measure the walls of a city which an army is

[61] *op. cit.* 25.15–27: "Omnium igitur honestarum artium, quae sive naturaliter aguntur sive a⟨d⟩ naturae imitationem proferuntur, materiam optinet rationis artificium geometria, principio ardua ac difficilis incessu, delectabilis ordine, plena prestantiae, effectu insuperabilis. manifestis enim rationi[bu]s executionibus declarat ⟨rat⟩ionalium materiam, ita ut geometria⟨m⟩ ine[o]sse artibus aut arte⟨s⟩ ex geometria esse intelligat⟨ur⟩."

[62] *op. cit.* 50.9–15: "in iudicando autem mensor[em] bonum virum et iustum agere debet neque ulla ambitione aut sordibus moveri, servare opinionem et arti et moribus. . . . quidam enim per imperitiam quidam per inpudentiam peccant: totum autem hoc iudicandi officium et hominem et artificem exigit egregium."

[63] Frontinus, *De arte mensoria* 15.7; 16.4; see Hinrichs (1992) for the most recent text (and German translation) of this passage.

[64] Vegetius, *Epitoma rei militaris* 2.7 and 4.22 respectively.

[65] *op. cit.* 4.30.

aiming to besiege: one of them involves measuring shadows. The procedure is not described in detail; Vegetius, however, claims that nobody will doubt its efficacy.[65] The leader of the army is also expected to work out how many soldiers will fit into a certain space, and how much space one should keep between rows when marching in formation without breaking ranks.[66] In fact, numeracy is one of the characteristics which should direct choice when recruiting troops. Many legions demand literate soldiers; expertise in written signs and acquaintance with reckoning and calculating are indeed to be sought, if only because soldiers take it in turns to keep records of things like days of leave awarded to their colleagues, and good numeracy helps avoid injustice and the giving of unwarranted leave with impunity.[67]

Another "technical" sector where mathematics may have played a role is public administration. The new fiscal system introduced by Diocletian, possibly around AD 297, involved periodic surveying and counting of people and resources, down to the number of cattle or of trees in a field, all in order to work out the corresponding tax liability. There is evidence that this new system was indeed enforced, at least in some parts of the empire.[68] Moreover, around AD 325–326, after the reunification of the empire under Constantine, a reform of the financial administration seems to have taken place. The prefects of the regional prefectures were made responsible for levying and distributing the *annona*, and new titles appear, such as *comites rei privatae* and *comites sacrae largitionis*. Local functionaries existed at the same time as central administrators bearing the same title.[69] An analysis of the extant evidence about the *comites rei privatae* and *comites sacrae largitionis* reveals, unsurprisingly, that the majority came from a rather high social background[70] and that several of

[66] *op. cit.* 3.15.
[67] *op. cit.* 2.19: "Praeter corporis robur, notarum vel computandi artem in tironibus eligendam … notarum peritia, calculandi computandique usus."
[68] Jones (1964), I 62 f.; Bowman (1980); Williams (1985), 119 ff.; Corcoran (1996).
[69] Delmaire (1989a), 11 ff.
[70] With a couple of exceptions, see Delmaire (1989a) and (1989b), especially 97–101 and 105–111.

them had received a substantial education in rhetoric. For some there is evidence of philosophical studies.[71]

An increase in the number of people involved with the administration of finances, therefore ultimately also with things like accounts, is established. What is not clear is how important, if at all, was mathematical expertise in these contexts. The question can be considered as part of the more general matter of professionalism and bureaucracy in the late Roman Empire.[72] At least some of the public offices would seem to have required certain skills: for instance, there were departments who looked after legal cases on behalf of the Emperor, and their employees had to be trained in the law or to rely on someone who was. The departments responsible for the maintenance and construction of public buildings, including aqueducts, and the board in charge of land-surveying which was still operative in the sixth century, must have included architects and surveyors among its staff. On the face of it, it would seem that specialized skills were desirable assets that had the potential to open doors. Current thinking about late Roman bureaucracy, however, inclines to the view that access to the administration and promotion within it were to a great extent a mere matter of connections, patronage and money. Top officers tended to be educated, but that signifies more that they belonged to a certain social group than that education was indispensable or even just useful for their job.[73] In other words, according to the prevalent view, it was not necessarily the case – in fact, it was hardly ever the case – that good mathematical skills facilitated access to top positions on the financial board.

[71] For instance, Fortunatianus, *comes rei privatae* in the East from AD 370 to 377, who is defined philosopher by Libanius, *Epistulae* 694; Iovinus, who was *comes sacrae largitionis* in the East *ca* 364–365, and to whom Libanius passed on a letter by Iamblichus, see Libanius, *Epist.* 577 (AD 357); Longinianus, *comes sacrae largitionis* in the West in 399, who may have been a NeoPlatonist, see his letter to Augustine in this latter's *Epistulae* 234.

[72] Among the many contributions, see MacMullen (1964); Brunt (1975) and (1983); Bowman (1976); Pedersen (1976); Saller (1980); Kelly (1998).

[73] Pedersen (1976); Brown (1978), (1980) and (1992); Kaster (1988); Bowersock (1990).

Given that the financial board *had* to carry out some function, however, and even allowing for malfunctioning, *someone* in that department must have had some calculating skills – if not the top people, then someone employed or owned by them. The picture is thus that of a group of leading officers, non-specialized but with good connections and usually a good general education, who may have changed posts quite easily in order to move up the ladder or to another department or geographical area, plus a "hard core" of more or less skilled employees, who would stay with a department through all sorts of changes, including imperial reversals of fortune. At present, very little is known about this "hard core" of lower bureaucrats: a ground-breaking study about *notarii* and *exceptores* has showed that they were quite literate, as well as being particularly able at their specific task. There is also some evidence that some of these categories at least, especially in late antiquity, exhibited some awareness of themselves as a group – that they manifested some kind of professional identity – but further study is definitely required.[74]

The lower officers in charge of dealing with the mathematics required for administrative or commercial tasks went under several names: *calculatores*, *tabularii*, *numerarii*, λογίσται, καθολικοί.[75] Those terms are not well-defined: they could denote secretaries or clerks or administrators with no particular responsibility for account-keeping or finances; then again they may have been specifically in charge of calculations and things to do with numbers.

An early source, Martial, describes a *calculator* and a stenographer (*notarius velox*) being surrounded by many pupils, and an inscription from *ca.* AD 144 commemorates a thirteen-

[74] Teitler (1985); Kelly (1998).

[75] The examples where λογίσται are mentioned are often difficult to interpret, because the duties involved are not clear, and in small local contexts could be quite general: see *P. Oxy.* 84.2 (AD 316), a receipt from a guild of metal workers for payment by a λογιστής of a certain amount of public money for some works done for the city. Johannes Lydus (sixth century) mentions that the καθολικοί were in charge of public accounts, *De magistratibus romanis* 3.7. Inscription 657 (late third century AD) in Reynolds & Ward Perkins records a "libra[r]ius notarius [rat]iocinator n(u)m[er]arius."